Journal of Uncertain Systems
Vol.6, No.2, pp.86-88, 2012
Online at: www.jus.org.uk

Efficient Geophysical Technique of Vertical Line Elements as a Natural Consequence of General Constraints Techniques

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Received 19 August 2011; Revised 15 April 2012

Abstract

One of the main objectives of geophysics is to find how density ρ and other physical characteristics depend on a 3-D location (x, y, z). In general, in numerical methods, a way to find the dependence $\rho(x, y, z)$ is to discretize the space, and to consider, as unknown, e.g., values $\rho(x, y, z)$ on a 3-D rectangular grid. In this case, the desired density distribution is represented as a combination of point-wise density distributions. In geophysics, it turns out that a more efficient way to find the desired distribution is to represent it as a combination of thin vertical line elements that start at some depth and go indefinitely down. In this paper, we show that the empirical success of such vertical line element techniques can be naturally explained if we recall that, in addition to the equations which relate the observations and the unknown density, we also take into account geophysics-motivated constraints.

Keywords: inverse problem, geophysics, vertical line elements, constraints

1 Formulation of the Problem: Need to Explain Empirical Success of Vertical Line Element Techniques

Geophysics: main objective. One of the main objectives of geophysics is to find out what is happening inside the Earth: whether there is oil and other minerals worth prospecting, whether there is a risk of earthquake, etc. Specifically, we are interested in finding out how different material characteristics c such as density, conductivity, speed of sound, etc., depend on the 3-D spatial location, i.e.,

- on the coordinates x and y that describe the 2-D (surface) spatial location, and
- on the depth z.

Need for indirect measurements. In principle, to find the value $c(x, y, z)$ of a physical characteristic c at location (x, y) and depth z, we can drill a borehole and directly measure the corresponding value. Sometimes, we need to do it. However, boreholes are very expensive, so it is desirable to find the values $c(x, y, z)$ without incurring these expenses.

To find these values, we measure auxiliary quantities which are related to the desired quantity $c(x, y, z)$ – and then determine the desired values $c(x, y, z)$ based on the results of these indirect measurements. For example, one way to indirectly measure the density $\rho(x, y, z)$ at different spatial locations is to measure the gravitational field $\vec{g}(x, y, z)$ at different spatial locations (x, y) and at different heights z. Gravity is caused by the mass, so the measured values $g(x, y, z)$ provide indirect information about the desired density values.

It is known how the observed gravity values are related to the desired density distribution

$$\vec{g}(\vec{r}\,) = \text{const} \cdot \int\int\int \frac{\rho(\vec{r}\,')}{\|\vec{r} - \vec{r}\,'\|^3} \cdot (\vec{r}\,' - \vec{r}),$$

where we denoted $\vec{r} = (x, y, z)$ and $\vec{r}\,' = (x', y', z')$. Thus, once we know the observed values $\vec{g}(\vec{r}\,)$, we can solve the corresponding system of linear equations and find the desired values $\rho(\vec{r}\,')$; see, e.g., [4].

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Traditional approach. The traditional approach to solving such systems of equations is to *discretize* the corresponding field $\rho(\vec{r}\,')$, i.e., to consider finitely many values $\rho(\vec{r}\,')$ corresponding to finitely many 3-D locations $\vec{r}\,'$. These may be locations on a regular rectangular grid, these may be irregularly place adaptive locations – as in finite elements method. A general distribution is thus represented as a combination of such point sources.

New approach: vertical line elements. In [3], it was shown that a new method is more effective, in which we represent the original distribution not as a combination of points sources, but as a combination of *vertical line elements*: narrow vertical cylinders of constant density that start at a certain depth and go down indefinitely. This approach has shown to be very successful, see, e.g., [1, 2]. A similar approach was shown to be very successful for magnetic measurements [3].

Natural question. A natural question is: why is this heuristic approach successful? Can we find a theoretical explanation for this success?

2 Our Explanation

Need to take constraints into account. Not all mathematical solutions of the above system of equations are geophysically meaningful. For a solution to be geophysically meaningful, it has to satisfy certain geophysics-motivated constraints.

One of the main constraints: density should increase with depth. One of the most important such constraints is that, in general, the density should increase with depth. This is not a universal rule: e.g., cavities or oil deposits are exactly the cases when lower-density areas are located below higher-density ones, but overall, this constraint must be satisfied – and we should only produce an answer with such a situation if this situation is dictated by observations.

How to formally describe such a constraint. Let us consider the simplest case when we use values on a grid. In this case, we have values $x_1, \ldots, x_n$, $y_1, \ldots, y_m$, $z_1, \ldots, z_p$, and we try to determine the values of the unknowns $\rho_{i,j,k} \stackrel{\text{def}}{=} \rho(x_i, y_j, z_k)$ corresponding to all possible triples (i, j, k) for which $i = 1, \ldots, n$, $j = 1, \ldots, m$, and $k = 1, \ldots, p$.

In these terms, to describe the general constraint that the density increases with depth, it is sufficient to require that when we go one step deeper, the density increases, i.e., that the following inequality

$$\rho_{i,j,k} \geq \rho_{i,j,k-1}$$

is satisfied for all i, j, and k.

Difficulty of using this constraint. We want to solve the above system of equations under these constraints. In general, the complexity of taking a constraint into account depends on how many unknowns a constraint has

- constraints that have only one unknown are easier to handle,
- constraints that use two unknown are more difficult to handle,
- constraints that use three or more unknowns are even more difficult to handle,
- etc.

Each of the above constraints contains two unknowns: $\rho_{i,j,k}$ and $\rho_{i,j,k-1}$. Thus, to simplify the handling of these constraints, it is desirable to reformulate them in such a way that each constraint includes only one unknown.

Natural reduction to easier-to-handle constraints. A natural way to reduce a constraint $x \geq y$ to a constraint with a single unknown is to represent the original constraint in the equivalent form $z \geq 0$, where $z \stackrel{\text{def}}{=} x - y$.

Thus, instead of the original unknowns $\rho_{i,j,k}$, we should consider new unknowns $\Delta\rho_{i,j,k} \stackrel{\text{def}}{=} \rho_{i,j,k} - \rho_{i,j,k-1}$. In terms of these new unknowns, the original unknowns have the form

$$\rho_{i,j,k} = \sum_{\ell \leq k} \Delta\rho_{i,j,\ell}.$$

This natural reduction leads exactly to vertical line elements technique. The meaning of each original unknown $\rho_{i,j,k}$ can be described if we consider the situation where this unknown is different from 0 and all other unknowns are equal to 0. In this case, we have density different from 0 at a single 3-D point (i, j, k).

To find out the meaning of a new unknown $\Delta\rho_{i,j,k}$, let us similarly consider a situation where this unknown is different from 0 and all other new unknowns are equal to 0. In this case, according to the above formula that describe the density in terms of the new unknowns, the density is different from 0 only at 3-D points (i, j, k), $(i, j, k + 1)$, $(i, j, k + 2)$, etc. In all these points, the density has the exact same value – the value $\Delta\rho_{i,j,k}$. Thus, we have a thin vertical element of constant density – i.e., exactly a vertical line element.

Acknowledgments

The authors are greatly thankful to Mark Baker and Diane Doser for valuable discussions.

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