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Mathematical Aspects of Grading Student's Homework in On-Line Web Applications

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Abstract

Existing tools for grading on-line homework are difficult to customize to the needs of new courses. ASP.NET and Visual Studio 2008 allow us to create very customized e-learning tools. In this paper a new tool for grading on-line homework will be presented. In order to efficiently grade on-line assignments, new methods which use fuzzy sets theory with non-associative operator were applied.

Keywords: grading homeworks, e-learning, fuzzy logic, non-associative operators

1 On-Line Tools for Grading Homework

In order to efficiently work with a large group of students, it is convenient to use on-line web applications. There are many advantages of such programs. Web applications are cross-platform, do not require installation, they are secure programs which are accessible through the web browser (e.g., Microsoft Internet Explorer, Mozilla Firefox, Apple Safari etc.). There are many tools for on-line grading which are available today:

- Webassign (http://webassign.net/)
- WebWorK (http://webwork.maa.org/)
- etc.

Existing systems for on-line grading are very complicated, difficult to customize and, in some cases, very expensive.

On the other hand, Visual Studio 2008 [5], ASP.NET [4], SQL Server Express 2008, and ADO.NET allow us to create very complex web applications very fast. Using C# programming language, it is possible to create very complex on-line assignments.

Sometimes, in order to create an assignment which is related to certain theory (e.g., integration) it is necessary to use a different mathematical theory (for example, in order to check if the indefinite integral is correct, it is necessary to check if the difference between the student's answer x_s and then correct answer x_c is constant). In other words, grading of on-line assignment requires application of complicated mathematical theories which are not necessarily available. In this paper, some mathematical problems which are related to the grading of on-line homework will be presented.

2 User Interface

The system which is discussed in this paper was created in asp.net in Summer 2008. The author used the web application in the Fall 2008 and Spring 2009 semesters at the University of Texas at El Paso in order to support teaching of several courses (Calculus 2, Calculus 3, Applied Analysis 1, Differential Equations). The system can be access from the author's web page http://andrzej.pownuk.com (in the section "For students"). Login web page is shown in the Fig. 1. After login to the system, the user can see the main students' web page, which is shown on the Fig. 2. On this web page, students can navigate to the page with the grades,

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solve homework, see additional information, see current score, and change user name and password. "Solve homework" page is shown on the Fig. 3. On that web page, students are able to solve on-line assignment and see their current grades.

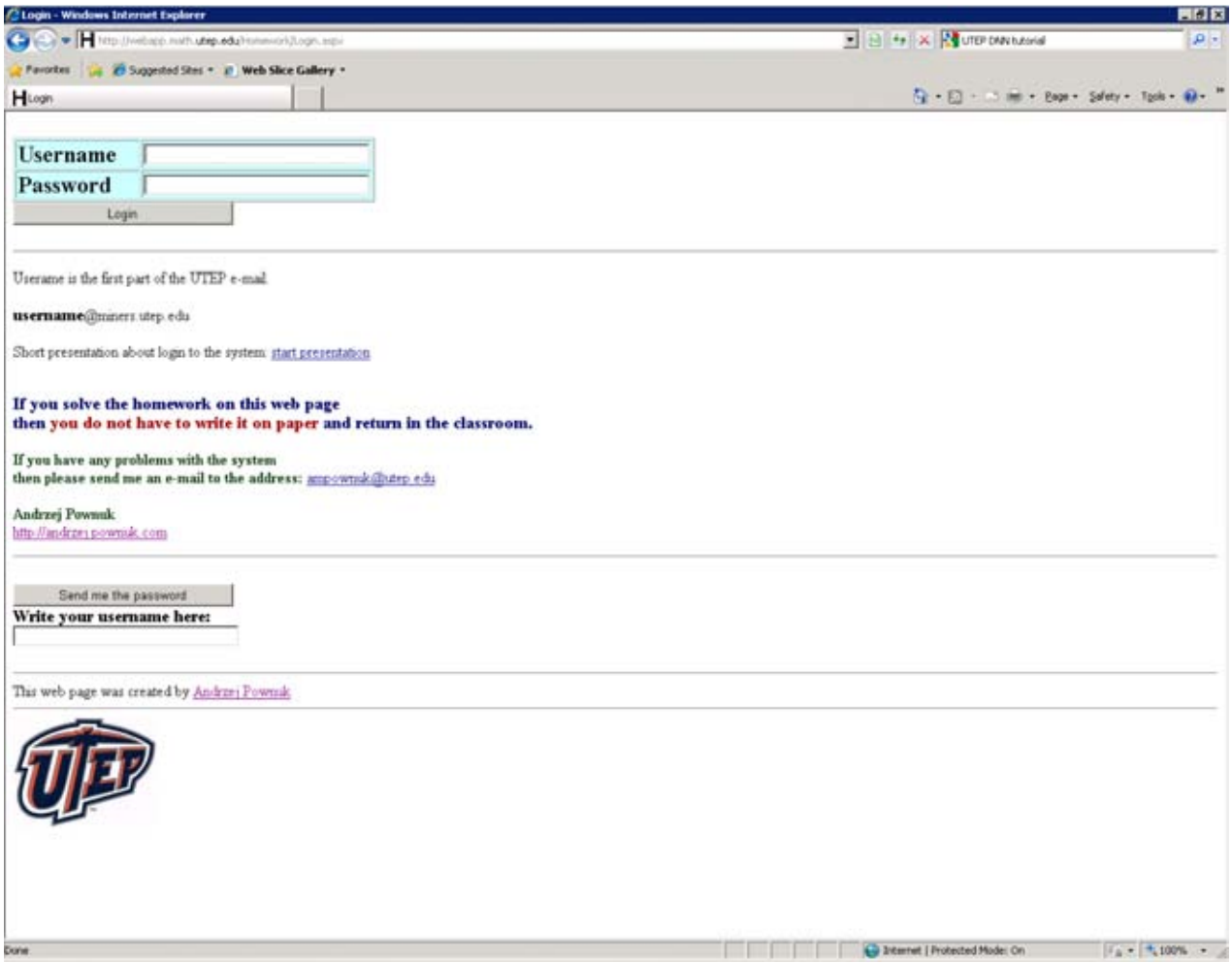


Figure 1: Login web page

3 Administrator Interface

After login to the system, an administrator can see the web page which is shown on the Fig. 4. Using this page, it is possible to add a new user to the system, to change password and other information about the user, to edit homework list in the system, to edit the list of tests, to edit messages to all students, to edit message to each particular group, to edit the list of semesters, to import students from cvs text files, to edit instructors' list, to edit administrators' list, to read log files, and to edit global setting of the program.

One of the most important page in the program is "edit group" page, which is shown on Fig. 5. Using this page, it is possible: to add new group, to edit members of the group, to edit the list of homework in each group, to edit the list of tests in each group, to edit grades in several different ways (e.g., the page shown on the Fig. 6), it is possible to export the list of grades, edit the link to the web page with additional information, edit file list which is available to each student in the group, send an e-mail with custom information and the list of grades with appropriate statistics.

On the "edit student" web page, it is possible to edit the first name, to edit the last name, to edit a student ID number, to edit a password, to edit an e-mail, to lock the user, to look at the student's scores, to

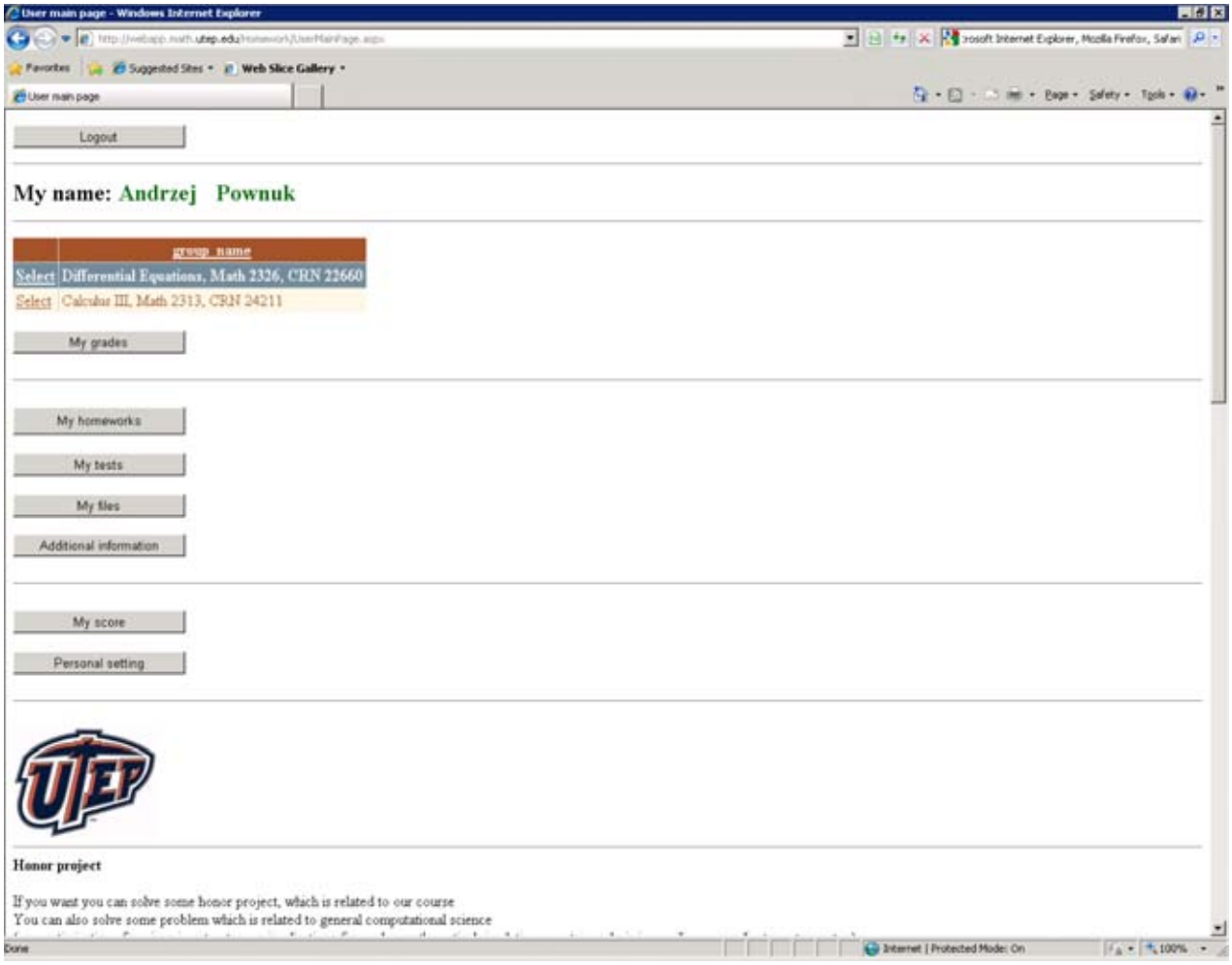


Figure 2: User main page

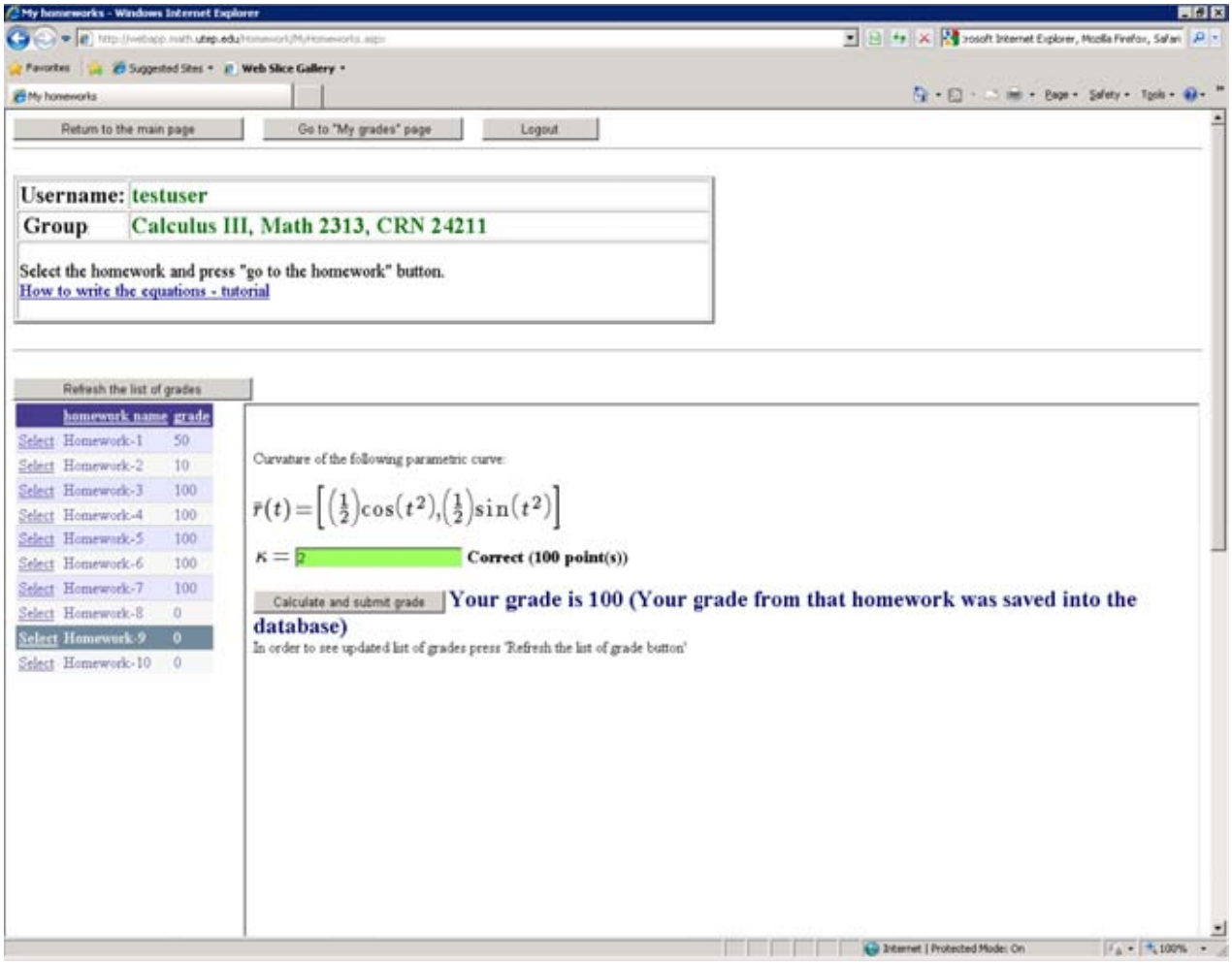


Figure 3: Solve homework page

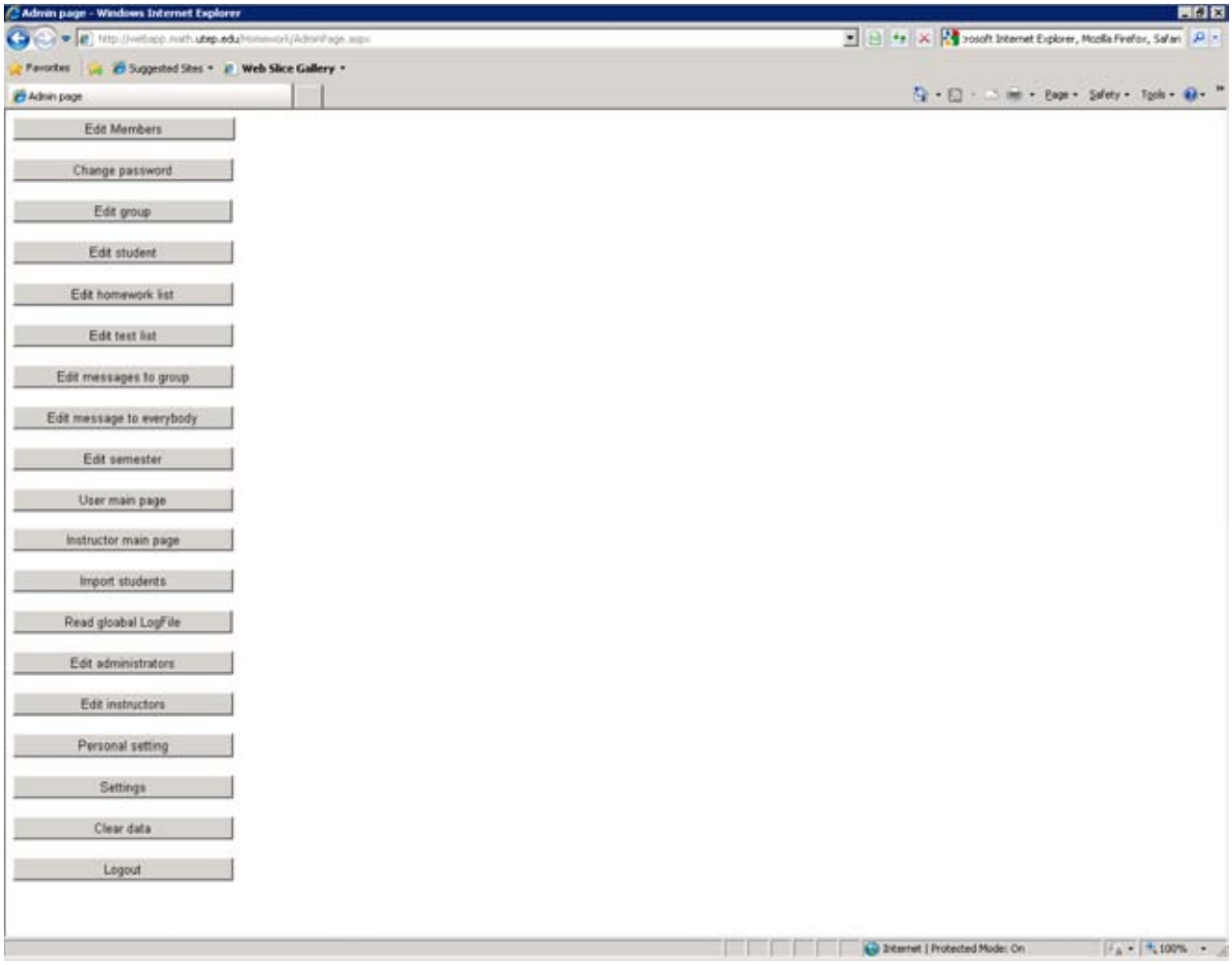


Figure 4: Admin main page

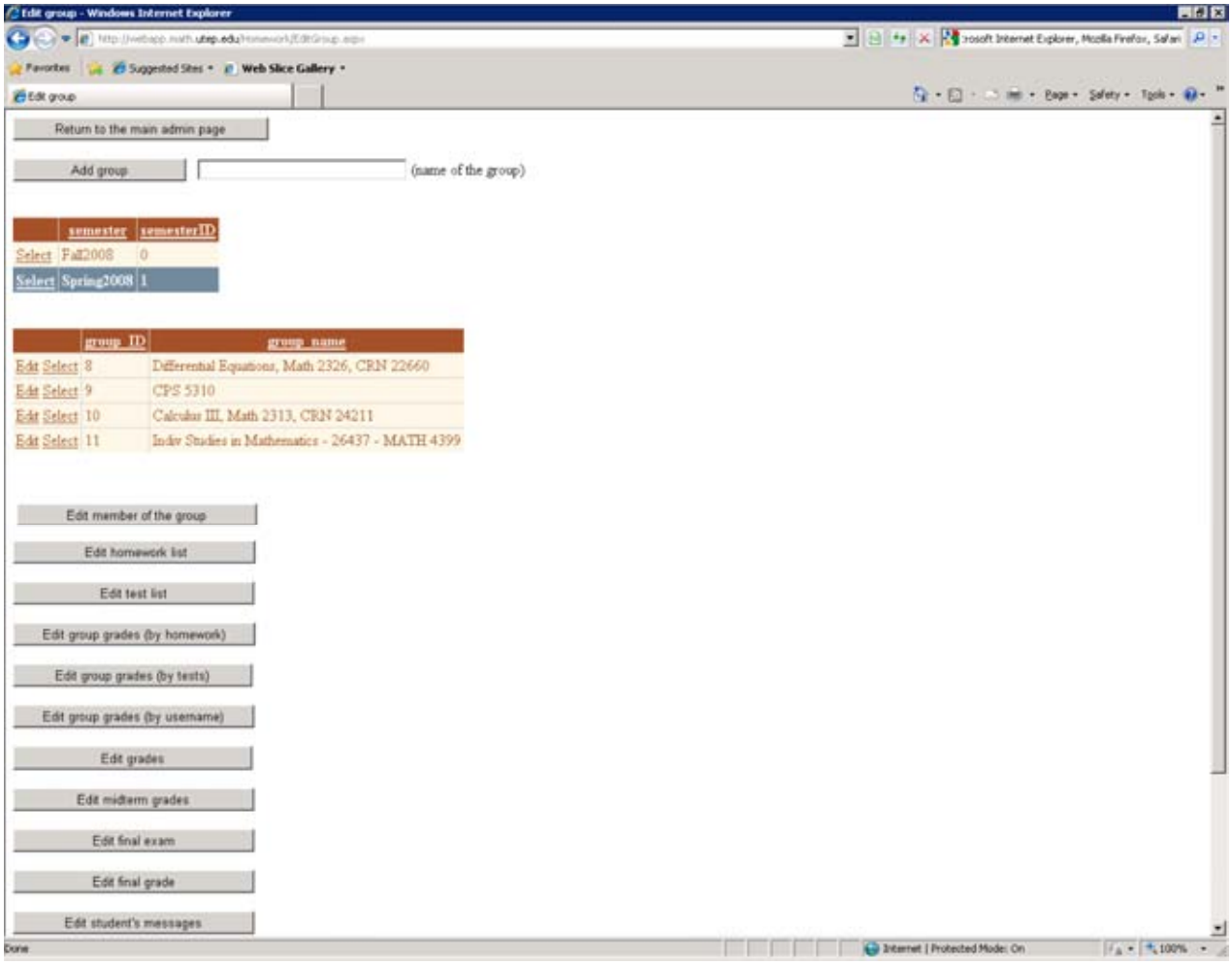
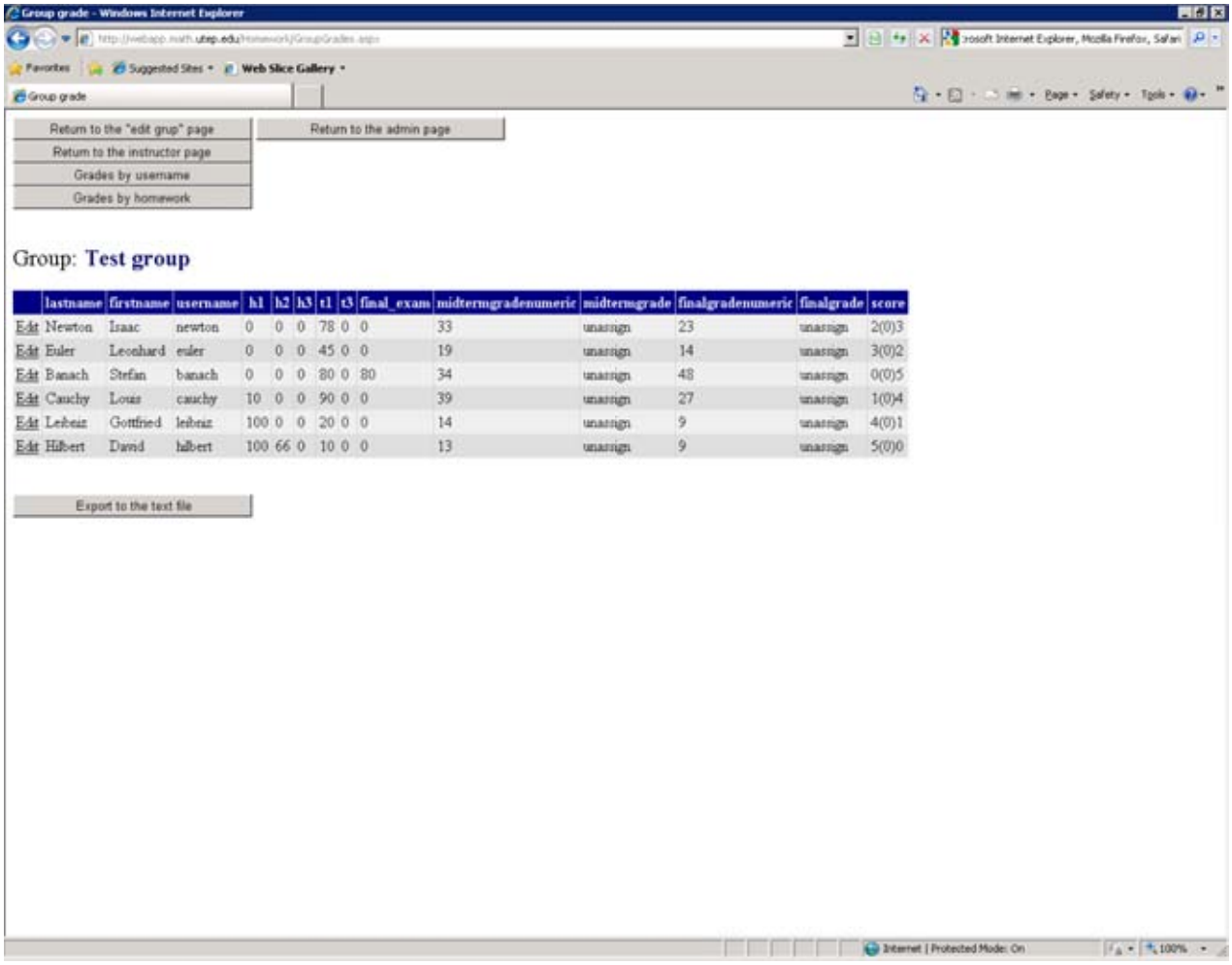


Figure 5: Edit group page



	lastname	firstname	username	h1	h2	h3	t1	t3	final_exam	midtermgradenumeric	midtermgrade	finalgradenumeric	finalgrade	score
Edit	Newton	Isaac	newton	0	0	0	78	0	0	33	unassign	23	unassign	2(0)3
Edit	Euler	Leonhard	euler	0	0	0	45	0	0	19	unassign	14	unassign	3(0)2
Edit	Banach	Stefan	banach	0	0	0	80	0	80	34	unassign	48	unassign	0(0)5
Edit	Cauchy	Louis	cauchy	10	0	0	90	0	0	39	unassign	27	unassign	1(0)4
Edit	Leibniz	Gottfried	leibniz	100	0	0	20	0	0	14	unassign	9	unassign	4(0)1
Edit	Hilbert	David	hilbert	100	66	0	10	0	0	13	unassign	9	unassign	5(0)0

Figure 6: Edit grades page

edit grades, and to send an e-mail to the student.

4 Homework Creation

In order to create a new homework, it is necessary to use Visual Studio 2008 (see Fig. 7) or Visual Web Developer Express which is available for free. There are three approaches for creation of a new homework:

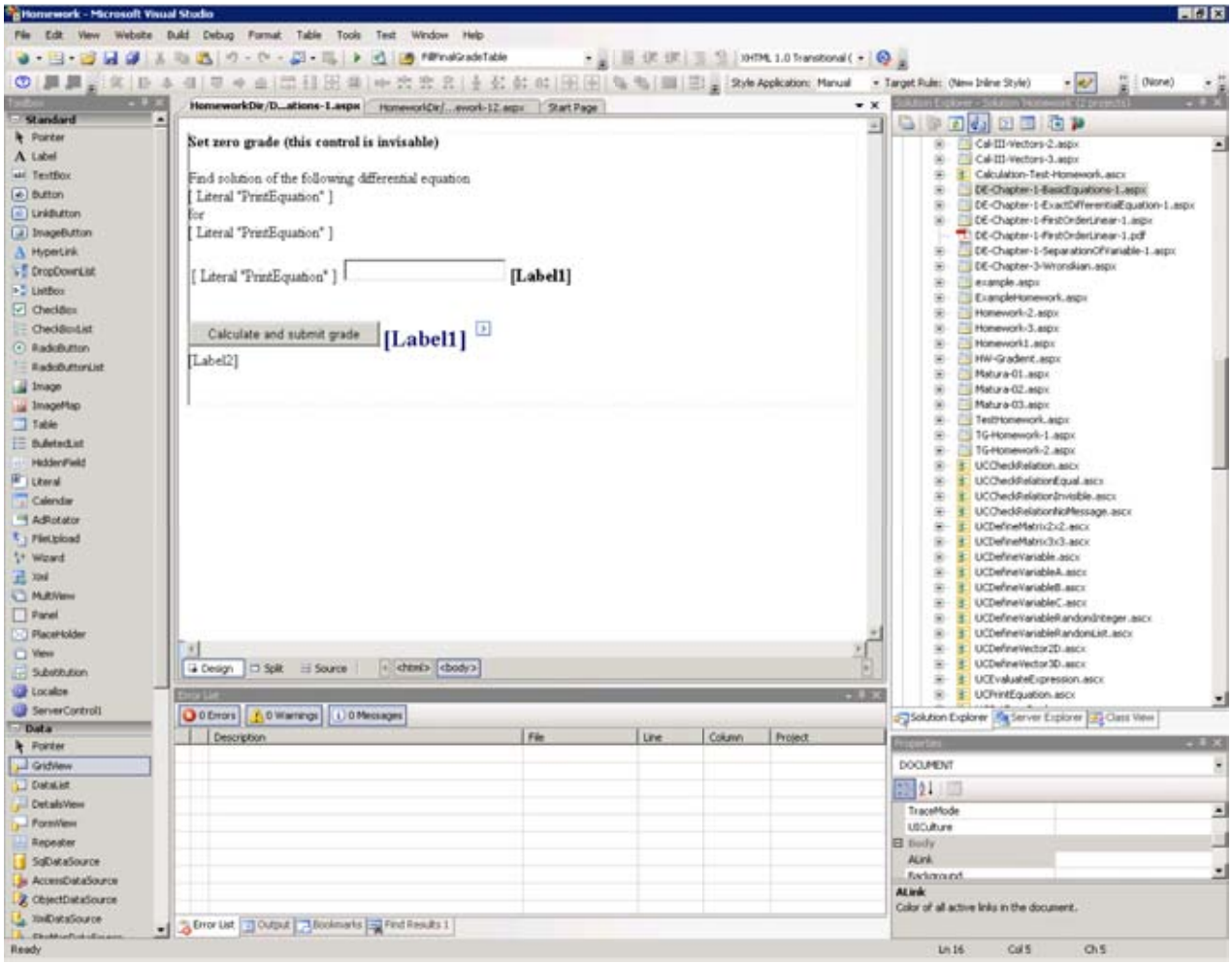


Figure 7: Visual Studio 2008

- the use of asp.net controls and C# language,
- the use of user controls (*ascx* files),
- mixed approach which use all the features.

4.1 Creation of the Homework by Using ASP.NET Controls and C# Language

Visual studio offers many different asp.net controls which can be used in order to create very complicated assignments. The most widely used are the following controls: the textbox control, button control, and label control. In order to evaluate arithmetical expressions, a special C# library was created. At this moment, it is possible to input the student answers in the following format:

- text string,

- natural numbers, real numbers, rational numbers,
- arithmetic expressions (which contain +, −,* , sin, cos, ln, etc.),
- complex numbers (and expression which contain complex numbers),
- symbolic expressions, e.g., $x^2 - 2y + 1$.

In order to verify the results, it is necessary to compare the student's answer x_s and the correct answer x_c. In the homework system, the following relations are available: $=$, $<$, $>$, constant difference (this relation is necessary in order to check indefinite integrals). It is possible to create homework assignment which use random questions, external databases, and ask complicated algorithmic questions.

4.2 Creation of the Homework by Using User Controls

Many homework assignments do not need any complicated calculations. In this situation, it is possible to use user controls (*ascx* files). A user control is a kind of composite control that works like an ASP.NET web page. One can add existing web server controls and markup to a user control, and define properties and methods for the control. One can then embed them in ASP.NET web pages, where they act as a unit. The most useful user controls print LaTeX equations, check the student's answers, and submit the grade to the database. Users controls significantly speed up the process of creation of a new homework assignment.

4.3 Mixed Approach

It is possible to extend the functionality of user controls by using custom C# procedures. However, in this case, it is necessary to take care of communication between the user controls and the standard asp.net code.

5 Mathematical Aspects of Grading

5.1 Numeric Answers

Let us denote the correct answer as x_c. The student's answer x_s is correct if

$$|x_c - x_s| < \varepsilon, \tag{1}$$

where $\varepsilon > 0$ is a given accuracy.

5.2 Complex Answers

If the correct answer x_c is a complex number, then equality relation can be checked in the following way:

$$|x_c - x_s| < \varepsilon, \tag{2}$$

where $|z| = \sqrt{Re(z)^2 + Im(z)^2}$ is the modulus of the complex number z.

5.3 Symbolic Answers

Let's assume that the correct answer x_c is an expression which contain variable v. Student's answer x_s is correct if

$$\forall v_i \in V, |x_c(v_i) - x_s(v_i)| < \varepsilon, \tag{3}$$

where v_i are some test values. It is very important to choose the test data correctly. For example, if $x_c = x$ and $x_s = y$ and $v_1 = (1,1), v_2 = (2,2), v_3 = (3,3)$ and

$$x_c(v_i) - x_s(v_i) = i - i = 0, \tag{4}$$

the answer will be $x_c = x_s$, which is incorrect, because, in general, $x \neq y$. If we choose $v_1 = (1,1), v_2 = (1,2), v_3 = (2,1), v_4 = (2,2)$ then

$$x_c(v_2) - x_s(v_2) = 1 - 2 = -1 \neq 0, \tag{5}$$

then $x_c \neq x_s$ which is correct.

From the mathematical point of view, the answers x_c and x_s are the same if a given relation $R(x_c, x_s)$ is satisfied. Examples of such relations are given in the formulas (1), (2), and (3). However, the process of grading may be, in general, much more complicated.

6 Uncertainty in Grading

Let us assume that in the homework assignment it is necessary to find the solution of the quadratic equation

$$ax^2 + bx + c = 0. \tag{6}$$

In order to solve the equation (6), it is necessary to find

$$\Delta = b^2 - 4ac. \tag{7}$$

- If $\Delta > 0$ then $x_1 = \dfrac{-b - \sqrt{\Delta}}{2a}$ and $x_2 = \dfrac{-b + \sqrt{\Delta}}{2a}$.
- If $\Delta = 0$ then $x_1 = x_2 = \dfrac{-b}{2a}$.
- If $\Delta < 0$ then the equation has no solution.

In order to check the knowledge of the student, it is necessary to answer the following question:

Does the student know how to solve the quadratic equation?

Using classical (Boolean) logic [1], the answer to this question is **yes** or **no**. In other words, the *truth value* of the relation $R(x_c, x_s)$ belongs to the following set $\{0, 1\}$, i.e.:

$$truth(R(x_c, x_s)) \in \{0, 1\}, \tag{8}$$

where $truth(R(x_c, x_s))$ is the truth value of the relation R. In this approach, it is possible to give the student only two grades: A or F. This solution is not always acceptable. Many teachers would like to use *the partial credit* in the evaluation process. In order to achieve this, it is necessary to extend the co-domain of the truth function into the whole interval $[0, 1]$, i.e.:

$$truth(R(x_c, x_s)) \in [0, 1]. \tag{9}$$

That lead to the concept of *multivalued logic* (for example, *fuzzy logic*, *Lukasiewicz logic* etc.). Let us consider a group of students which give different answers $x_{s1}, x_{s2}, \ldots, x_{sm}$ to the question x_c. Now it is possible to create a so-called membership function $\mu_{x_c}(x_s)$

$$\mu_{x_c}(x_s) = truth(R(x_c, x_s)). \tag{10}$$

This is a definition of the membership function of *a fuzzy set* [6]. The value of the membership function can then be transformed into the actual grade in the form of a letter (A, B, C, D, E) or in the form of number (e.g., $\{0, 1, \ldots, 100\}$ or $\{2, 3, 4, 5\}$, etc.):

$$grade_L(x_s) = G_L(\mu_{x_c}(x_s)) \in \{F, D, C, B, A\}; \tag{11}$$

$$grade_{100}(x_s) = G_{100}(\mu_{x_c}(x_s)) \in \{0, 1, \ldots, 100\}; \tag{12}$$

$$grade_{2,5}(x_s) = G_{2,5}(\mu_{x_c}(x_s)) \in \{2, 3, 4, 5\}. \tag{13}$$

Almost all algorithms $y = f(x)$ can be decomposed into a sequence of simpler tasks $f_1, \ldots, f_n$.

$$y = f(x) = (f_n \circ f_{n-1} \circ \ldots \circ f_1)(x) \tag{14}$$

In the case of a quadratic equation, the function f transforms the quadratic equation $ax^2 + bx + c = 0$ into the solution of a quadratic equation x_1, x_2:

$$f(\text{“}ax^2 + bx + c = 0\text{”}) = \{x_1, x_2\}. \tag{15}$$

The problem can be spit into several parts. Function f_1 transforms the polynomial equation $ax^2+bx+c=0$ into there numbers $\{a,b,c\}$.

$$f_1(\text{“}ax^2+bx+c=0\text{”})=\{a,b,c\}. \tag{16}$$

The second function f_2 calculates the number Δ:

$$f_2(\{a,b,c\})=\Delta=b^2-4ac. \tag{17}$$

The function f_3 is checking the sign of the number Δ :

$$f_3(\Delta)=\{-,0,+\}. \tag{18}$$

If $\Delta>0 \Leftrightarrow f_3(\Delta)=\text{“}+\text{”}$, then

$$x_1=f_4(a,b,c)=\frac{-b-\sqrt{b^2-4ac}}{2a}, \tag{19}$$

$$x_2=f_5(a,b,c)=\frac{-b+\sqrt{b^2-4ac}}{2a}. \tag{20}$$

If $\Delta=0 \Leftrightarrow f_3(\Delta)=$ “0”, then

$$x=f_6(a,b,c)=\frac{-b}{2a}. \tag{21}$$

If $\Delta<0 \Leftrightarrow f_3(\Delta)=\text{“}-\text{”}$, then

$$x=f_7(a,b,c)=\emptyset. \tag{22}$$

In order to check the above-described method it is necessary to check all steps of the algorithm. Depending on the values of the number Δ ,it is necessary to consider the following sequences of functions:

- if $\Delta>0$, then it is necessary to evaluate the functions f_1,f_2,f_3,f_4,f_5,
- if $\Delta=0$, then it is necessary to evaluate the functions f_1,f_2,f_3,f_6,
- if $\Delta<0$, then it is necessary to evaluate the functions f_1,f_2,f_3,f_7.

In order to estimate the value of the function $\mu_{x_c}(x_s)$, it is possible to use several different approaches.

In the classical logic, the problem is solved correctly if all the intermediate steps are solved correctly.

$$truth(R(x_c,x_s))=truth(\psi(f_1,x_s))\ \textbf{AND}\ldots\textbf{AND}\ truth(\psi(f_n,x_s)) \tag{23}$$

where $truth(\psi(f_i,x_s))$ is *true* if the i-th step of the calculation is correct and *false* if the i-th step of the calculation is incorrect. According to the formula (23), the answer is correct if all the steps of calculations are correct. The answer is incorrect if at least one step is incorrect. In that case, $truth(R(x_c,x_s))\in[0,1]$. In each particular case, it is necessary to define the meaning of the statement (23). Each teacher knows how to calculate grade by using partial credits. The presented approach has a clear interpretation and – in some special cases – is wildly used in the education today.

Other approaches may be based on t-norms [2]. Non-associative operations are discussed in the paper [3]. In this paper, several different ways of defining the **AND** operator will be presented.

7 Weight Approach

In order to assign the grade for each student answer x_s, it is possible to assign certain weights w_i to each the values of the functions f_i. An example of such weights are shown in the Table 1. Weights describe the difficulty of each step of the calculations and have to be defined by the teacher.

The grade can be calculated by the following formula:

$$grade_{100}(x_s)=\sum_i w_i(x_s). \tag{24}$$

The weights are different for different signs of Δ.

Let's assume that we would like to solve the following quadratic equation $x^2-4=0$, and the student's answer is the following: $a=1,b=0,c=-4$, $\Delta=16$, $x_1=2,x_2=-1$. The grading process is shown in the Table 2. After taking into account partial credit, the grade will be 70%.

Table 1: Example of weights

f	f_1	f_2	f_3	f_4	f_5	f_6	f_7
$\Delta > 0$	5	30	5	30	30		
$\Delta = 0$	5	30	5			60	
$\Delta < 0$	5	30	5				60

Table 2: Partial credit

Condition	Logical value	Weight
f_1	*true*	5
f_2	*true*	30
f_3	*true*	5
f_4	*true*	30
f_5	*true*	0
f_6	*null*	0
f_7	*null*	0
Total		70

8 Modified Weight Approach

In the pure weight approach, if somebody would make a mistake at the beginning of calculation, then all the next steps – even if logically correct – will be wrong. Because of this, in a good grading model, it is necessary to check not only correctness of each step, but also its logical relation to the previous step(s). In this approach, it is possible to give partial credit for incorrect but reasonable answers. In this case, the table of weights 3 has two more rows.

Let's assume that we would like to solve the following quadratic equation $x^2 - 4 = 0$, and the student's answer is the following: $a = 1, b = 0, c = -4$, $\Delta = 25$, $x_1 = \frac{-5}{2}, x_2 = \frac{5}{2}$. The grading process is shown in the Table 4. After taking into account the partial credit, the grade will be 47%. *The theory of fuzzy sets* provide a very realistic model of the grading process. The result of the test x_c in each particular group of students can be interpreted as a *fuzzy set* with the membership function $\mu_{x_c}(x_s)$.

In general, in the grading process, it is possible to apply any reasonable combinations of nonlinear functions, not necessarily classical t-norms.

9 Conclusions

ASP.NET and Visual Studio 2008 are efficient tools for creating very complicated and customized homework assignments. In order to speed up the process of creation of on-line assignment, it is possible to use asp.net user controls.

Table 3: An example of weights for partial credit with logical answers

f	f_1	f_2	f_3	f_4	f_5	f_6	f_7
$\Delta > 0$	5	30	5	30	30		
$\Delta > 0$, logical answer	0	20	2	20	20		
$\Delta = 0$	5	30	5			60	
$\Delta = 0$, logical answer	0	20	2			40	
$\Delta < 0$	5	30	5				60
$\Delta < 0$, logical answer	0	20	2				40

Table 4: Partial credit with logical answers

Condition	Logical value	Weight	Logical answer	Weight
f_1	true	5	null	0
f_2	false	0	false	0
f_3	false	0	true	2
f_4	false	0	true	20
f_5	false	0	true	20
f_6	null	0	true	0
f_7	null	0	null	0
Total		5		42

The process of grading can be described by using the theory of fuzzy set. In order to efficiently use partial credit, it is convenient to use non-associative **AND** operators. The grading process can be very complex and can consist of any reasonable (i.e., somehow justified) nonlinear functions.

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