

## Transmission Line Maintenance Scheduling Considering both Randomness and Fuzziness

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Received 10 August 2009; Revised 1 March 2011

#### Abstract

The failure possibility of transmission lines in power system may change from time to time since it will be affected by weather, environment and operating conditions, especially under short-time period. Impacts of various conditions on failure possibility can be described using fuzzy words. In this paper, credibility theory was introduced to model the short-term line maintenance-scheduling problem in power system combining randomness and fuzziness. Benders decomposition was proposed to solve proposed new two-fold random fuzzy model, by breaking the primitive two-fold uncertainty model into a master problem and several sub-problems. The sub-problem is developed as a random fuzzy expected value problem. Credibility theory and DC load flow are adopted to solve the sub-problem. The master problem, which is formulated as a deterministic multi-objective integer-programming problem, is solved by an improved Balas implicit enumeration method considering the characters of transmission line maintenance. The test results on IEEE-RBTS and IEEE-RTS demonstrate the feasibility of proposed method which could accommodate system's security and economy objectives.

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**Keywords:** power system, transmission line maintenance scheduling, credibility theory, random fuzzy programming, generalized Benders decomposition

### ♦ NOMENCLATURE

Θ	Nonempty set
$P(\Theta)$	Power set of $\Theta$
Pos	Possibility measure of fuzzy event
$(\Theta, P(\Theta), Pos)$	Possibility space
$\xi_{con}$	Random fuzzy variable representing uncertainty of component condition-dependent outage
R	Set of real numbers
Cr	Credibility measure of fuzzy event
Pr	Probability measure of random event
$E_{\it pro-fuz}$	Expected value of random fuzzy variable
$E_{_{pro}}$	Expected value of random variable
$X_{kt}$	Transmission line $k$ maintenance state in period $t$ , 0 represents generator $k$ is being maintained
	in period t, 1 represents otherwise
$e_k$	Earliest time for maintenance of transmission line $k$ to begin
$l_k$	Latest time for maintenance of transmission line k to begin
$d_k$	Maintenance duration of transmission line k
r <sub>ij</sub>	Resource $j$ needed for maintenance by transmission line $i$

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$\beta_{jt}$	Resource available for line maintenance resource $j$
$B_r$ $ENS_{pro-fuz}$	Random fuzzy power energy not supplied
$P_{NG^{*1}}$	Active power of generators injection vector
$D_{NC^{*1}}$	Curtailment of power vector
$C_{NC^{*1}}$	Power load vector
$T_{NL^{*1}}$	Active power of branch vector
${ ilde T}_{ m max}$	Normal limit of active power on transmission line vector
$A_{_{NL^*N}}$	Distribution coefficient of DC power flow model represents the relationship between active
	power on node and branch
NL	Transmission line number
NG	Set of generating nodes
NC	Set of load nodes
$\alpha, \beta, \chi, \lambda, \eta, \pi, \mu$	Multipliers of corresponding constraint

## **1** Introduction

Rapid development changes the structure of power grid, and brings transmission management an urgent requirement in China. Short term transmission line maintenance scheduling problem is to determine the appropriate time for which line should be taken off for preventive maintenance during a dispatcher planning horizon for one or two weeks. It is an important tool to guarantee power system security in power control center. Comparing to research on unit maintenance scheduling, research on transmission line maintenance scheduling has a shorter history. The transmission line is originally considered as a constraint in unit maintenance scheduling [1, 19, 4]. The first attempt to find a suitable tool for transmission line maintenance scheduling was made by Marwali and Shahidehpour [13], who proposed an integrated generator and transmission maintenance scheduling considering network constraints. Subsequently, Marwali and Shahidehpour further developed a comprehensive approach for generator and transmission maintenance scheduling constraints [15, 18].

Previous researches only consider the randomness properties of the maintenance problem [14, 16]. Short-term line maintenance scheduling should use proximate available information to predict operational risk and determine maintenance schedule for the coming days. The failure possibility of transmission lines, used in long-term transmission line maintenance scheduling, are average values based on long-term statistic records. Such failure possibility values may be unreasonable for the risk assessment in short-term operations. Failure possibility may change from time to time since weather, environment and operating conditions will affect failure probability of transmission lines under short-time period. For instance, failure possibility could be much higher in adverse weather conditions than usual [2]. The impacts of various conditions on failure possibility can be described using fuzzy words based on experiences of operators (such as most adverse, fairly adverse or less adverse, etc.), which can be modeled using a fuzzy membership function.

The basic concepts of the credibility theory and the prospect of its fist application in power system were introduced in our preliminary work [6, 7]. In this paper, credibility theory [10] is introduced to model short-term line maintenance scheduling combining randomness and fuzziness in Section 2. Consequently, a two-fold uncertainty integer programming model (primitive two-fold uncertainty model) is build up. In Section 3, credibility theory combining Bender's decomposition method is used to break primitive two-fold uncertainty model into master-problem (integer program) and sub-problem (random fuzzy program). Test results are included in Section 4.

## 2 Random Fuzzy Maintenance Scheduling Model

Short-term line maintenance scheduling considering both randomness and fuzziness is a mixed integer two-fold uncertainty optimization problem in applied mathematics. Traditionally, probability theory and fuzzy theory belong to two independent mathematics research areas. Accordingly, stochastic optimization and fuzzy optimization are also usually used separately to study their corresponding uncertainty phenomenon. Accordingly, constrained by the

limitation of mathematic tools, traditional line maintenance scheduling has to take into account only one kind of uncertainty, randomness [15, 14, 16] or fuzziness [5], as shown in Fig.1.



Figure 1: Transmission line maintenance scheduling considering single uncertainty aspect

The first attempt to find a suitable mathematic tool for two-fold uncertainty analysis was made by Kwakernaak [8] in 1978. Credibility theory was proposed as a new branch of mathematics for studying the behavior of fuzzy phenomena by Liu Baoding in 2004, which is based on measure theory. Moreover, probability theory also can be deduced based on measure theory. Thus, two-fold uncertainty combining randomness and fuzziness could be calculated together. As a result, in power system, two-fold uncertainty short-term short term transmission line maintenance scheduling problem can be modeled and solved, as shown in Fig.2.



Figure 2: Transmission line maintenance scheduling considering two-fold uncertainty

This is the method we shall employ here since our main goal is to explore the ways of evaluating two-fold uncertainty in transmission lines maintenance. Weather, environment and operating conditions may affect failure probability of transmission lines. However, the impacts of weather, environment and operating conditions on failure probability are difficult to be quantified. Here, we introduce fuzzy function to describe it. Firstly, we classified transmission lines into several groups according to their failure possibilities [6], such as:

- Most possible failure (a<sub>0</sub>,b<sub>0</sub>,c<sub>0</sub>,...);
- Second most possible failure (a<sub>1</sub>,b<sub>1</sub>,c<sub>1</sub>,...);
- .......
- Normal possibility (a<sub>2</sub>,b<sub>2</sub>,c<sub>2</sub>,...);
- .....
- Second most impossible failure (a<sub>3</sub>,b<sub>3</sub>,c<sub>3</sub>,...);
- Most impossible failure (a<sub>4</sub>,b<sub>4</sub>,c<sub>4</sub>,...);

••••

Here, the a, b, c ...refer to various conditions, including locations, line lengths, weather, environment, and other operation conditions. Each component can be assigned to a specific group based on operators' experiences and the conditions to which it has been exposed. The possibility of system component outages in each group can be modeled using a random fuzzy variable.

Let condition-dependent outage of a transmission line be represented by a random fuzzy variable  $\xi_{con,i}$ , which is a function from the possibility space  $(\Theta, P(\Theta), Pos)$  to the set of random variables.

In order to build up two-fold uncertainty line maintenance scheduling model, following definition is used. **Definition 1** Let  $f: \mathfrak{R}^n \to \mathfrak{R}$  be a measure function, and  $\xi_1, \xi_2, \dots, \xi_n$  random fuzzy variables on the possibility space  $(\Theta_i, P(\Theta_i), Pos_i)$ ,  $i = 1, 2, \dots, n$ . Then  $\xi = f(\xi_1, \xi_2, \dots, \xi_n)$  is a random fuzzy variable, defined as

$$\xi(\theta_1, \theta_2, \cdots, \theta_n) = f(\xi_1(\theta_1), \xi_2(\theta_2), \cdots, \xi_n(\theta_n))$$
(1)

According to Definition 1, power system operation risk is also a random fuzzy variable. Expected value [11, 9, 21, 12] can be used as an important tool to perform random fuzzy optimizing model.

**Definition 2** Let  $\xi$  be a random fuzzy variable. Then the expected value of  $\xi$  is defined by

$$E_{pro-fuz}[\xi] = \int_0^{+\infty} C_r \{\theta \in \Theta \mid E_{pro}[\xi(\theta))] \ge r\} d_r - \int_{-\infty}^0 C_r \{\theta \in \Theta \mid E_{pro}[\xi(\theta)] \le r\} d_r$$
(2)

where

$$E_{pro}[\xi(\theta))] = \int_{0}^{+\infty} P_r \{ E_{pro}(\xi(\theta)) \ge r \} d_r - \int_{-\infty}^{0} P_r \{ E_{pro}(\xi(\theta)) \le r \} d_r.$$

According to (1) $\sim$ (2), primitive model for short-term line maintenance scheduling considering both randomness and fuzziness can be formulated as follows.

Its objective is to minimize maintenance cost and not supplied energy cost due to random fuzzy element outage. Here, hour is used as the unit of time.

$$Min \ E_{pro-fuz} \Big[ \sum_{t} \sum_{k=1}^{N} C_{kt} (1 - x_{kt}) + \sum_{t} \psi_{pro-fuz,t} \Big].$$
(3)

The constraints could be classified into two types: one is the constraints related to the transmission lines maintenance strategy, the other is the constraints related to the random fuzzy operation risk of whole power system.

1) Constraints related to the transmission lines maintenance strategy.

Maintenance time constraints:

$$\begin{cases} x_{kt} = 1 & t < e_k \text{ or } t > l_k + d_k \\ x_{kt} = 0 & S_k \le t \le S_k + d_k \\ x_{kt} \in \{0,1\} & e_k \le t \le l_k \\ \sum_{t} x_{kt} = l_k - e_k - d_k & e_k \le t \le l_k. \end{cases}$$
(4)

Maintenance resource constraints: for transmission lines maintenance resource j:

$$\sum_{t}\sum_{i}r_{ij}(1-x_{kt}) \le \beta_{jt}.$$
(5)

Maintenance crew constraints: for maintenance line r:

$$\sum_{k \in N_R} (1 - x_{kt}) \le b_r .$$
(6)

Transmission lines not to be maintained because of bad climates:

$$x_{kt} = 1 \qquad T_{bad1} \le t \le T_{bad2} \quad . \tag{7}$$

2) Constraints related to the random fuzzy operation risk of whole power system Power not served constraints, where  $\varepsilon_t$  is the maximum limit of random fuzzy:

$$E_{pro-fuz}[ENS_{pro-fuz}(\xi_{FOR,i})] < \varepsilon_t \quad .$$
(8)

Power balance constraints:

$$\sum_{j \in NG} P_i + \sum_{j \in NC} D_j = \sum_{j \in NC} C_j .$$
(9)

DC Load flow model:

$$A(P+D-C) = T \tag{10}$$

$$P_{\min} \le P \le P_{\max} \tag{11}$$

$$T \le \tilde{T}_{\max}$$
 (12)

$$0 \le D \le C \tag{13}$$

## 3 Credibility Theory Based Solution Methodology

### 3.1 Master Problem and Slave Problem Based on Benders Decomposition

Primitive model  $(3) \sim (13)$  is a mixed two-fold uncertainty integer programming model. Benders decomposition method could be used to break original problem into master and slave problem. By this way, the algorithm was changed to perform iterations between sub-problems and the master problem. Sub-problems get a set of maintenance parameters from the mater problem, and then send their results to the master problem. Then the mater problem will combine these new constraints with previous ones to find a more optimal solution. The iteration proceeds until an optimality solution is gotten.

Benders decomposition was first introduced to solve units maintenance scheduling by Yellen and Al-Khamis. Subsequently, Benders decomposition was further developed to solve maintenance scheduling for different circumstance by Silva, Morozowski [19, 4], Shahidehpour [13, 14, 15, 16, 18], EL-Sharkh, El-Keib [5] and et al. However, those issues mainly focus on single uncertainty, such as randomness or fuzziness. Here, we introduce Benders decomposition to solve twofold uncertainty short-term transmission line maintenance scheduling combining both random and fuzziness.

For solving the random fuzzy primitive model in Section II, following Theorem is needed:

**Theorem 1** Assume that  $\xi$  and  $\eta$  are random fuzzy variables from the possibility space  $(\Theta, P(\Theta), Pos)$ . Then for any real numbers *a* and *b*, we have [10, 11, 9]

$$E_{pro-fuz}[a\xi + b\eta] = aE_{pro-fuz}[\xi] + bE_{pro-fuz}[\eta].$$
<sup>(14)</sup>

Based on the above Theorem, primitive problem  $(3) \sim (13)$  is equivalent to following program

$$Min\left[\sum_{t=1}^{T}\sum_{i=1}^{N}C_{kt}(1-x_{kt})\right] + Min\left\{E_{pro-fuz}\left[\sum_{t}\psi_{pro-fuz,t}\right]|(8) \sim (13)\right\}$$

$$s.t.\left\{-(4) \sim (7)\right.$$
(15)

The dual problem o  $r_{step}$  Benders iteration is

$$F_{t}^{r_{step}} = \max_{\substack{\alpha,\beta,\lambda \ P_{pmin} \leq P \leq P_{max} \\ 0 \leq D \leq C}} \min_{\substack{\beta \neq \alpha, \beta \neq S = T_{max} \\ 0 \leq D \leq C}} \left\{ \begin{array}{l} E_{pro-fuz} [\varphi_{t}^{r_{step}}] \left( \sum_{j \in NG} P_{i} + \sum_{j \in NC} D_{j} - \sum_{j \in NC} C_{j} \right) + \\ E_{pro-fuz} [\beta_{t}^{r_{step}}] \left( A(P + D - C) - T \right) + \\ E_{pro-fuz} [\chi_{t}^{r_{step}}] \left( E_{pro-fuz} [ENS_{pro-fuz}^{r_{step}} (\xi_{FOR,i})] - \varepsilon \right) + \\ E_{pro-fuz} [\lambda_{t}^{r_{step}}] (T - \tilde{T}_{max}) \end{array} \right\}.$$
(16)

Hence, primitive problem  $(3) \sim (13)$  become

$$Min \left[\sum_{t=1}^{52} \sum_{i=1}^{N} C_{kt} (1 - x_{kt})\right] + F_{t}^{r_{step}}$$

$$s.t. \left\{ \begin{array}{c} (4) \ \sim \ (7) \end{array} \right.$$
(17)

The sub-problem of  $r_{step}$  Benders iteration is feasible, if and only if following problem's optimal solution is no more than  $\varepsilon_t$ 

$$\begin{array}{ll} \text{Min} \quad E_{pro-fuz}[ENS_{pro-fuz}(\xi_{FOR,i})] \\ \text{s.t.} \{ & (9) \sim (13) \end{array}$$
(18)

Its duel function is

$$G_{t}^{r_{itep}} = \max_{\substack{\eta, \pi, \mu \\ T_{min} \leq P \leq P_{max} \\ T_{min} \leq T \leq T_{max} \\ 0 \leq D \leq C}} \min_{\substack{P \in P_{itz} \leq P_{itz} \\ T_{min} \leq T \leq T_{max} \\ 0 \leq D \leq C}} \left\{ \frac{E_{pro-fitz} [ENS_{pro-fitz} (\xi_{FOR,i})]}{\sum_{j \in NG} P_{i} + \sum_{j \in NC} D_{j} - \sum_{j \in NC} C_{j}} \right\} \\ + E_{pro-fitz} [\pi_{t}^{r_{step}}] (A(P + D - C) - T) \\ + E_{pro-fitz} [\mu_{t}^{r_{step}}] (T - \tilde{T}_{max})$$

$$(19)$$

Then the infeasible Benders cut is formulated as follow:

$$E_{pro-fuz}[ENS_{pro-fuz}(\xi_{FOR,i})] + \sum_{k} E_{pro-fuz}[\mu_{kt}^{r_{step}}] \cdot \tilde{T}_{k,\max} \cdot (x_{kt}^{r} - x_{kt}) \le \varepsilon_{t}$$
(20)

where

$$E_{pro-fuz}[\mu_{kt}^{r_{sep}}] = E_{pro-fuz}[\frac{\partial ENS_{pro-fuz}}{\partial C_{k}^{l}}].$$
<sup>(21)</sup>

The feasible Benders cut is generated as

$$z \ge \sum_{t} \left\{ E_{pro-fuz} [\psi_{pro-fuz,t}] + \sum_{k} [C_{kt} (1 - x_{kt}^{r}) + E_{pro-fuz} [\lambda_{kt}^{r}] \cdot \tilde{T}_{k,\max} \cdot (x_{kt}^{r} - x_{kt})] \right\}$$
(22)

where

$$E_{pro-fuz}[\lambda_{kt}^{r_{step}}] = E_{pro-fuz}[\frac{\partial \psi_{pro-fuz,t}}{\partial C_k^l}].$$
<sup>(23)</sup>

Then, primitive problem (3) $\sim$ (13) become

$$\begin{aligned}
&Min \quad z \\
&st. \\
&\begin{cases} x_{kt} = 1 & t < e_{k} \text{ or } t > l_{k} + d_{k} \\
&x_{kt} = 0 & S_{k} \le t \le S_{k} + d_{k} \\
&x_{kt} \in \{0,1\} & e_{k} \le t \le l_{k} \\
&\sum_{t} x_{kt} = l_{k} - e_{k} - d_{k} & e_{k} \le t \le l_{k} \\
&\sum_{t} \sum_{r} r_{ij}(1 - x_{kt}) \le \beta_{jt} \\
&\sum_{t} \sum_{r} (1 - x_{kt}) \le b_{r} \\
&x_{kt} = 1 & T_{start-no-main,k} \le t \le T_{end-no-main,k} \\
&z \ge \sum_{t} \left\{ E_{pro-fitz} [\Psi_{pro-fitz,t}] + \sum_{k} [C_{kt}(1 - x_{kt}^{r}) + E_{pro-fitz}[\lambda_{kt}^{r}] \cdot \tilde{T}_{k,\max} \cdot (x_{kt}^{r} - x_{kt})] \right\} \\
&E_{pro-fitz} [ENS_{pro-fitz}(\xi_{FOR,i})] + \sum_{k} E_{pro-fitz} [\mu_{kt}^{r_{stop}}] \cdot \tilde{T}_{k,\max} \cdot (x_{kt}^{r} - x_{kt}) \le \varepsilon_{t}.
\end{aligned}$$
(24)

Hence, primitive problem  $(3) \sim (13)$  could be decomposed into master problem and sub-problems. Master problem is a mixed integer program which could be solved by Balas implicit enumeration method. The operation sub-problem is solved by an approach based on credibility theory.

The whole flow chart of short-term transmission line maintenance scheduling is shown in Fig.3.



Figure 3: Flow chart of short-term transmission line maintenance scheduling

# **3.2 Operation Slave Problem Solved by Credibility Theory and Active Power Security Correlation**

 $E_{pro-fuz}[ENS_{pro-fuz}]$  is estimated as follows:

Step 1. Let e = 1.

Step 2. Generate  $\theta_k$  from transmission line outage possibility space and generator and transformers outage probability space.

Step 3. Compute  $E_{pro}[ENS(\theta_k)]$  based on active power rescheduling, as shown in Fig.4.

Step 4. Let  $a = Min_{1 \le k \le N} E_{pro}[ENS(\theta_k)], b = Max_{1 \le k \le N} E_{pro}[ENS(\theta_k)].$ 

Generate  $r \in [a, b]$ ,  $k = 1, 2, \dots, N$ 

If  $r \ge 0$ , then

$$e = e + Cr\{\theta \in \Theta \mid E_{pro}[ENS(\theta_k)] \ge r\}.$$
(25)

If r < 0, then

$$e = e + Cr\{\theta \in \Theta \mid E_{\text{pro}}[ENS(\theta_k)] \le r\}.$$
(26)

Step 5.

$$E_{pro-fuz}[ENS_{pro-fuz}] = a \lor 0 + b \land 0 + e \bullet (b-a) / N .$$
(27)

Likewise, 
$$E_{pro-fuz}\left[\frac{\partial \psi_{pro-fuz,t}}{\partial C_k}\right]$$
,  $E_{pro-fuz,t}\left[\psi_{pro-fuz,t}\right]$  and  $E_{pro-fuz}\left[\frac{\partial ENS_{pro-fuz,t}}{\partial C_k}\right]$  could also be solved through Step

 $1 \sim$  Step 5. Thus, (21) and (23) could be resolved. Then infeasible cut (20) and feasible cut (22) could be calculated to solve extended master problem (24).



Figure 4: Flow diagram for calculating random fuzzy expected value of electric energy not supplied

### 3.3 Master Problem Resolved by Improved Balas Implicit Enumeration Method

The Balas implicit enumeration method is an integer optimal programming method [20]. It will take a long time to find the optimal solution of the master problem by using Balas implicit enumeration method. In this section, an improved Balas method was presented to solve the master problem based on the characteristic of maintenance scheduling.

The operation of maintenance of a transmission line has the characteristic of continuity. In other words, when a transmission line is determined to be maintained, the transmission line will keep its off-state until the maintenance of the transmission line is finished. The duration time for maintenance of each transmission line can be given according to power system's requirement. This character of the transmission line maintenance scheduling was used to modify the forward and backward search method of the Balas algorithm.

An example of a three-weeks maintenance scheduling is used to illustrate the improved Balas method, as shown in Fig.4. We set 0 and 1 represent on and off states of the transmission line, respectively. The maintenance duration of Line A, Line B and Line C was set as 2 weeks.

1) Improvement of forward search method. For example, when we got  $x_1 = 1$  for Line A during forward search, we can directly get  $x_2 = 1$  and  $x_3 = 0$  because Line A has fixed maintenance duration of 2 weeks. The other partial solution 101 and 111 can be excluded by the characteristic of transmission line maintenance.

2) Improvement of backward search method. For example, in general Balas algorithm, the trace-back route for Line C arrives at Node 9 first, and the forward searching process will continue to detect the other feasible branch. If we make use of the characteristic of unit maintenance, the trace-back route 1 will go straight to Node 7 to search the other feasible branch.

As shown in Fig.5, our improved method needs to detect only 13 branches instead of all 18 branches needed in general Balas searching method. As a result, the efficiency of optimization process is improved.



Figure 5: Modified forward detection and backup trace part of implicit enumeration method

## 4 Test Cases

Based on the described method, a FORTRAN software was developed. IEEE-RBTS [1] and IEEE-RTS [17] are uesed to illustrate its performance.

### 4.1 IEEE-RBTS Test

IEEE-RBTS [1] which is a smaller scale system is used to analysis the random fuzzy operation risk under different cases. The network of IEEE-RBTS is depicted in Fig.6.



Figure 6: IEEE-RBTS diagram

The cost of power energy not supplied is set as 4\$/kWh, and  $\varepsilon_t$  is 5% of load for each hour. Maintenance cost will increase 10\$ per hour while line maintenance is brought forward or delayed,. The maintenance schedule of IEEE-RBTS on Tuesday of the 51<sup>st</sup> week, whose day peak load is also peak load of the year, is studied.

(1) Influence of different random fuzzy function

According to credibility theory, fuzziness of random fuzzy variable  $\xi_{FOR}^{Line}$  could be represented by different types of functions. Accordingly, the fuzziness of random fuzzy variables of our proposed algorithm could also use different types of fuzziness definition. 3 cases are used to study the influence of different random fuzzy function on the performance of our algorithms.

We classified the possibility of transmission lines outage into 4 levels, as shown in Table 1.

Case 1) the fuzzy form of transmission lines outage is simulated by a trapezoidal fuzzy variable;

Case 2) the fuzzy form of transmission lines outage is simulated by a triangle fuzzy variable;

Case 3) the fuzzy form of transmission lines outage is simulated by a rectangle fuzzy variable.

	Transmission line number	Case 1	Case 2	Case 3
1 <sup>st</sup> level	4,5,8,9	(0.0010,0.0015,0.0020,0.0025)	(0.0010, 0.0020, 0.0025)	(0.0010, 0.0025)
2 <sup>nd</sup> level	1,6	(0.0015,0.0020,0.0025,0.0030)	(0.0015, 0.0025, 0.0030)	(0.0015, 0.0030)
3 <sup>rd</sup> level	2,7	(0.0050,0.0055,0.0060,0.0065)	(0.0050, 0.0060,0.0065)	(0.0050, 0.0065)
4 <sup>th</sup> level	3	(0.0040,0.0045,0.0050,0.0060)	(0.0040, 0.0050,0.0060)	(0.0040, 0.0060)

Table1: Various representation of random fuzzy variable to represent failure possibility

The calculated maintenance schedule using different representation of random fuzzy variable to describe failure possibility is shown in Table 2. The highlighted items show the transmission lines whose planned schedule need to be modified. The acquired risk and economy index using different representation of random fuzzy variable is given in Table 3.

Table 2: Maintenance schedules by using different representation of random fuzzy variable

Transmission	Planned maintenance	Adjusted maintenance schedule
line number	schedule (hour)	(hour)
	Case 1 - Case 3	Case 1 - Case 3
1	$10$ th $\sim$ $13$ th	8th~11th
2	10th~13th	10th~13th
3	10th~13th	10th~13th

Table 3: Risk and economy indexes by using different representation of random fuzzy failure possibility

	Case 1	Case 2	Case 3
energy not supplied (kWh)	16788.27	16839.31	16790.96
cost of energy not supplied (\$)	67153.09	67357.25	67163.80
Maintenance cost (\$)	60030.00	60030.00	60030.00
Total cost (\$)	127183.09	127387.25	127193.80

Table 2 indicates that there is no significant difference between the transmission line maintenance schedules calculated by different fuzzy membership form. But the values of risk and economy indices calculated by different fuzzy membership form are different, as shown in Table 3.

(2) Influence of different random fuzzy power not served maximum limit

Ten cases (Case1- Case10) are used to study the influence of maximum limit of random fuzzy power not served  $\varepsilon_t$  on maintenance. In case 1-10,  $\varepsilon_t$  is set as 1%, 2% ...10% of load for each hour respectively. Maintenance

schedules calculated using different  $\varepsilon_t$  are shown in Table 4, and risk and economy index calculated using different  $\varepsilon_t$  are shown in Table 5. The highlighted items show the transmission lines whose planned schedule need to be modified.

Transmissi	Planned maintenance	Adjusted maintenance schedule (hour)		
on line number	schedule (hour)	Case1-3	Case 4	Case 5~-Case 10
1	10th~13th	$2nd$ $\sim$ $5th$	$7$ th $\sim$ 10th	$8$ th $\sim$ 11th
2	10th~13th	$10$ th $\sim$ $13$ th	$10$ th $\sim$ $13$ th	10th~13th
3	10th~13th	$10$ th $\sim$ 13th	10th~13th	10th~13th

Table 4: Maintenance schedules using different  $\varepsilon_{t}$ 

	Planned	Adjusted maintenance schedule		
	maintenance schedule	Case1-3	Case 4	Case 5~-Case 10
energy not supplied (kWh)	31190.44	6450.39	10245.62	16788.27
cost of energy not supplied (\$)	124761.70	25801.55	40982.50	67153.09
Maintenance cost (\$)	60000.00	60260.00	60060.00	60030.00
Total cost (\$)	184761.73	86061.55	101042.50	127183.09

Table 5: Risk and economy indexes using different  $\varepsilon_{t}$ 

From Table 5, we can see that with the increase of maximum limit of random fuzzy power not served  $\varepsilon_{t}$ , energy not supplied increase. Consequently, the cost of energy not supplied and maintenance cost also increase.

(3) Influence of piece price of power energy not supplied and maintenance schedule adjusting

The above ten cases were used to study the influence of change of piece price of power energy not supplied and maintenance schedule adjusting. The piece price of power energy not supplied is changed from 4 \$/kWh to 0.4 \$/kWh, and maintenance schedule adjusting cost is changed from 10\$ per hour to 200\$ per hour, if line maintenance is brought forward or delayed. The calculated maintenance schedule is shown in Table 6, and the risk and economy indices are shown in Table 7.

Tronomissi	Planned	Adjusted maintenance schedule (hour)			
on line number	maintenance schedule (hour)	Case1-3	Case 4	Case 5- 10	
1	$10$ th $\sim$ $13$ th	$2nd{\sim}5th$	$3rd\sim 6th$	$8$ th $\sim$ 11th	
2	10th~13th	$10$ th $\sim$ $13$ th	10th~13th	10th~13th	
3	10th~13th	$10$ th $\sim$ 13th	$10$ th $\sim$ 13th	10th~13th	

Table 6: Maintenance schedules calculated using different  $\varepsilon_{t}$ 

Table 7: Risk and economy calculated using different  $\varepsilon_{t}$ 

	Planned	Ad	justed mainten	ance schedule
	maintenance schedule	Case1-3	Case 4	Case 5- 10
energy not supplied (kWh)	31190.44	6450.39	6431.37	16788.27
Total cost (\$)	72476.17	67780.16	66972.55	67315.31

From Table 5, we can see that the decrease of maintenance cost variation after adjusting is smaller than the increase of the variation of energy not served cost. Thus, total cost will increase with the increase of  $\varepsilon_{t}$ . From Table 7, we can see that after changing the piece price of power energy not supplied and maintenance schedule adjusting, the change trend of total cost shows different. With the increase of  $\varepsilon_{t}$ , it firstly shows decrease, and then increase.

Furthermore, from Table 5 and Table 7, we can see that the energy not supplied decrease after coordination by control center. Accordingly, the security level of the power system also increases. By this way, the operation control center could control the secure level.

### 4.2 IEEE-RTS Test

The network of IEEE-RTS [17] which is a larger scale system is depicted in Fig.7. The performance of our proposed method is further studied using a maintenance schedule of IEEE-RTS for a whole week, as shown in Table 8. The calculated risk and economy indices are shown in Table 9. Here, the cost of power energy not supplied is set as 5/kWh,  $\varepsilon_{t}$  is 0.5% of load for each hour.



Figure 1: IEEE-RTS diagram

Table 8: Maintenance schedules of IEEE-RTS

Transmission line number	Planned maintenance schedule (hour)	Adjusted maintenance schedule(hour)
2	$25 { m th}{\sim}72 { m nd}$	25th~72nd
3	$37 \mathrm{th}{\sim}84 \mathrm{th}$	94th~141st
4	$73 \mathrm{rd}{\sim} 120 \mathrm{th}$	106th~153rd
5	85th~132nd	89th~136th
6	$97$ th $\sim$ 144th	99th~146th

	Planned	Adjusted
	maintenance schedule	maintenance schedule
energy not supplied (kWh)	76987.05	49696.59
cost of energy not supplied (\$)	384935.1	248483
Maintenance cost (\$)	1200000	1221820
Total cost (\$)	1584935.1	1470303

Table 9: Risk and economy indexes of IEEE-RTS

As shown in Table 9, after adjusting by power control center, energy not supplied and total cost reduced 35.45% and 7%, respectively. It shows the coordination by control center can improve the security and economical level of power operation.

## 5 Conclusion

A new approach of short-term transmission line maintenance scheduling based on credibility theory is proposed in this paper. This new method can combine two-fold random fuzzy uncertainty in line maintenance scheduling. Furthermore, Bender decomposition was introduced to solve our new proposed algorithm considering both randomness and fuzziness. The introduction of credibility theory to solve the line maintenance-scheduling problem considering both randomness and fuzziness is a new meaningful try in the research field of power system.

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