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Fuzzy Availability Computation of Semi-Markovian Mechatronic System Using Hybrid Approach

Rajesh Prabhu Gaonkar*, Uday Amonkar, Milind Sakhardande, Vivek Kamat
Goa College of Engineering, Farmagudi, Ponda, Goa - 403401, INDIA

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Abstract

This paper presents availability computation procedure of a semi-Markovian mechatronic system under the uncertainties that involve randomness as well as fuzziness. Conversion of semi-Markov to Markov model is stated first and hybrid approach is then used for fuzzy availability computation. Sometimes statistical data is available about failure and repair rates, but in cases where such data is scarce or not available, one has to rely on subjective information or judgment provided by the expert. As a result, the problem in which some data is probabilistic and some is of fuzzy nature is resolved in this paper using the hybrid approach.

Keywords: semi-Markov model, fuzzy availability, triangular fuzzy number (TFN), hybrid approach

1 Introduction and Literature Investigation

Availability analysis is an important research topic in engineering and lots of investigations are ongoing worldwide in this area. In the real practical situations, the data required for availability analysis sometimes cannot be recorded or collected precisely due to human errors, machine errors, or some other unexpected situations. Availability calculation mainly necessitates failure and repair rate data. Lots of uncertainty is involved in the precise capturing of this data. Such data may sometimes be available and can be modeled statistically using probability theory. But sometimes it may be vague, ambiguous, imprecise, incomplete, fuzzy or in the form of linguistic terms and variables. In most of the cases, some of the data is readily available and some data is in the form of fuzziness. This paper attempts to demonstrate the availability calculation procedure when failure and repair rate data available with the analyst is of different nature, i.e., probabilistic and possibilistic (fuzzy).

Verma et al. [7] presented two approaches to model fuzzy availability of a deteriorating system. This paper uses fuzzy numbers and interval of confidence to include uncertainties in the transition rates of the Markov model. First approach, assumes five linguistic variables with triangular membership functions allocated to all failure and repair rates. Second approach, assumes transitions from one state to another state using fuzzy variables. Verma et al. [8] proposed the semi-Markovian approach for availability modeling of a deteriorating system under fuzziness. The semi-Markov model is converted to Markov model and fuzzy availability is found out assuming linguistic variables with failure and repair rates as triangular membership functions. Further, Verma et al. and Prabhu Gaonkar et al. also demonstrated procedure to compute fuzzy availability using vertex method [9] and fuzzy simulation [5].

This paper initially focuses on the uncertainty issues in availability computation. Fuzzy sets and triangular fuzzy number concept is briefly explained next. Four state semi-Markov model for mechatronic system and its conversion to Markov model from the literature is stated in Section 4 and availability expression is presented. Concept of hybrid approach is introduced in Section 5. Subsequent section discuss about fuzzy availability computation for two cases and discussion of results. The two cases of hybrid nature are: Case I: failure rates fuzzy and repair rates probabilistic and Case II: failure rates probabilistic and repair rates fuzzy. The paper ends with conclusion in Section 7.

2 Addressing Uncertainty

Some of the most well established uncertainty theories include various probability theories [1] (e.g., Kolmogoroff's probability theory), evidence theory (Dempster, Shafer), possibility theory (Zadeh), fuzzy set theory (Zadeh), rough

* Corresponding author. Email: rpg@gec.ac.in (R. Prabhu Gaonkar).

set theory (Pawlak) and interval arithmetic. Additionally, combinations such as fuzzy-probability theory are widely investigated. The most important causes of uncertainty in engineering problems are lack of information and ambiguity.

In availability computation, major hurdle is to handle and estimate model parameters that are uncertain in nature. In general, parameter uncertainty is: (i) due to randomness, i.e., due to variability phenomena or the fact that all the factors affecting the system can not be modeled. (ii) incompleteness or vagueness, i.e., information regarding the values of the model parameters are lacking. Probability theory deals with randomness, whereas possibility theory deals with the vagueness. To tackle randomness, the most commonly used approach is to collect the data and carry out statistical analysis. Probability theory accomplishes this work well. In case, statistical data is lacking or information about model parameters is incomplete, expert human judgment is the lone way to get the required information. These expert opinions have been used to build up probability distributions using Bayesian probability theory or Bayesian framework. Some researchers [2] have questioned some aspects of this theory. Subjective information of experts may have deficiencies like ambiguity, vagueness, imprecision, etc. Such types of uncertainties have been handled well by a fuzzy set theory.

3 Fuzzy Set Theory and Triangular Fuzzy Number

Zadeh advocated the concept of grades of membership or the concept of possibility values of membership [3, 4]. If $X = \{x\}$ represents a fundamental set and x are the elements of this fundamental set, to be assessed according to an uncertain postulation and assigned to a subset A of X, the set $A = \{x, \mu_A(x) / x \in X\}$ is referred to as the uncertain set or fuzzy set of X. $\mu_A(x)$ is the membership function of the uncertain set A. The membership function $\mu_A(x)$ for a fuzzy set A can be defined as: $\mu_A(x) : X \to [0,1]$. If the membership function of fuzzy number A is given by:

$$\mu_A(x) = \begin{cases} 0, & x < L \\ \dfrac{(x-L)}{(M-L)}, & L \le x \le M \\ \dfrac{(R-x)}{(R-M)}, & M \le x \le R \\ 0, & x > R, \end{cases} \tag{1}$$

then A is referred to as triangular fuzzy number, denoted by $A = (L, M, R)$ and is depicted in Figure 1. α - cut of this TFN is

$$A_\alpha = [L + \alpha\,(M - L),\ R - \alpha\,(R - M)] \qquad \forall \alpha \in [0,1]. \tag{2}$$

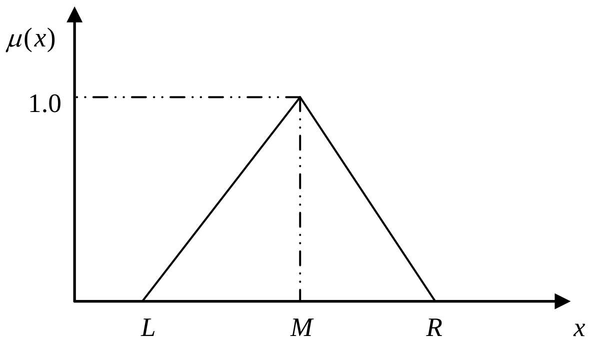

Figure 1: Triangular fuzzy number (TFN)

4 Model Description and Availability Expression

Markov Analysis is a powerful modeling and analysis technique with strong applications in the time-based reliability and availability analysis. It is a method of analyzing repairable systems with constant failure and repair rates. Systems are described by state transition diagrams and may be used to model systems that exhibit strong dependencies. The reliability behavior of a system is represented using a state-transition diagram, which consists of a set of discrete states that the system can be in, and defines the rate at which transitions between those states take place. Markov states represent all possible conditions the system can exist in. The system can only ever be in one state at a time. A single state must be set up as the initial starting state. Transition rates represent the rate at which the Markov diagram moves from one state to another. For example, the transition rate from a working state to a failed state is represented by the failure rate whereas the transition from a failed state to working state is represented by the repair rate. As such, Markov models consist of comprehensive representations of possible chains of events, i.e., transitions, within systems, which in the case of reliability and availability analysis correspond to sequences of failures and repair. The underlying assumption here is, the probability that a system will undergo transition from one state to another state depends only on the current state of the system and not on any previous states the system has experienced. The transition probability is not dependent on the past state or history of the system. This is same as memoryless property of exponential distribution. Therefore exponential failure or repair times satisfy the Markovian property. Assumption of exponential distribution is valid for the failure events of many engineering problems, mainly those in which all components are in useful life period. The real problem arises when it comes to repair process, whether exponential distribution is valid. If repair times are not exponentially distributed the additional techniques are required to evaluate the time dependent values.

One technique to tackle the problem of system with non-exponential distribution (non-Markovian or semi-Markovian models or problems) is method, commonly known as 'device of stages' [8, 9, 5]. The method is based on the logic or conclusion that if repair time is not exponentially distributed, then the state, from which that particular transition is commencing, can be divided into a number of sub-states with each transition exponentially distributed. The essence of the problem is to deduce the number of sub-states, the way they are connected and their numerical parameters in order to represent the state being considered. This process of dividing a system into sub-states (referred as 'stage') is known as 'method of stages'. When number of states is combined in series, it is known as 'series method' or when combined in parallel, it is known as 'parallel method'.

Most of the systems nowadays are mechatronic systems. Mechatronic system is the one that have mechanical and electronic components. A mechatronic system may fail due to two types of failures i.e. degradation failure (after it becomes critical) or shock which is a sudden failure. These two types of failure mechanisms are considered in development of a semi-Markov model in the literature [8, 9, 5]. Degradation failure is a failure mechanism that evolves over time and will typically develop to a critical failure over time. E.g. wear, vibration, cumulative damage, noise, leakage, contamination, mechanical defect, material deterioration and failure etc. Degradation is considered in two categories namely critical and non-critical. Critical failure is one that causes complete loss of the functional capability of the system. In non-critical degradation, system functions at a lower functional level. Shock is a sudden failure mechanism that is not dependent on time. E.g. sudden failures like electrical, instrument failure, control failures, no or faulty indicators and alarms, etc. Shock mechanism is considered as critical failure as it causes immediate loss of function.

For the system under description, it is assumed that all the repair times follow special erlangian distribution (non-Markovian) and all the failure times follow exponential distribution (failure rates constants). These two assumptions compose a semi-Markov model as stated in literature [8, 9, 5]. This model is shown in Figure 2.

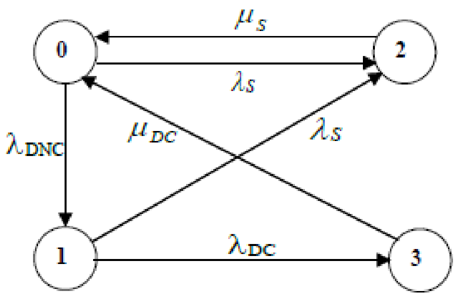

Figure 2: Semi-Markov model

- State 0 : Fully operating state
- State 1 : Degradation non-critical state
- State 2 : Shock critical state
- State 3 : Degradation critical state
- λ_{DNC} : Degradation non-critical failure rate
- λ_{DC} : Degradation critical failure rate
- λ_S : Shock failure rate
- μ_S : Shock repair rate
- μ_{DC} : Degradation critical repair rate

As mentioned in earlier paragraphs, the semi-Markov problem is conventionally solved using device of stages technique. Stages in series method have been considered in this paper to solve the model/problem.

Let 'η' and 'ρ' be the series stage device parameters. These parameters represent 'number of stages' and the 'repair rate' respectively. The standard equations cited in the literature [8, 9, 5] for 'η' and 'ρ' are as follows:

$$\text{Number of stages: } \eta = \frac{M_1^2}{M_2 - M_1^2}, \tag{3}$$

$$\text{Repair rate: } \rho = \frac{M_1}{M_2 - M_1^2} \tag{4}$$

where M_1 and M_2 are the first two moments of the distribution being modeled. We consider erlangian distribution for the repair times. The first moment (M_1) and the second moment (M_2) of erlangian distribution is as follows:

$$\text{First moment: } M_1 = \mu , \tag{5}$$

$$\text{Second moment: } M_2 = \mu^2 + \sigma^2 \tag{6}$$

where μ is the 'mean' or the 'average value' and σ^2 is the 'variance'. Variance is defined as the average value of the quantity 'square of the (distance from mean)'. This average is taken over the whole distribution. The 'standard deviation (σ)' is defined as 'square root of (the variance)'.

For an illustration, we consider that repair time from state 2 to state 0 follow a special erlangian distribution with mean value (μ) = 21 hours and standard deviation (σ) = 12 hours. Substituting these values in equation (5) and (6), we obtain M_1 = 12 and M_2 = 585. Putting these values in equation (3) and (4), we obtain the number of stages $(\eta) \approx 3$ and repair rate (ρ_S) = 0.145833 repairs/hour $\approx$ 1280 repairs/ year. Similarly, considering suitable values of mean (μ) and standard deviation (σ) as 20 and 13 hours respectively for the repair time from state 3 to state 0, we obtain number of stages $(\eta) \approx 2$ and repair rate (ρ_{DC}) $\approx$ 1040 repairs/year. Using this derived information, semi-Markov model is converted into a Markov model. This model now has in total seven states. State two is broken into three states and state 3 into two states. All the transition rates (failure as well repair rates), at present are constants, that satisfies Markovian property as explained in the beginning of this section. The Markov model thus obtained is shown in Figure 3.

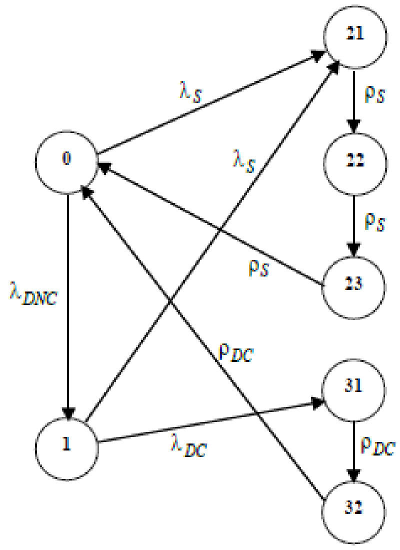

Figure 3: Converted Markov model

Stochastic transitional matrix (T) of the Markov model depicted in Figure 3 is

$$T = \begin{bmatrix} -(\lambda_{DNC}+\lambda_S) & \lambda_{DNC} & \lambda_S & 0 & 0 & 0 & 0 \\ 0 & -(\lambda_{DC}+\lambda_S) & \lambda_S & 0 & 0 & \lambda_{DC} & 0 \\ 0 & 0 & -\rho_S & \rho_S & 0 & 0 & 0 \\ 0 & 0 & 0 & -\rho_S & \rho_S & 0 & 0 \\ \rho_S & 0 & 0 & 0 & -\rho_S & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\rho_{DC} & \rho_{DC} \\ \rho_{DC} & 0 & 0 & 0 & 0 & 0 & -\rho_{DC} \end{bmatrix} . \tag{7}$$

The solution of the model can be obtained by solving following equations:

$$[P_0 \quad P_1 \quad P_{21} \quad P_{22} \quad P_{23} \quad P_{31} \quad P_{32}] \cdot T = 0, \tag{8}$$

$$\sum_{for\ all\ i} P_i = 1 \tag{9}$$

where P_i is probability of being in state i (i = 0, 1, 21, 22, 23, 31, 32).

State 0 and state 1 are operating states and remaining states are failed states. Availability of this Markov model is given by equations:

$$A(t) = P_0 + P_1, \tag{10}$$

$$P_0 = \left\{ 1 + \frac{\lambda_{DNC}}{\lambda_S + \lambda_{DC}} + 3 \cdot \frac{\lambda_S}{\rho_S} \cdot \left(\frac{\lambda_S + \lambda_{DC} + \lambda_{DNC}}{\lambda_S + \lambda_{DC}} \right) + 2 \cdot \frac{\lambda_{DNC} \cdot \lambda_{DC}}{\rho_{DC} \cdot \lambda_{DC} \cdot (\lambda_S + \lambda_{DC})} \right\}^{-1}, \tag{11}$$

$$P_1 = \left(\frac{\lambda_{DNC}}{\lambda_S + \lambda_{DC}} \right) \cdot P_0. \tag{12}$$

5 Hybrid Approach

The representation of variability by a probability distribution function rather than fuzzy numbers, even when faced with lack of information may lead to un-conservative estimation of the availability [2]. It is rather appropriate to consider model parameter uncertainty in terms of possibilities rather than probabilities when sufficient information does not exist. If a given model involves some parameters that are justifiably represented by probability density function (in particular, sufficient data exists to substantiate these probability density functions), while others are considered to be more adequately represented by fuzzy numbers, a method has been derived to combine these two modes of representation of uncertainty in the estimation of availability. As the name suggests, hybrid approach combines probabilistic and fuzzy variables. It is very much useful and most practical where some variables are represented by probability distribution functions and some by fuzzy variables. The steps in hybrid approach [2, 6, 10] are as follows:

Step 1: Let $P_1, P_2, \ldots, P_n$ are the 'n' model parameters, each being represented by a probability density functions. Get 2^n vertices for the 'n' probability density functions as per Rosenblueth's Point Estimate Method (RPEM) [6] as shown in Figure 4. The end points will be: $P_{1-}, P_{1+}, \ldots, P_{n-}, P_{n+}$.

Step 2: Let $F_1, F_2, \ldots, F_m$ are 'm' model parameters each represented by a fuzzy number. Select a α value of the membership function as shown in Figure 5. The end points of the α cut of fuzzy variables are selected as: $F_{1-}, F_{1+}, \ldots, F_{m-}, F_{m+}$.

The various combinations of end points constitute different extreme scenarios of the fuzzy variables there by giving 2^m combinations where 'm' is the number of fuzzy variables.

Step 3: Calculate Availability (A) at each α level by considering 2^n vertices for probability density functions and each of the 2^m combinations of fuzzy variables (2^m reliability indices at each α level):

$$A = f(P_1, P_2, \ldots, P_n, F_1, F_2, \ldots, F_m). \tag{13}$$

Step 4: Repeat this procedure at all α levels.

Step 5: Build fuzzy availability by selecting the inferior and superior value of availability values at each α level.

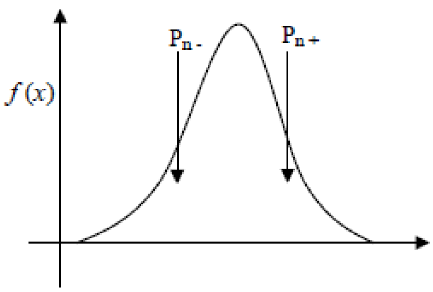

Figure 4: Probability density function of P_n

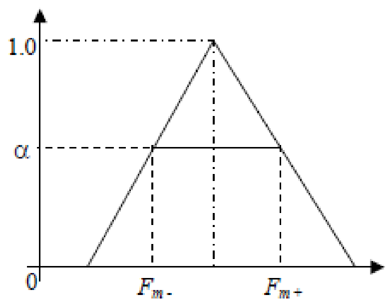

Figure 5: Fuzzy number F_m

The result of combining probability density functions with the fuzzy numbers is a random fuzzy number. The hybrid approach takes advantage of the rich information provided by probability density functions, but retains the conservative character of fuzzy calculus to account for those parameters for which a representation by unique probability density functions is not justified by the available data [2, 6].

6 Fuzzy Availability Computation, Results and Discussion

As the intent of this paper is to compute availability with uncertainties that involve randomness as well as fuzziness, two cases are formulated in this section. Both the cases take care of the uncertainties of both types. Fuzzy availability is computed using the availability expression obtained in Section 4. Equations (10), (11) and (12) have been used to compute fuzzy-random availability. While computing fuzzy-random availability, arithmetic's and Monte Carlo simulation is employed.

Case I - Failure Rates Fuzzy and Repair Rates Probabilistic:

In this case, failure rates are assumed as triangular fuzzy numbers and repair rates as uniformly distributed (UD). Hybrid approach methodology (explained earlier) is used in this case. As we have three failure rates, which are in TFN form, the various combinations of end points of the fuzzy numbers will give $2^3 = 8$ combinations. As repair rates are uniformly distributed, they are obtained using Monte Carlo simulation. At each α level, lower and upper availability is computed for all 8 combinations and the ones with least value and highest value amongst them are taken as lower and upper bounds of availability at that α level. Final availability value is a fuzzy random number.

Case II - Failure Rates Probabilistic and Repair Rates Fuzzy:

Now, failure rates are uniformly distributed and repair rates are considered as triangular fuzzy numbers. Here also, availability computation is done using hybrid approach. Various combinations of end points of fuzzy repair rates are $2^2 = 4$ and failure rates are determined using Monte Carlo simulation. Using similar procedure as explained in earlier case, lower and upper bound of availability is calculated at each α level.

The input data for both the cases is given in Table 1. Lower and upper bounds/values of availability at each α – cut/level are given in Table 2 and plotted in Figure 6.

Table 1: Input data

		Case I	Case II
Failure Rates	Nature	TFN	UD
	λ_{DNC}	[13,14,15]	[13,15]
	λ_{DC}	[35,36,37]	[35,37]
	λ_S	[45,46,47]	[45,47]
Repair Rates	Nature	UD	TFN
	ρ_{DC}	[1020,1060]	[1020,1040,1060]
	ρ_S	[1270,1290]	[1270,1280,1290]

Table 2: Fuzzy availability output

α	Case I		Case II	
	Lower	Upper	Lower	Upper
0	0.899825	0.905052	0.899827	0.90505
0.1	0.900018	0.90486	0.899897	0.904983
0.2	0.900211	0.904668	0.899969	0.904917
0.3	0.900404	0.904476	0.900039	0.904849
0.4	0.900597	0.904285	0.900111	0.904783
0.5	0.90079	0.904093	0.900182	0.904715
0.6	0.900983	0.903901	0.900252	0.904647
0.7	0.901176	0.90371	0.900323	0.90458
0.8	0.901369	0.903519	0.900393	0.904512
0.9	0.901563	0.903328	0.900463	0.904445
1	0.901756	0.903137	0.900534	0.904378

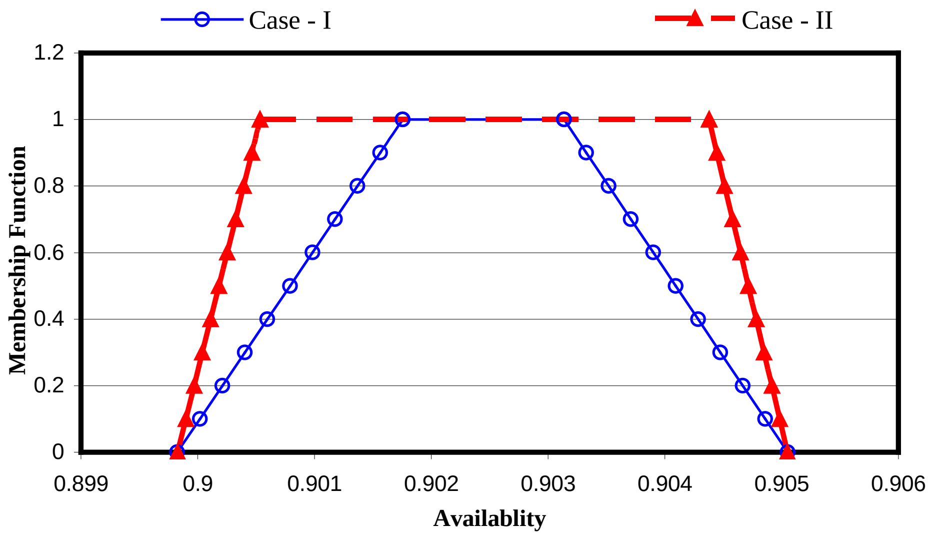

Figure 6: Fuzzy availability obtained in case I and II

One may observe the shape of fuzzy random availability in Figure 6. Fuzzy distribution of availability in both the cases is of the shape of trapezoidal fuzzy number (TrFN). Trapezoidal fuzzy number is having plateau at $\alpha = 1$ for some range of availability values. This is in spite of some data is in the form of TFN. It is because remaining data is random/ probabilistic. If these results are compared with the availability values obtained in literature [8, 9, 5], it is observed that availability values take the shape of TFN. This is for the reason that all the data considered in literature is of fuzzy nature (TFN). It is also quite logical that when some data is random and some is in the form of TFN, more availability range will have $\alpha = 1$, i.e., certainty zone.

7 Conclusion

This paper demonstrated a methodology of computing availability of a semi-Markovian mechatronic system when some data is random and some is of fuzzy type. Two combinations of nature of failure and repair rates have been considered. Recently evolved methodology such as hybrid approach is attempted to arrive at the availability value of the system. It is worth mentioning that using this distinct computation procedure, availability/reliability of any system/ model can be evaluated when data is random (probabilistic) and/or fuzzy (possibilistic).

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