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Credibilistic Bi-Matrix Game

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Abstract

In this paper, we propose a new approach to defining bi-matrix game under the light of fuzzy environment. The main aim is to define the credibilistic bi-matrix game whose pay-off elements are characterized by fuzzy variables and its uncertainty is measured by credibility theory. The quadratic programming plays the major role to obtain the solution of credibilistic bi-matrix game for the players. The methodology is exhibited with two numerical examples.

Keywords: bi-matrix game, credibility measure, fuzzy constraints, quadratic programming

1 Introduction

Game theory [1] attempts to capture mathematical behavior in strategic situations in which one's success in making choices depends on the choices of others. Traditional applications of game theory attempt to find equilibria in these games. In an equilibrium strategy, each player of the game has adopted a strategy that they dislike to change. Many equilibrium concepts have been developed (most famously the Nash equilibrium) to capture this idea. These equilibrium concepts are motivated in different ways depending on the field of applied mathematics although they often overlap or coincide. This methodology is not without criticisms and debates to continue over the appropriateness of particular equilibrium concepts, the appropriateness of equilibria altogether the usefulness of mathematical models. It's usefulness in the context of our present day socio-economic problems has come to be widely recognized. Many business and economic situations are concerned with a problem of planning activity. In each case, there are limited resources at your disposal and your problem is to use of these resources to yields the maximum production or to minimize the cost of production or to give the maximum profit etc. It is impossible to predict the future with complete certainty, the structure of the business world will radically be difficult from that it is. There have been nothing like uncertainty, there would be no speculation on the stock market etc. Unfortunately we do not live in a world where success can be predicted with complete certainty. So we desire to deal with uncertainty. For example, when the players have lack of full information about the other players (or even their own) payoff functions, payoffs may be specified as fuzzy variables according to an expert system. If the fuzzy payoffs are known to all players, then according to appropriate optimal criteria, the game can be solved by credibility approach.

The all possible values of the game arranged in a matrix known as pay-off matrix. If two players have different pay-off matrices (i.e. sum is may not be zero), the game known as bi-matrix game. In this paper we have considered the elements of the bi-matrix game are fuzzy variables. The concept of fuzzy set was initiated by Zadeh [16] via membership function in 1965 and the fuzzy variable was proposed by Kaufmann [8]. In order to measure a fuzzy event, Zadeh [17] has proposed the concept of possibility measure in 1978. In 2004, the pioneering work of Liu [10] in the field of credibility theory was being perfected and become a strong tool to deal with incomplete and uncertain situation. The credibility measure plays the same role as probability in probability space. The credibility measure is more powerful than the other two fuzzy measures possibility and necessity in fuzzy set. In this paper, we have derived the quadratic programming problem which depends

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upon the confidence level for an equivalent credibilistic bi-matrix game. The confidence level came into picture due to incomplete information i.e. how much the given data have been satisfied to the pay-off matrix. For example, 1 for exact satisfaction and 0 for complement of satisfaction and in between is the uncertainty and can be handle through fuzzy measure. Quadratic programming is a technique for determining an optimum value of interdependent constraints in view of the available resources.

Several methodologies have been proposed to solve bi-matrix game with fuzzy elements but, no studies have been made in bi-matrix game on fuzzy environment based on credibility measure. In this paper, fuzzy variable has been utilized to incorporate the value of the bi-matrix game and then credibility measure has been used for these pay-offs. The concepts in bi-matrix game and credibility theory to yields a new method referred to herein as credibilistic bi-matrix game.

2 Preliminaries

Definition 2.1 [11] *Let Θ be a nonempty set, $P(\Theta)$ be the power set of Θ and Cr be a credibility measure on the element of $P(\Theta)$. Then the triplet $(\Theta, P(\Theta), Cr)$ is called a credibilty space.*

The mathematical framework of credibilty measure Cr is define on the credibility space just as measurable function on a probability space and the fuzzy variable in credibility space is same as random variable in probability space.

Definition 2.2 [11] *A fuzzy variable ξ is a function from a credibility space $(\Theta, P(\Theta), Cr)$ to the set of real numbers $\Re$. Its membership function μ is derive from the credibility measure on credibilistic space by*

$$\mu(x) = Cr\{\theta \in \Theta | \xi(\theta) = x\},\ x \in \Re.$$

The credibility measure is a mapping from the credibility space to the $[0,1]$ and derive from the membership function μ is defined by

$$Cr\{\xi \in \Re\} = \frac{1}{2}\left[\sup_{x\in\Re}\mu(x) + 1 - \sup_{x\in\Re^c}\mu(x)\right],$$

where $\Re^c$ is the complement of the set of real numbers $\Re$.

Let us consider the trapezoidal fuzzy variable $\xi = (a,b,c,d)$ with $a \le b < c \le d$. Then the membership function $\mu(x)$ is defined by Liu [11] as follows:

$$\mu(x) = \begin{cases} \frac{x-a}{b-a}, & \text{if } a \le x \le b \\ 1, & \text{if } b \le x \le c \\ \frac{x-d}{c-d}, & \text{if } c \le x \le d \\ 0, & \text{otherwise} \end{cases}$$

and the fuzzy measure namely credibility is defined by Liu [10] as follows,

$$\text{Cr}\{\xi \le 0\} = \begin{cases} 1, & \text{if } d \le 0 \\ \frac{2c-d}{2(c-d)}, & \text{if } c \le 0 \le d \\ \frac{1}{2}, & \text{if } b \le 0 \le c \\ \frac{a}{2(a-b)}, & \text{if } a \le 0 \le b \\ 0, & \text{otherwise.} \end{cases}$$

Definition 2.3 [12] *The fuzzy variables $\xi_1, \xi_2, \cdots, \xi_n$ are said to be independent if,*

$$\text{Cr}\left\{\bigcap_{i=1}^{n}\{\xi_i \in \mathbb{B}_i\}\right\} = \min_{1\le i\le n}\text{Cr}\{\xi_i \in \mathbb{B}_i\}$$

for any Borel set $\mathbb{B}_1, \mathbb{B}_2, \cdots, \mathbb{B}_n$ of $\Re$. The fuzzy variables $\xi_1, \xi_2, \cdots, \xi_n$ are said to be identically distributed if,

$$\text{Cr}\{\xi_i \in \mathbb{B}\} = \text{Cr}\{\xi_j \in \mathbb{B}\} \quad \text{for all} \quad i,j = 1,2,\cdots,n \quad \text{and } i \ne j$$

for any Borel set $\mathbb{B}$ of $\Re$.

Definition 2.4 [13] *Let ξ be a fuzzy variable. Then the expected value of ξ is defined by,*

$$E[\xi] = \int_0^{\infty} \mathrm{Cr}\{\xi \geq x\}dx - \int_{-\infty}^{0} \mathrm{Cr}\{\xi \leq x\}dx$$

provided that at least one of the two integrals is finite.

Example 2.1 [13] *Let $\xi = (a, b, c, d)$ be a fuzzy variable with $c \leq a < b \leq d$. Then we have,*

$$E[\xi] = \frac{1}{4}(a + b + c + d),$$
$$E[x\xi + y\zeta] = xE[\xi] + yE[\zeta],$$

where x and y are the real numbers and ζ is a fuzzy variable on the same credibilistic space $(\Theta, P(\Theta), Cr)$.

Theorem 2.1 [10] *Let us consider the trapezoidal fuzzy variable $\xi = (a, b, c, d)$ and confidence level $\alpha \in (0, 1]$ we have,*
(i) $\mathrm{Pos}\{\xi \leq 0\} \geq \alpha$, *if and only if* $(1 - \alpha)a + \alpha b \leq 0$;
(ii) $\mathrm{Nec}\{\xi \leq 0\} \geq \alpha$, *if and only if* $(1 - \alpha)c + \alpha d \leq 0$;
(iii) when $\alpha \leq \frac{1}{2}$, $\mathrm{Cr}\{\xi \leq 0\} \geq \alpha$, *if and only if* $(1 - 2\alpha)a + 2\alpha b \leq 0$;
(iv) when $\alpha > \frac{1}{2}$, $\mathrm{Cr}\{\xi \leq 0\} \geq \alpha$, *if and only if* $(2 - 2\alpha)c + (2\alpha - 1)d \leq 0$;

Proof : For prove see the reference [10].

Theorem 2.2 [10] *Let $\xi_k = (a_k, b_k, c_k, d_k)$ be a trapezoidal fuzzy variable for $k = 1, 2, \cdots, n$ and a function $g(x, \xi)$ can be written as*

$$g(x, \xi) = h_1(x)\xi_1 + h_2(x)\xi_2 + \cdots + h_n(x)\xi_n + h_0(x).$$

If two functions defined as $h_k^+(x) = h_k(x) \vee 0$ and $h_k^-(x) = -h_k(x) \wedge 0$ for $k = 1, 2, \cdots, n$, then there exist a confidence level α,
i) when $\alpha < 1/2$, $\mathrm{Cr}\{g(x, \xi) \leq 0\} \geq \alpha$, *if and only if,*

$$(1 - 2\alpha)\sum_{k=1}^{n}[a_k h_k^+(x) - d_k h_k^-(x)] + 2\alpha \sum_{k=1}^{n}[b_k h_k^+(x) - c_k h_k^-(x)] + h_0(x) \leq 0;$$

ii) when $\alpha \geq 1/2$, $\mathrm{Cr}\{g(x, \xi) \leq 0\} \geq \alpha$, *if and only if,*

$$(2 - 2\alpha)\sum_{k=1}^{n}[c_k h_k^+(x) - b_k h_k^-(x)] + (2\alpha - 1)\sum_{k=1}^{n}[d_k h_k^+(x) - a_k h_k^-(x)] + h_0(x) \leq 0.$$

Proof : For prove see the reference [10].

2.1 Bi-Matrix Game

In this subsection, first we define the bi-matrix game whose pay-off elements are characterized by real numbers. Let $X \equiv \{1, 2, \cdots, m\}$ be a set of strategies for the player I and $Y \equiv \{1, 2, \cdots, n\}$ be a set of strategies for player II. Let $\Re^n$ be the $n-$dimensional Euclidean space and $\Re^n_+$ be its non-negative orthant. And $e^T = \{1, 1, \cdots, 1\}$ be a vector of element '1' whose dimension is specified as per specific context. Mixed strategies of players I and II are represented by weights to their strategies $S^X = \{x \in \Re^m_+, e^T x = 1\}$ and $S^Y = \{y \in \Re^n_+, e^T y = 1\}$ respectively.

By considering real numbers a_{ij} and b_{ij} as the expected reward for the player I and II corresponding to the proposed strategy i and j respectively, then the pay-off matrices for bi-matrix game can be defined as follows

$$(A,B)=\begin{pmatrix} (a_{11},b_{11}) & (a_{12},b_{12}) & . & . & . & (a_{1n},b_{1n}) \\ (a_{21},b_{21}) & (a_{22},b_{22}) & . & . & . & (a_{2n},b_{2n}) \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ (a_{m1},b_{m1}) & (a_{m2},b_{m2}) & . & . & . & (a_{mn},b_{mn}) \end{pmatrix}.$$

Definition 2.5 *A pair $(x^*,y^*)\in S^X\times S^Y$ is said to be the Nash equilibrium strategy of the bi-matrix game $BG=(S^X,S^Y,A,B)$ if,*

$$\begin{aligned} & x^TAy^*\le {x^*}^TAy^* \\ \textit{and}\quad & {x^*}^TBy\le {x^*}^TBy^*. \end{aligned}$$

Theorem 2.3 *Let $BG=(S^X,S^Y,A,B)$ be the given bi-matrix game. A necessary and sufficient condition that (x^*,y^*) be an equilibrium strategy of BG is that it is a solution of the following quadratic programming problem,*

$$\begin{aligned} \max\quad & x^T(A+B)y-v-w \\ \textit{subject to,}\quad & Ay-ve \le 0 \\ & B^Tx-we \le 0 \\ & e^Tx-1 = 0 \\ & e^Ty-1 = 0 \\ & v,w \in \Re \\ & x,y \ge 0. \end{aligned}$$

Proof : For prove see the reference [2].

Lemma 2.1 *If v^* be the value of the game for the player I where v^* is given by,*

$$v^*={x^*}^TAy^*=\max_x\{x^TAy^*:e^Tx=1,x\ge 0\}$$

and w^* is the value of the game for player II where w^* is given by,

$$w^*={x^*}^TBy^*=\max_y\{{x^*}^TBy:e^Ty=1,y\ge 0\},$$

and if we assume that ${x_i}'=\frac{x_i}{w}$ and ${y_j}'=\frac{y_j}{v}$ (for $i=1,2,\cdots,m;j=1,2,\cdots,n$), then the **Theorem 2.3** *reduces into the following quadratic programming problem,*

$$\begin{aligned} \max\quad & e^Tx'+e^Ty'+{x'}^T(A+B)y' \\ \textit{subject to,}\quad & \sum_{j=1}^{n}a_{ij}y_j' \le 1 \quad \textit{for}\quad i=1,2,\cdots,m \\ & \sum_{i=1}^{m}b_{ij}x_i' \le 1 \quad \textit{for}\quad j=1,2,\cdots,n \\ & x',y' \ge 0 \end{aligned}$$

where (x^*,y^*) and (v^*,w^*) are the equilibrium strategies and equilibrium outcome of the bi-matrix game BG respectively.

Proof : Since $(x^*, y^*) \in S^X \times S^Y$ be an equilibrium strategy of the bi-matrix game BG if and only if x^* and y^* are simultaneously solutions of the following two problems:

$$\begin{cases} \max & x^T A y^* \\ \text{subject to,} & e^T x = 1 \\ & x \geq 0, \end{cases} \tag{2.1}$$

and

$$\begin{cases} \max & {x^*}^T B y \\ \text{subject to,} & e^T y = 1 \\ & y \geq 0. \end{cases} \tag{2.2}$$

Here $(x^*, y^*) \in S^X \times S^Y$ be an optimal strategy of the BG that satisfy the conditions of (2.1) and (2.2). Also $e^T x' = \frac{1}{w}$ and $e^T y' = \frac{1}{v}$, that is, $e^T {x'}^* = \frac{1}{w^*}$ and $e^T {y'}^* = \frac{1}{v^*}$. So the constraints of the **Theorem 2.3** can be written into the following form:

$$\begin{aligned} Ay^* &\leq v^* e \qquad \text{because} \quad x^T A y^* \leq v^* x^T e = v^* \\ \text{Hence} \quad Ay &\leq ve \quad \text{or} \quad Ay' \leq e. \\ \text{i.e.,} \quad \sum_{j=1}^{n} a_{ij} y_j' &\leq 1 \quad \text{for} \quad i = 1, 2, \cdots, m. \end{aligned}$$

and

$$\begin{aligned} {x^*}^T B &\leq w^* e \qquad \text{because} \quad {x^*}^T B y \leq w^* y^T e = w^* \\ \text{Hence} \quad B^T x &\leq we \quad \text{or} \quad B^T x' \leq e. \\ \text{i.e.,} \quad \sum_{i=1}^{m} b_{ij} x_i' &\leq 1 \quad \text{for} \quad j = 1, 2, \cdots, n. \end{aligned}$$

Therefore the objective function of the **Theorem 2.3** can be modified into the following form,

$$\begin{aligned} & \max \quad x^T (A+B) y \quad - \quad v - w \\ \text{or} \qquad & \max \quad \left(\frac{x}{w}\right)^T (A+B) \left(\frac{y}{v}\right) \quad + \quad \frac{1}{w} + \frac{1}{v} \\ \text{Hence,} \qquad & \max \quad {x'}^T (A+B) y' \quad + \quad e^T x' + e^T y'. \end{aligned}$$

3 Credibilistic Bi-Matrix Game

In many applied situation, we can't be sure about rewards of our strategies. So the rewards are vague. Therefore to introduce the bi-matrix game we choose the rewards as fuzzy variables and then measured by credibility. Let the fuzzy variable ξ_{ij} denotes the pay-off that player I gain or the fuzzy variable ζ_{ij} be the pay-off that player II gain when the player I plays the pure strategy i and player II plays the pure strategy j. Then the credibilistic bi-matrix game can be represented as fuzzy pay-off matrix as follows

$$(\xi, \zeta) = \begin{pmatrix} (\xi_{11}, \zeta_{11}) & (\xi_{12}, \zeta_{12}) & \cdot & \cdot & \cdot & (\xi_{1n}, \zeta_{1n}) \\ (\xi_{21}, \zeta_{21}) & (\xi_{22}, \zeta_{22}) & \cdot & \cdot & \cdot & (\xi_{2n}, \zeta_{2n}) \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ (\xi_{m1}, \zeta_{m1}) & (\xi_{m2}, \zeta_{m2}) & \cdot & \cdot & \cdot & (\xi_{mn}, \zeta_{mn}) \end{pmatrix}.$$

Definition 3.1 *Let ξ_{ij} and ζ_{ij} $(i = 1, 2, \cdots, m; j = 1, 2, \cdots, n)$ be independent fuzzy variables. Then (x^*, y^*) is called an expected credibilistic Nash equilibrium strategy to the credibilistic bi-matrix game $BG = \{S^X, S^Y, \xi, \zeta\}$ if,*

$$E[x^T \xi y^*] \leq E[{x^*}^T \xi y^*] \leq E[{x^*}^T \xi y],$$
$$E[{x^*}^T \zeta y] \leq E[{x^*}^T \zeta y^*] \leq E[x^T \zeta y^*].$$

Definition 3.2 *Let ξ_{ij} and ζ_{ij} $(i = 1, 2, \cdots, m; j = 1, 2, \cdots, n)$ are different independent fuzzy variables, $\alpha \in (0, 1]$ and $w, v \in \Re$ be the predetermined level of the fuzzy pay-offs. Then (x^*, y^*) is called a α-credibilistic equilibrium strategy to the credibilistic bi-matrix game $BG = \{S^X, S^Y, \xi, \zeta\}$ if,*

$$\max\{v|\ \mathrm{Cr}\{x^T \xi y^* \geq v\} \geq \alpha\} \leq \max\{v|\ \mathrm{Cr}\{{x^*}^T \xi y^* \geq v\} \geq \alpha\} \leq \max\{v|\ \mathrm{Cr}\{{x^*}^T \xi y \geq v\} \geq \alpha\},$$

$$\max\{w|\ \mathrm{Cr}\{{x^*}^T \zeta y \geq w\} \geq \alpha\} \leq \max\{w|\ \mathrm{Cr}\{{x^*}^T \zeta y^* \geq w\} \geq \alpha\} \leq \max\{w|\ \mathrm{Cr}\{x^T \zeta y^* \geq w\} \geq \alpha\}.$$

3.1 Quadratic Programming Problem

To derive the solution of the credibilistic bi-matrix game, we are to solve the following fuzzy quadratic programming problem:

$$\max \quad e^T x^{'} + e^T y^{'} + x^{'T} (\xi + \zeta) y^{'}$$

$$\begin{aligned} \text{subject to,} \quad & \sum_{j=1}^{n} \xi_{ij}\ y_j^{'} \ \leq \ 1 \quad \text{for} \quad i = 1, 2, \cdots, m. \\ & \sum_{i=1}^{m} \zeta_{ij}\ x_i^{'} \ \leq \ 1 \quad \text{for} \quad j = 1, 2, \cdots, n. \\ & y^{'} \geq 0, x^{'} \geq 0. \end{aligned}$$

Since the fuzzy variables are present in the above quadratic programming problem, traditional method is not applicable. To find the solution of the above problem we have to introduce the fuzzy expected operator for the objective function and for the constraints and fuzzy measurable function named as credibility with confidence level α. So the fuzzy quadratic programming problem becomes

$$\max \quad E[e^T x^{'} + e^T y^{'} + x^{'T} (\xi + \zeta) y^{'}]$$

$$\begin{aligned} \text{subject to,} \quad & \mathrm{Cr}\{\sum_{j=1}^{n} \xi_{ij}\ y_j^{'} \ \leq \ 1\} \geq \alpha \\ & \mathrm{Cr}\{\sum_{i=1}^{m} \zeta_{ij}\ x_i^{'} \ \leq \ 1\} \geq \alpha \\ & y^{'} \geq 0, x^{'} \geq 0. \end{aligned}$$

With the help of the **Theorem 2.2**, we can easily find that $h^+(x) = x^{'}, h^+(y) = y^{'}$ and $h^-(x) = 0, h^-(y) = 0$ because $y^{'} \geq 0, x^{'} \geq 0$, and the credibility theory, the above problem can be written into crisp quadratic programming problem which depends upon the confidence level $\alpha \in (0, 1]$.

$$\max \ \frac{1}{4} \sum_{i=1}^{m} \sum_{j=1}^{n} \{(a1_{ij} + a2_{ij}) + (b1_{ij} + b2_{ij}) + (c1_{ij} + c2_{ij}) + (d1_{ij} + d2_{ij})\} x_i^{'} y_j^{'} + \sum_{i=1}^{m} x_i^{'} + \sum_{j=1}^{n} y_j^{'}$$

subject to,

i) when $\alpha < 1/2$,

$$(1-2\alpha)\sum_{j=1}^{n} a1_{ij}\ y_j^{'} + 2\alpha \sum_{j=1}^{n} b1_{ij}\ y_j^{'} \le 1,$$
$$(1-2\alpha)\sum_{i=1}^{m} a2_{ij}\ x_i^{'} + 2\alpha \sum_{i=1}^{m} b2_{ij}\ x_i^{'} \le 1;$$

ii) when $\alpha \ge 1/2$,

$$(2-2\alpha)\sum_{j=1}^{n} c1_{ij}\ y_j^{'} + (2\alpha-1) \sum_{j=1}^{n} d1_{ij}\ y_j^{'} \le 1,$$
$$(2-2\alpha)\sum_{i=1}^{m} c2_{ij}\ x_i^{'} + (2\alpha-1) \sum_{i=1}^{m} d2_{ij}\ x_i^{'} \le 1,$$

where $\xi_{ij} = (a1_{ij}, b1_{ij}, c1_{ij}, d1_{ij})$ and $\zeta_{ij} = (a2_{ij}, b2_{ij}, c2_{ij}, d2_{ij})$ are trapezoidal fuzzy numbers corresponding to the credibilistic bi-matrix game. When $ck_{ij} = bk_{ij}$ $(k = 1, 2)$, the trapezoidal fuzzy number becomes triangular fuzzy number, and the same idea can also be implemented for triangular fuzzy number.

4 Numerical Examples

Example 1: Let us consider 2×2 fuzzy pay-off matrices for the credibilistic bi-matrix game whose elements are $\xi_{11} = (1, 5, 6, 7)$, $\xi_{12} = (2, 5, 10, 12)$, $\xi_{21} = (3, 5, 8, 11)$ and $\xi_{22} = (2, 9, 12, 13)$ and $\zeta_{11} = (2, 3, 5, 7)$, $\zeta_{12} = (3, 5, 11, 12)$, $\zeta_{21} = (1, 4, 7, 9)$ and $\zeta_{22} = (2, 4, 10, 12)$ with confidence level $\alpha = 1/4$ and $\alpha = 3/4$. Then the quadratic programming problem for the credibilistic bi-matrix game is
i) when $\alpha = 1/4$,

$$\begin{array}{rrcl}
\max & 9x_1^{'}y_1^{'} + 15x_1^{'}y_2^{'} + 12x_2^{'}y_1^{'} & + & 16x_2^{'}y_2^{'} + x_1^{'} + x_2^{'} + y_1^{'} + y_2^{'} \\
& \text{subject to,} \quad 6y_1^{'} + 7y_2^{'} & \le & 2 \\
& 8y_1^{'} + 11y_2^{'} & \le & 2 \\
& 5x_1^{'} + 5x_2^{'} & \le & 2 \\
& 8x_1^{'} + 6x_2^{'} & \le & 2 \\
& x_1^{'}, x_2^{'}, y_1^{'}, y_2^{'} & \ge & 0.
\end{array}$$

Using Wolfe's modified simplex method, we have obtained $x_1^{'} = 0.4$, $x_2^{'} = 0$, $y_1^{'} = 0.25$ and $y_2^{'} = 0$. Hence $v = 4$ i.e, value of the credibilistic bi-matrix game for the player I with strategy $(1, 0)$ and $w = 2.5$ i.e, value of the credibilistic bi-matrix game for the player II with strategy $(1, 0)$;
ii) when $\alpha = 3/4$,

$$\begin{array}{rrcl}
\max & 9x_1^{'}y_1^{'} + 15x_1^{'}y_2^{'} + 12x_2^{'}y_1^{'} & + & 16x_2^{'}y_2^{'} + x_1^{'} + x_2^{'} + y_1^{'} + y_2^{'} \\
& \text{subject to,} \quad 13y_1^{'} + 22y_2^{'} & \le & 2 \\
& 19y_1^{'} + 25y_2^{'} & \le & 2 \\
& 12x_1^{'} + 16x_2^{'} & \le & 2 \\
& 23x_1^{'} + 22x_2^{'} & \le & 2 \\
& x_1^{'}, x_2^{'}, y_1^{'}, y_2^{'} & \ge & 0.
\end{array}$$

Using Wolfe's modified simplex method, we have obtained $x_1^{'} = 0.125$, $x_2^{'} = 0$, $y_1^{'} = 0.1053$ and $y_2^{'} = 0$. Hence $v = 10$ i.e., value of the credibilistic bi-matrix game for the player I with strategy $(1, 0)$ and $w = 8$ i.e, value of the credibilistic bi-matrix game for the player II with strategy $(1, 0)$.

Example 2: Let us consider 2×2 fuzzy pay-off matrices whose elements are trapezoidal numbers for the credibilistic bi-matrix game,

$$\xi = \begin{pmatrix} (175,180,190,195) & (150,156,158,160) \\ (80,90,95,100) & (175,180,190,195) \end{pmatrix}$$

and

$$\zeta = \begin{pmatrix} (160,165,170,175) & (140,145,148,150) \\ (70,75,78,82) & (160,165,170,175) \end{pmatrix}.$$

Then the quadratic programming problem for the credibilistic bi-matrix game is

$$\max \quad 352.5x_1^{'}y_1^{'} + 301.75x_1^{'}y_2^{'} + 167.5x_2^{'}y_1^{'} + 352.5x_2^{'}y_2^{'} + x_1^{'} + x_2^{'} + y_1^{'} + y_2^{'}$$

subject to,

$$\begin{aligned} \text{when} \quad \alpha < \frac{1}{2}, \\ \{(1-2\alpha)175 + 2\alpha 180\}y_1^{'} + \{(1-2\alpha)150 + 2\alpha 156\}y_2^{'} \leq 1 \\ \{(1-2\alpha)80 + 2\alpha 90\}y_1^{'} + \{(1-2\alpha)175 + 2\alpha 180\}y_2^{'} \leq 1 \\ \{(1-2\alpha)160 + 2\alpha 165\}x_1^{'} + \{(1-2\alpha)70 + 2\alpha 75\}x_2^{'} \leq 1 \\ \{(1-2\alpha)140 + 2\alpha 145\}x_1^{'} + \{(1-2\alpha)160 + 2\alpha 165\}x_2^{'} \leq 1 \\ x_1^{'}, x_2^{'}, y_1^{'}, y_2^{'} \geq 0. \end{aligned}$$

$$\begin{aligned} \text{when} \quad \alpha \geq \frac{1}{2}, \\ \{(2-2\alpha)190 + (2\alpha-1)195\}y_1^{'} + \{(2-2\alpha)158 + (2\alpha-1)160\}y_2^{'} \leq 1 \\ \{(2-2\alpha)95 + (2\alpha-1)100\}y_1^{'} + \{(2-2\alpha)190 + (2\alpha-1)195\}y_2^{'} \leq 1 \\ \{(2-2\alpha)170 + (2\alpha-1)175\}x_1^{'} + \{(2-2\alpha)78 + (2\alpha-1)82\}x_2^{'} \leq 1 \\ \{(2-2\alpha)148 + (2\alpha-1)150\}x_1^{'} + \{(2-2\alpha)170 + (2\alpha-1)175\}x_2^{'} \leq 1 \\ x_1^{'}, x_2^{'}, y_1^{'}, y_2^{'} \geq 0. \end{aligned}$$

i) when $\alpha = 0.2$,

$$\max \quad 352.5x_1^{'}y_1^{'} + 301.75x_1^{'}y_2^{'} + 167.5x_2^{'}y_1^{'} + 352.5x_2^{'}y_2^{'} + x_1^{'} + x_2^{'} + y_1^{'} + y_2^{'}$$

subject to,

$$\begin{aligned} 177y_1^{'} + 152.399994y_2^{'} \leq 1 \\ 84y_1^{'} + 177y_2^{'} \leq 1 \\ 162x_1^{'} + 72x_2^{'} \leq 1 \\ 142x_1^{'} + 162x_2^{'} \leq 1 \\ x_1^{'}, x_2^{'}, y_1^{'}, y_2^{'} \geq 0. \end{aligned}$$

The solution of the above non-linear programming problem is

$$y^{'} = (0.001327763, 0.005019592) \ \ and \ \ x^{'} = (0.005617978, 0.001248439),$$

which leads to the following solution of the original problem:

$$y^{*} = (0.209184, 0.790816) \ \ and \ \ w = 145.636363$$

$$x^{*} = (0.818182, 0.181818) \ \ and \ \ v = 157.545938.$$

ii) when $\alpha = 0.7$,

$$\max \quad 352.5x_1^{'}y_1^{'} + 301.75x_1^{'}y_2^{'} + 167.5x_2^{'}y_1^{'} + 352.5x_2^{'}y_2^{'} + x_1^{'} + x_2^{'} + y_1^{'} + y_2^{'}$$

subject to,

$$\begin{aligned} 192y_1^{'} + 158.8y_2^{'} &\leq 1 \\ 97y_1^{'} + 192y_2^{'} &\leq 1 \\ 172x_1^{'} + 79.599998x_2^{'} &\leq 1 \\ 148.8x_1^{'} + 172x_2^{'} &\leq 1 \\ x_1^{'}, x_2^{'}, y_1^{'}, y_2^{'} &\geq 0. \end{aligned}$$

The solution of the above non-linear programming problem is

$$y^{'} = (0.001547035, 0.004426758) \quad and \quad x^{'} = (0.005208709, 0.001307814),$$

which leads to the following solution of the original problem:

$$y^* = (0.258970, 0.741030) \quad and \quad w = 153.456069$$

$$x^* = (0.799308, 0.200692) \quad and \quad v = 167.397832.$$

Hence for different values of α, the decision maker can obtain the different strategies and the corresponding values of the credibilistic bi-matrix game. For appropriate value of α, the decision maker can obtained the required strategy and the corresponding value of the credibilistic bi-matrix game.

5 Conclusion

In this paper, we have proposed the bi-matrix game under the light of fuzzy environment. The ambiguous information on the elements of the bimatrix game is interpreted as fuzzy variable and represented as credibilistic bi-matrix game. The credibilistic bi-matrix game can be written into the form of fuzzy quadratic programming problem with the help of bi-matrix game theory. The credibility theory plays the major role to convert the credibilistic bi-matrix game into quadratic programming and then it is solved by Wolfe's modified simplex method. Then we have obtained the strategy as well as the value of the game for the players. Finally we have concluded that the method which has discussed here is highly applicable to the real world game problem (decision making) where the data are imprecise/inexact.

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