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Chaos Control of a Non-linear Finance System

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Abstract

The usefulness of chaos control for a kind of nonlinear chaos finance system is studied. First, the selection of feedback gain matrix control, its revision of gain matrix control, linear feedback control and speed feedback control are used to control chaos to unstable equilibrium point. Then the Lyapunov direct method and Routh-Hurwith are used to analyze and study the gain's scope of controlled system and the asymptotic stability at the equilibrium point. At last, the validity of the four controlling methods is proved through theoretical analysis and numerical simulations.

Keywords: chaos finance system, chaos control, Lyapunov method, linear feedback control, speed feedback control

1 Introduction

Nonlinear chaotic dynamical system research is popular problem in the nonlinear science field. Nonlinear chaotic systems have been extensively studied within scientific, engineering and mathematical communities [3, 5, 6]. Since the chaotic phenomenon in economics was first found in 1985, great impact has been imposed on the prominent economics at present, because the occurrence of the chaotic phenomenon's in the economic system means that the macro economic operation has in itself the inherent indefiniteness. Researches on the complicated economic system by applying nonlinear methods have been fruitful [7, 8, 9]. In the field of finance, stocks and social economics, due to the interaction between nonlinear factors, with the evolution process from low dimensions to high dimensions, the diversity and complexity have manifested themselves in the internal structure of the system and there exists extremely complicated phenomenon and external characteristics in such a kind of system. So it has become more and more important to make a systematic and deep study in the internal structural characteristics in such a complicated economic system.

In this paper, we study the chaos control of a nonlinear finance system. The selection of gain matrix control, the revision of gain matrix control [4], the speed feedback control and the linear feedback control [1] are used to suppress chaos to unstable equilibrium points. The Lyapunov direct method [2, 10] and Routh-Hurwith are used to analyze the asymptotic stability at the equilibrium point. Moreover, numerical simulations are applied to verify the effectiveness of chosen controllers.

This article is organized as follows. In Section 2, a new chaotic finance system is introduced. The speed feedback control and linear feedback control is considered in Section 3. In Section 4, we discuss the selection of feedback gain matrix control and it's revision of feedback gain matrix control. Four numerical examples are given to demonstrate the effectiveness of the proposed methods in Section 5. Finally the conclusion is given in Section 6.

2 The No-linear Finance Chaotic System

The nonlinear finance chaotic system can be described by the following differential equation:

$$\begin{cases} \dot{x} = z + (y-a)x \\ \dot{y} = 1 - by - x^2 \\ \dot{z} = -x - cz \end{cases} \tag{1}$$

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where variable x represents the interest rate in the model; variable y represents the investment demand and variable z is the price exponent. The parameter a is the saving, b is the per investment cost, c is the elasticity of demands of commercials. And they are positive constants.

When c-b-abc≤0, system (1) has only equilibrium point (0, 1/b, 0). When c-b-abc>0, system (1) has another two equilibrium points $(\pm\theta,(1+ac)/c,\mp\theta/c)$, where $\theta=\sqrt{(c-b-abc)/c}$.

We have a transformation of system (1) at the equilibrium point $(\theta,(1+ac)/c,-\theta/c)$:

$$\begin{cases} X = x-\theta \\ Y = y-(1+ac)/c \\ Z = z+\theta/c. \end{cases} \tag{2}$$

Then system (1) is rewritten as follows:

$$\begin{cases} X = X/c+\theta Y+XY+Z \\ Y = -X^2-2\theta X-bY \\ Z = -X+cZ. \end{cases} \tag{3}$$

For system (3), it has three equilibrium points

$$p_0(0,0,0),\ p_1(-2\theta,0,2\theta/c),\ p_2(-\theta,\theta^2/b,\theta/c).$$

When c-b-abc>0, c+b-1/c>0, then the balance point P_0 (0, 0, 0) of system (3) under the conditions is the stable convergence.

When c-b-abc>0, c+b-1/c<0, the balance point P_0 (0, 0, 0) of system (3) under the conditions is the saddle.

When c-b-abc=0, 0<c<1, then according to Ref [4], the equilibrium point P_0 (0, 0, 0) is the unstable point of the non-hyperbola.

When c-b-abc=0, c>1, According to the central manifold theorem, the balance point P_0 (0, 0, 0) is also gradually inclined to be stable.

When $bc^4+b^2c^3-2ab^2c^2+(2ab-2-3b^2)c+3b=0$, Bifurcation occurs at $p_1(-2\theta,0,2\theta/c)$, and it is unstable equilibrium point.

When a=0.9, b=0.2, c=1.2, the system (3) has a new chaotic attractor, as shown in Fig.1.

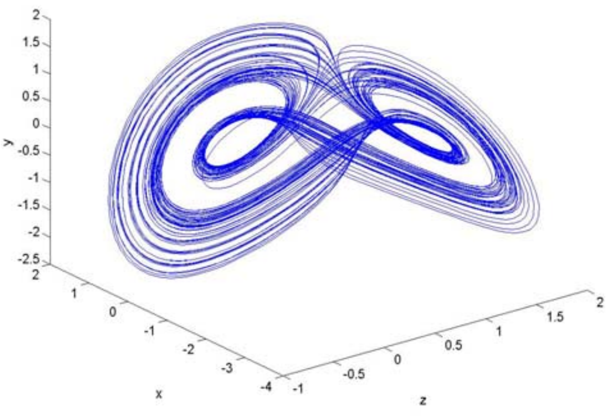


Figure 1: The new attractor of system (3)

3 Controlling Chaotic Attractor to Equilibrium P_0

In this section, we will control chaotic system (3) to equilibrium point P_0 (0, 0, 0) by using two different feedback methods, suppose that the controlled system is the following form:

$$\begin{cases} \dot{X} = X/c + \theta Y + XY + Z + u_1 \\ \dot{Y} = -2\theta X - bY - X^2 + u_2 \\ \dot{Z} = -X - cZ + u_3. \end{cases} \tag{4}$$

In which u_i (i=1, 2, 3) are external control inputs. By using the suitable input function u_i (i=1, 2, 3), the chaotic trajectory (x, y, z) of system (4) will be dragged to P_0 (0, 0, 0).

3.1 Linear Feedback Control

We assume that u_3=0, u_1, u_2 are of the nonlinear forms, $u_1= -k_1x$, $u_2= -k_2y$, where k_i (i=1,2) are feedback coefficients. We can get the controlled system (3) as follows:

$$\begin{cases} \dot{X} = X/c + \theta Y + Z + XY - k_1 X \\ \dot{Y} = -2\theta X - bY - X^2 - k_2 Y \\ \dot{Z} = -X - cZ. \end{cases} \tag{5}$$

Theorem 1 When $k_1 = k_2 + b + 1/c > 0$, $k_2 > 1/c + \theta/2 - b > 0$, the controlled system (5) is asymptotically stable at equilibrium point P_0 (0, 0, 0).

Proof: we divide system (5) into subsystems:

$$\begin{cases} \dot{X} = X/c + \theta Y + Z + XY - k_1 X \\ \dot{Y} = -2\theta X - bY - X^2 - k_2 Y \end{cases} \tag{6}$$

and

$$\dot{Z} = -X - cZ. \tag{7}$$

For system (6), we choose the Lyapunov function for to-be controlled system (6) as:

$$V_1(X,Y) = (X^2 + Y^2)/2.$$

The time derivative of V along the trajectories of system (6) is

$$\dot{V}_1 = (1/c - k_1)X^2 - (b + k_2)Y^2 - \theta XY + XZ.$$

Let $\left\|(X,Y)^T\right\| = \sqrt{X^2 + Y^2}$. When $k_1 = k_2 + b + 1/c > 0$,

$$\begin{aligned} \dot{V}_1 &\le (1/c - k_1)X^2 - (b + k_2)Y^2 + \theta|X|\|Y\| + |X|\|Z\| \\ &\le (1/c - k_1)X^2 - (b + k_2)Y^2 + \theta(X^2 + Y^2)/2 + \sqrt{X^2 + Y^2}\,|Z| \\ &= -(b + k_2 - \theta/2)(X^2 + Y^2) + \sqrt{X^2 + Y^2}\,|Z|. \end{aligned}$$

When $\sqrt{X^2 + Y^2} \ge |Z|/(k_2 + b - \theta/2 - \varepsilon)$, then $\left\|(X,Y)^T\right\| \ge |Z|/(k_2 + b - \theta/2 - \varepsilon)$,

$$\dot{V}_1 = -\varepsilon(X^2 + Y^2) = -\varepsilon\left\|(X,Y)^T\right\|^2.$$

Let $a_1(r) = \varepsilon r^2$. Then $\chi_1(r) = r/(k_2 + b - \theta/2 - \varepsilon)$.

Let $\underline{a_1}(r) = \overline{a_1}(r) = r^2/2$. Then $\gamma_1(r) = \underline{a_1}^{-1} \times \overline{a_1}^{-1} \times \chi_1(r) = r/(k_2 + b - \theta/2 - \varepsilon)$.

For system (7), let $V_2(X,Y) = Z^2/2$. The time derivative of V along the trajectories of system (7) is

$$\dot{V}_2 = -cZ^2 - XZ \le -cZ^2 + |Z|\sqrt{X^2 + Y^2}.$$

When $|Z| \ge \sqrt{X^2 + Y^2}/c - \varepsilon = \left\|(X,Y)^T\right\|/c - \varepsilon$, $\dot{V}_1 \le -\varepsilon Z^2$, let $a_2(r) = \varepsilon r^2$. Then $\chi_2(r) = r/c - \varepsilon$. Let $\underline{a_2}(r) = \overline{a_2}(r) = r^2/2$. Then $\underline{\gamma_2}(r) = a_2^{-1} \times \overline{a_2} \times \chi_2(r) = r/(c - \varepsilon)$.

When $k_2 > 1/c + \theta/2 - b > 0$， we can get the gain function

$$\gamma_1(\gamma_2(r)) = r/(c - \varepsilon)(k + b - \theta/2 - \varepsilon) < r.$$

According to Ref [11], system (5) is asymptotically stable at equilibrium point $p_0(0,0,0)$.

3.2 Speed Feedback Control

We assume that $u_1 = u_2 = 0$, and u_3 is the speed form $u_3 = -k_1\dot{X}$, where k_1 is a speed feedback coefficient, the controlled chaotic system (4) is rewritten as:

$$\begin{cases} \dot{X} = X/c + \theta Y + XY + Z \\ \dot{Y} = -2\theta X - bY - X^2 \\ \dot{Z} = -X - cZ - k_1(X/c + \theta Y + Z + XY). \end{cases} \quad (8)$$

The Jacobi matrix of system (8) is

$$\begin{bmatrix} 1/c & \theta & 1 \\ -2\theta & -b & 0 \\ -k_1/c - 1 & -k_1\theta & -c - k_1 \end{bmatrix}. \quad (9)$$

The characteristic equation of matrix (9) is

$$\lambda^3 + p_1\lambda^2 + p_2\lambda + p_3 = 0$$

where

$$p_1 = b + c - 1/c + k_1,$$
$$p_2 = bc - b/c + bk_1 + 2\theta^2,$$
$$p_3 = 2c - 2b - 2abc.$$

When $a = 0.9, b = 0.2, c = 1.2$, it is easy to see that $p_3 = 1.568 > 0$, according to Routh-Hurwitz criterion,

$$p_1 > 0, p_1 p_2 > p_3, \quad (10)$$

i.e., $k_1 > 0.4937$.

The real parts λ of Jacobi matrix (9) with the equilibrium point $p_0(0,0,0)$ are all negative, when $a = 0.9, b = 0.2, c = 1.2$ and k_1 satisfy (10), the controlled chaotic system (8) is asymptotically stable at equilibrium point $p_0(0,0,0)$.

4 Controlling Chaotic Attractor to Equilibrium P_1

In this section, we will control chaotic system to equilibrium $p_1(-2\theta, 0, 2\theta/c)$ by using different feedback methods. Suppose that the control system is the following form:

$$\begin{cases} \dot{X} = X/c + \theta Y + XY + Z + u_1 \\ \dot{Y} = -2\theta X - bY - X^2 + u_2 \\ \dot{Z} = -X - cZ + u_3 \end{cases} \quad (11)$$

where

$$u_1 = -k_{11}(X - \overline{X}), u_2 = -k_{22}(Y - \overline{Y}), u_3 = -k_{33}(Z - \overline{Z}).$$

There $k_{ii}(i = 1,2,3)$ are feedback coefficients.

4.1 The Selection of Gain Matrix Control

Diverted the controlled system to the equilibrium point $p_1(-2\theta, 0, 2\theta/c)$, then the controlled system is rewritten as

$$\begin{cases} \dot{X} = X/c + XY + \theta Y + Z - k_{11}(X + 2\theta) \\ \dot{Y} = -X^2 - 2\theta X - bY - k_{22}Y \\ \dot{Z} = -X - cZ - k_{33}(Z - 2\theta/c). \end{cases} \quad (12)$$

Let $x = X + 2\theta, y = Y, z = Z - 2\theta/c$,

Then system (12) is rewritten as

$$\begin{cases} \dot{x} = x/c + xy - \theta y + z - k_{11}x \\ \dot{y} = -x^2 + 2\theta x - by - k_{22}y \\ \dot{z} = -x - cz - k_{33}z. \end{cases} \tag{13}$$

Then the equilibrium point $p_1(-2\theta, 0, 2\theta/c)$ of system (12) turns into (0, 0, 0) of system (13).

Theorem 2 When $k_{ii}(i=1,2,3)$ are positive, the equilibrium $p_1(-2\theta, 0, 2\theta/c)$ of system (12) is asymptotically stability when $a=0.9, b=0.2, c=1.2$.

Proof: The Jacobi matrix of system (13) is

$$\begin{bmatrix} 1/c - k_{11} & -\theta & 1 \\ 2\theta & -b - k_{22} & 0 \\ -1 & 0 & -c - k_{33} \end{bmatrix}. \tag{14}$$

The characteristic equation of matrix (6) is

$$\lambda^3 + B_1\lambda^2 + B_2\lambda + B_3 = 0,$$

where

$$B_1 = k_{11} + k_{22} + k_{33} + b + c + 1/c,$$
$$B_2 = bc - b/c + 2\theta^2 + c(k_{11} + k_{22}) + b(k_{11} + k_{33}) + k_{11}k_{22} + k_{11}k_{33} + k_{22}k_{33} - k_{22}/c - k_{33}/c,$$
$$B_3 = 2\theta^2(c + k_{33}) + bk_{11}(c + k_{33}) + ck_{11}k_{33} + k_{11}k_{22}k_{33} - bk_{33}/c - k_{22}k_{33}/c.$$

According to Routh-Hurwitz criterion, the real parts λ are negative if and only if $B_1 > 0, B_3 > 0, B_1B_2 > B_3$, $k_{ii}(i=1,2,3) > 0$.

When $a=0.9, b=0.2, c=1.2$ and $k_{11} = k_{33} = 0, k_{22} > 0$, the controlled chaotic system (12) is asymptotically stable at equilibrium point $P_1(-2\theta, 0, 2\theta/c)$.

With similar method, when k_{ii} get different feedback coefficients, we can get the same effect and then P_2 is asymptotically stable.

4.2 Revision of Gain Matrix Control

We assume that $u_2 = u_3 = 0, u_2 = k_1(-x - cz) + k_2(\overline{x} - x)$, where k_1, k_2 are feedback coefficients, then system (11) are is rewritten as:

$$\begin{cases} \dot{X} = X/c + \theta Y + XY + Z + k_1(-X - cZ) - k_2(X - \overline{X}) \\ \dot{Y} = -2\theta X - bY - X^2 \\ \dot{Z} = -X - cZ. \end{cases} \tag{15}$$

Diverted the controlled system (15) to equilibrium point $P_1(-2\theta, 0.2\theta/c)$, then system (15) is rewritten as:

$$\begin{cases} \dot{X} = X/c + \theta Y + XY + Z + k_1(-X - 2\theta) - k_2(X + 2\theta) \\ \dot{Y} = -2\theta X - bY - X^2 \\ \dot{Z} = -X - cZ. \end{cases} \tag{16}$$

Let $x = X + 2\theta, y = Y, z = Z - 2\theta/c$. Then system (16) is rewritten as

$$\begin{cases} \dot{x} = x/c + xy - \theta y + z - k_1x - k_2x \\ \dot{y} = -x^2 + 2\theta x - by \\ \dot{z} = -x - cz. \end{cases} \tag{17}$$

Theorem 3 When $k_{ii}(i=1,2)$ are positive, the equilibrium $p_1(-2\theta, 0, 2\theta/c)$ of system (16) is asymptotically stability when $a=0.9, b=0.2, c=1.2$.

Proof: The Jacobi matrix of system (17) is

$$\begin{bmatrix} 1/c-k_1-k_2 & -\theta & 1 \\ 2\theta & -b & 0 \\ -1 & 0 & -c \end{bmatrix}. \tag{18}$$

The characteristic equation of matrix (18) is

$$\lambda^3 + B_1\lambda^2 + B_2\lambda + B_3 = 0,$$

where

$$B_1 = k_1 + k_2 + k_3 + b + c - 1/c,$$
$$B_2 = bc - b/c + bk_1 + bk_2 + ck_2 + 2\theta^2,$$
$$B_3 = bck_2 + 2c\theta^2.$$

According to Routh-Hurwitz criterion, the real parts λ are negative if and only if $B_1 > 0, B_3 > 0, B_1B_2 > B_3$, $k_i\,(i = 1, 2) > 0$.

When $a = 0.9, b = 0.2, c = 1.2$ and $k_1, k_2 > 0$, the controlled chaotic system (16) is asymptotically stable at equilibrium point $P_1(-2\theta, 0, 2\theta/c)$.

5 Numerical Results

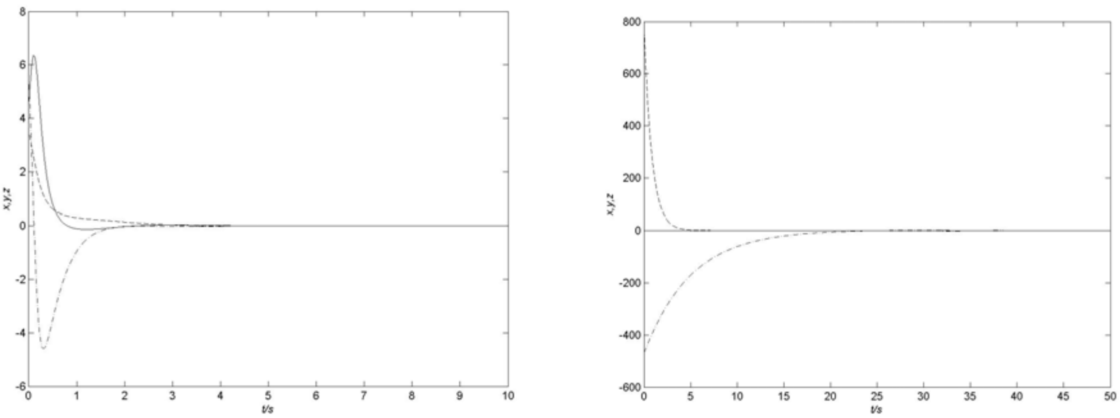
To verify the effectiveness and feasibility of the control approach, by using Matlab program, the numerical simulations have been completed. In the simulations, we choose the parameters a=0.9, b=0.2, c=1.2.

5.1 Linear Feedback Control

The initial states of the controlled system (5) are selected as (4.5, 5.5, 3.5), and the corresponding feedback coefficients are given by k_1=1.5, k_2=2.5, the behaviors of the states (x, y, z) of the controlled chaotic system (5) with time are displayed in Fig.2 (a).

5.2 Speed Feedback Control

The initial states of the controlled system (8) are selected as $(900, 500, 900)$, and the corresponding coefficient is given by $k_1 = 1.5$, the behaviors of the states (x, y, z) of the controlled chaotic system (5) with time are displayed in Fig.2 (b).



(a) (b)

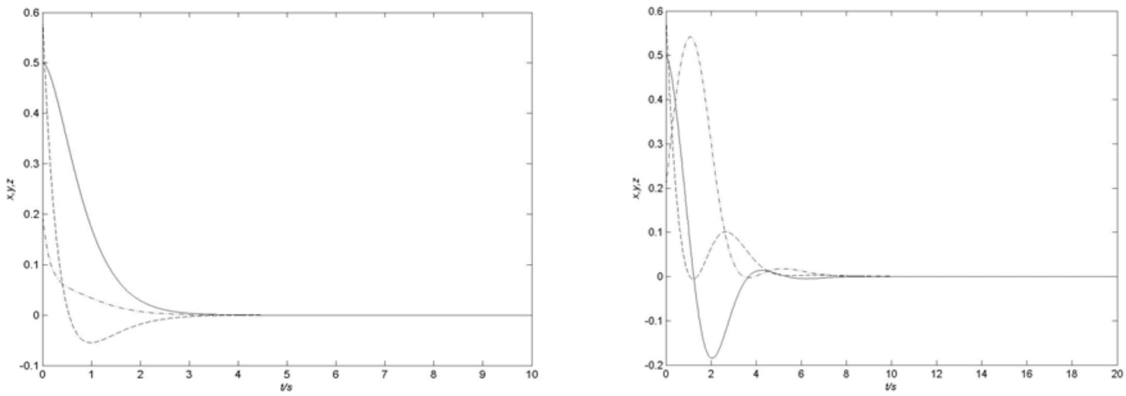
Figure 2: The behaviors of the states (x, y, z) of the controlled chaotic system (5)

5.3 The Selection of Gain Matrix Control

The initial states of the controlled system (13) are selected as $(0.5, 0.2, 0.6)$, and the corresponding feedback coefficient is given by $k_1 = k_3 = 2, k_2 = 6.75$, the behaviors of the states (x, y, z) of the controlled chaotic system (13) with time are displayed in Fig.3(a).

5.4 The Revision of Gain Matrix Control

The initial states of the controlled system (17) are selected as $(2, 5.5, 2)$, and the corresponding feedback coefficient are given by $k_1 = 0.5, k_2 = 1.5$,the behaviors of the states (x, y, z) of the controlled chaotic system (17) with time are displayed in Fig.3(b).



(a) (b)

Figure 3: The behaviors of the states (x, y, z) of the controlled chaotic system

6 Conclusion

The control problem of a chaotic finance system is investigated, the selection of gain matrix control and its revision of gain matrix control are used to suppress chaos to unstable equilibrium point $P_1(-2\theta, 0, 2\theta / c)$,while speed feedback control and linear feedback control are used to suppress chaos to unstable equilibrium point $P_0(0, 0, 0)$ .The Routh-Hurwitz criterion is used to study the conditions of the asymptotic stability of the controlled chaotic system. Furthermore, Numerical simulations are presented to verify the effectiveness of the proposed controllers.

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References

[1] Cai, G.L., and J.J. Huang, A new finance chaotic attractor, *International Journal of Nonlinear Science*, vol.3, no.3, pp.213–220, 2007.

[2] Cai, G.L., and Z.M. Tan, Chaos synchronization of a new chaotic system via nonlinear control, *Journal of Uncertain Systems*, vol.1, no.3, pp.235–240, 2007.

[3] Cai, G.L., Z.M. Tan, W.H. Zhou, and W.T. Tu, Dynamical analysis of a new chaotic system and its chaotic control, *Acta Physica Sinica*, vol. 56, no.11, pp. 6230–6237, 2007.

[4] Ding, J., and H.X. Yao, Chaos control of a kind of non-linear finance system, *Journal of Jiangsu University* (Natural Science Edition), vol.25, no.6, pp.500–504, 2004.

[5] Jia, Q., Adaptive control and synchronization of a new hyperchaotic system with unknown parameters, *Physics Letter A*, vol. 362, pp.424–429, 2007.

[6] Liu, L., S.Y. Chen, and C.X. Liu, Experimental confirmation of a new reversed butterfly-shaped attractor, *Chinese Physics*, vol.16, pp.1897–1900, 2007.

[7] Ma, J.H., and Y.S. Chen, Study for the bifurcation topological structure and the global complicated character of a kind of non-linear finance system (I), *Applied Mathematics and Mechanics*, vol.22, no.3, pp.1119–1128, 2001.

[8] Ma, J.H., and Y.S.Chen, Study for the bifurcation topological structure and the global complicated character of a kind of non-linear finance system (II), *Applied Mathematics and Mechanics*, vol.22, no.12, pp.1236–1242, 2001.

[9] Yang, M.Z., B. Cai, and G.L. Cai, Projective synchronization of a modified three-dimensional chaotic finance system, *International Journal of Nonlinear Science*, vol.10, no,1, pp.32–38, 2010.

[10] Yu, H.J., G.L. Cai, and Y.X. Li, Dynamic analysis and control of a new hyperchaotic finance system, *Nonlinear Dynamics*, DOI: 10.1007/s11071-011-0137-9, 2011. (In Press)