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A Simple and Efficient Goal Programming Model for Computing of Fuzzy Linear Regression Parameters with Considering Outliers

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Abstract

Fuzzy linear regression has been studied by several researchers since the past three decades. Existing outliers in the data set, causes the result of fuzzy linear regression be incorrect. In this paper, a simple and efficient model is suggested for computation of fuzzy linear regression with outliers. The proposed method is based on Goal Programming technique and for estimation upper and lower fuzzy bands, two separated linear programming models are calculated. The proposed method minimizes the estimation error between observed and estimated values and has better performance in comparison with previous approaches. The proposed model is less sensitive to outliers and also, we do not need to select any parameters beforehand. The performance of proposed model is illustrated by solving several examples and comparing the results with the previous studies.

Keywords: fuzzy linear regression, fuzzy linear programming, outlier

1 Introduction

The purpose of regression model is analyzing the relationship between dependent and independent variables based on the given data. In 1982, Tanaka et al. [20] introduced Fuzzy Linear Regression (FLR). He modeled the procedure of parameter estimation as a linear programming problem, where the inputs are crisp and the output is a fuzzy number. In order to estimate regression parameters, they applied linear programming and minimized the total spread of the fuzzy parameters subject to covering the observed values by estimated values. Although their approach is improved by many researchers [3, 8, 15, 17], this approach is still one of the most frequently and simplest methods for estimating parameters of fuzzy regression. Generally, there are two approaches in fuzzy regression analysis. First, Linear programming based on which tries to minimize the fuzziness of the model by minimizing the total spreads of its fuzzy coefficients, subject to including the data points of each sample within a specified feasible data interval [19, 20, 21]. Ge and Wang [7] tried to determine the relationship between threshold value and input data when data contains a considerable level of noise or uncertainty. They used the threshold value to measure degrees of fitness in fuzzy linear regression. Eventually, they showed that the parameter *h* is inversely proportional to the input noise. Also, many researchers recommended a combination of fuzzy regression models with some other approaches, like Monte-Carlo methods [1] to improve the result obtained from ordinary LFR. Second, Least squares method, which minimizes the sum of squared errors in the estimated value, based on their specifications [1, 4, 6, 7, 22]. This approach is indeed a fuzzy extension of the ordinary least squares, which obtains the best fitting to the data, based on the distance measure under fuzzy consideration, applying information included in the input–output data set.

One of the important problem associated with the Tanaka approach is the influence of outliers on the predicted upper and lower fuzzy bands. The Tanaka model is very sensitive to outliers and also the outliers make the fuzzy linear regression not to be able correct predicting. There are many studies which discuss about handling the problem of outliers [2, 3, 9, 11, 15]. More of the mentioned models need for selecting some parameters beforehand. As there

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could not be a systematic method for defining parameters, the problem for these models is to select the parameters in advance. This paper presents a simple model for computation of fuzzy linear regression with and without outliers.

The organization of this paper is as follows. In Section 2, the fuzzy linear regression is introduced. Section 3 explains the proposed model. The numerical examples and results are reported in Section 4. Finally, conclusions are included in the last section.

2 Fuzzy Linear Regression

Tanaka et al. [20] proposed the fuzzy linear regression (FLR) model in the case of crisp input and fuzzy output data set as follow:

$$\hat{Y} = A_0 + A_1 x_1 + A_2 x_2 + \ldots + A_k x_k \tag{1}$$

where $A_j = (\alpha_j, c_j),\ j = 0,1,\ldots,k$ is assumed to be a symmetric triangular fuzzy number with center a_j and half-width $c_j, c_j \geq 0$. To estimate A_j, Tanaka et al. [20] applied following model:

$$\begin{aligned}
&\min \quad \sum_{j=0}^{k} c_j \\
&\textit{subject to:} \\
&\quad \sum_{j=0}^{k} (a_j + (1-H)c_j)x_{ij} \geq \bar{y}_i + (1-H)e_i \qquad i = 1,2,\ldots,n \\
&\quad \sum_{j=0}^{k} (a_j - (1-H)c_j)x_{ij} \leq \bar{y}_i - (1-H)e_i \qquad i = 1,2,\ldots,n \\
&\quad a_j = \textit{free} \quad c_j \geq 0,\ j = 0,1,\ldots,k.
\end{aligned} \tag{2}$$

In model (2), n is the number of observations and $H \in [0,1]$ is the threshold level to be chosen by decision maker. Later, the model (2) has been modified by other researchers [16, 19]. They suggested the objective function should be as $\min \sum_{i=0}^{n} \sum_{j=0}^{k} c_j x_{ij}$ to prevent of being c_i ' s =0.

As mentioned, Tanaka et al. [20] approach is very sensitive to outliers. In the other words, if the outliers exist in the data set, the Tanaka model is not able to predict upper and lower fuzzy bands, correctly. Chen [3] and Peters [15] proposed the models to handle the outliers' problem. Chen [3] discussed the outlier problem by applying the following model:

$$\begin{aligned}
&\min \quad \sum_{i=0}^{n} \sum_{j=0}^{k} c_j x_{ij} \\
&\textit{subject to:} \\
&\quad \sum_{j=0}^{k} (a_j + (1-H)c_j)x_{ij} \geq \bar{y}_i + (1-H)e_i \qquad i = 1,2,\ldots,n \\
&\quad \sum_{j=0}^{k} (a_j - (1-H)c_j)x_{ij} \leq \bar{y}_i - (1-H)e_i \qquad i = 1,2,\ldots,n \\
&\quad \sum_{j=0}^{k} c_j x_{ij} - e_i \leq k \\
&\quad a_j = \textit{free} \quad c_j \geq 0,\ j = 0,1,\ldots,k
\end{aligned} \tag{3}$$

where k is a limiting value which should be assigned by decision maker. The problem with Chen [3] approach is how to choose k value. Although Chen [3] proposed some methods for selecting k, but there is still some problem in defining the suitable k value. The Peters [15] model is presented as follow:

$$\begin{aligned}
&\max \quad \bar{\lambda} \\
&\textit{subject to:} \\
&\qquad \sum_{j=0}^{k} (a_j + c_j) \geq \bar{y}_i - (1-\lambda_i)e_i \qquad i = 1, 2, ..., n \\
&\qquad \sum_{j=0}^{k} (a_j - c_j) \leq \bar{y}_i + (1-\lambda_i)e_i \qquad i = 1, 2, ..., n \\
&\qquad \bar{\lambda} = (\lambda_1 + \lambda_2 + ... + \lambda_n) / n \\
&\qquad \sum_{j=0}^{k} c_j x_{ij} \leq P_0(1-\bar{\lambda}) \\
&\qquad 0 \leq \lambda_i \leq 1, i = 1, ..., n, \bar{\lambda} \geq 0 \\
&\qquad a_j = \textit{free} \quad c_j \geq 0,\ j = 0, 1, ..., k.
\end{aligned} \tag{4}$$

The problem with Peters [15] model is also selecting P_0 and the selection of different P_0's would result in different outcomes.

3 Proposed Model

In this section, the proposed model is explained. This model applies Goal Programming (GP) technique for estimating the fuzzy regression linear parameters.

Let y_{iU} , $\bar{y}_i$ and y_{iL} be the upper, center and lower points of *i*th observed fuzzy data, respectively and $\hat{y}_{iU}$, $\hat{y}_{iL}$ be the upper and lower point of the *i*th predicted interval. Moreover, $\hat{y}_L$ and $\hat{y}_U$ are predicted fuzzy upper and lower bands which are shown in Figure 1. In this model, it is allowed $\hat{y}_{iL}$ to be larger than y_{iL} , but must be smaller than $\bar{y}_i$, and $\hat{y}_{iU}$ is allowed smaller than y_{iU} but must be greater than $\bar{y}_i$. In other words, $\bar{y}_i$ is considered as upper band of $\hat{y}_{iL}$ and lower band of y_{iL} simultaneously. In fact, the objective of the proposed model is minimization of the sum of deviations of $\hat{y}_{iL}$ from $\bar{y}_i$ and y_{iL} , and the sum of deviations of $\hat{y}_{iU}$ from $\bar{y}_i$ and y_{iU} . In other word, to obtain $\hat{y}_{iL}$, $\bar{y}_i$ is selected as upper point (instead of $\bar{y}_i + (1-H)e_i$ as the other models), and to obtain $\hat{y}_{iU}$, $\bar{y}_i$ is selected as lower point (instead of $\bar{y}_i - (1-H)e_i$). Note that the fuzzy bands $\hat{y}_L$ and $\hat{y}_U$ are calculated, separately here in: First, a GP model is solved to find lower fuzzy band ($\hat{y}_L$), then, another model is implemented to get upper fuzzy band ($\hat{y}_U$). In previous studies, the estimated FLR parameters are affected by outliers, because the upper and lower points of fuzzy data are used, simultaneously. Since, in proposed model $\bar{y}_i$, which is less sensitive than outliers, is used instead of upper and lower points of fuzzy data, the model can estimate the FLR parameters with least error. So, the main difference of proposed model and previous models could be illustrated to be the way of handling outliers without selecting any parameters in advance.

To obtained $\hat{y}_L$ band, the model (5) is solved.

$$\begin{aligned}
&\min \sum_{i=0}^{n} (d_{iU}^{+} + d_{iU}^{-} + d_{iL}^{+} + d_{iL}^{-}) \\
&\textit{subject to:} \\
&\qquad \sum_{j=0}^{k} (a_j + (1-H)c_j)x_{ij} + d_{iU}^{+} - d_{iU}^{-} = \bar{y}_i, \qquad i = 1, 2, ..., n \\
&\qquad \sum_{j=0}^{k} (a_j - (1-H)c_j)x_{ij} + d_{iL}^{+} - d_{iL}^{-} = \bar{y}_i - (1-H)e_i \qquad i = 1, 2, ..., n \\
&\qquad d_{iU}, d_{iL} \geq 0, i = 1, 2, ..., n \\
&\qquad a_j = \textit{free} \quad c_j \geq 0,\ j = 0, 1, ..., k.
\end{aligned} \tag{5}$$

In the first constraint, the $\bar{y}_i + (1-H)e_i$ is replaced by $\bar{y}_i$ in order the upper point to be the $\bar{y}_i$ values when predicting $\hat{y}_L$. In the model (5), $\left|d_{iU}^+ - d_{iU}^-\right|$ is the distance between $\bar{y}_i$ and $\hat{y}_{iL}$ and $|d_{iL}^+ - d_{iL}^-|$ is the distance between lower point of H-certain observed interval and $\hat{y}_{iL}$. Thus, the sum of two deviations should be minimized. The $\hat{y}_U$ band is obtained by solving GP model below:

$$\begin{aligned}
&\min \sum_{i=0}^{n} (d_{iU}^+ + d_{iU}^- + d_{iL}^+ + d_{iL}^-) \\
&\textit{subject to:} \\
&\quad \sum_{j=0}^{k} (a_j + (1-H)c_j)x_{ij} + d_{iU}^+ - d_{iU}^- = \bar{y}_i + (1-H)e_i, \qquad i = 1,2,\ldots,n \\
&\quad \sum_{j=0}^{k} (a_j - (1-H)c_j)x_{ij} + d_{iL}^+ - d_{iL}^- = \bar{y}_i \qquad i = 1,2,\ldots,n \\
&\quad d_{iU}, d_{iL} \geq 0, i = 1,2,\ldots,n \\
&\quad a_j = \textit{free} \quad c_j \geq 0,\ j = 0,1,\ldots,k.
\end{aligned} \tag{6}$$

In each models (5) and (6), there is one estimated band for lower and upper points. The upper line of model (5) (lower band) and the lower line of model (6) (upper band) are located around $\bar{y}_i$'s and could be eliminated to decrease the predicted error (see Figure. 1). Thus, the lower and upper fuzzy bands are:

$$\hat{y}_L = (\alpha_{0L} - c_{0L}) + (\alpha_{1L} - c_{1L})x \tag{7}$$
$$\hat{y}_U = (\alpha_{0U} + c_{0U}) + (\alpha_{1U} + c_{1U})x \tag{8}$$

where α_{jL} and c_{jL} are the estimated values for $\hat{y}_L$, and α_{jU} and c_{jU} are the estimated values for $\hat{y}_U$.

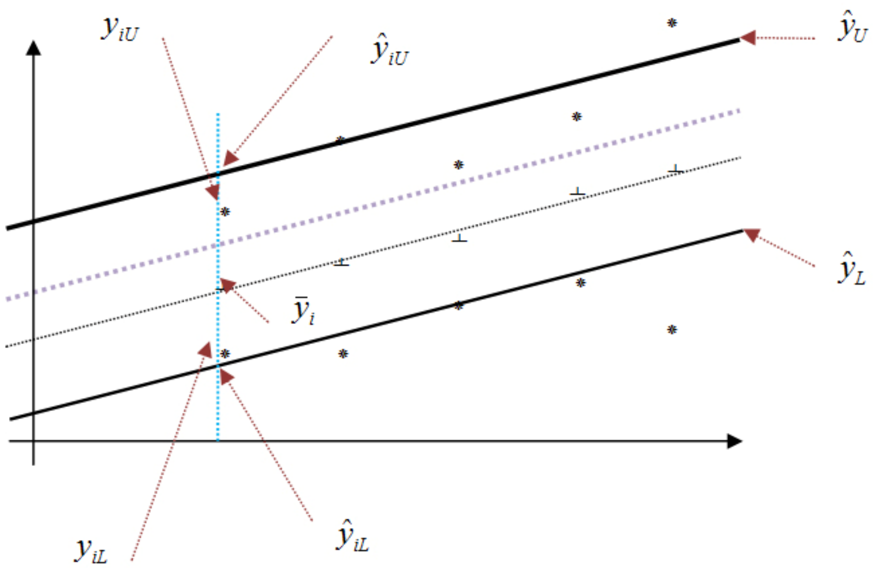

Figure 1: The graphical explanation of the proposed model

There are three cases in the linear fuzzy regression analysis:

(a) Constant spread;
(b) Increasing spread;
(c) Decreasing spread.

In the case (a), the predicted interval should lay between two parallel lines. Since, in the proposed model, it is probable that the slops of $\hat{y}_L$ and $\hat{y}_U$ lines to be different, the means of α_j's and c_j's $(j \neq 0)$ can be used as the slops of $\hat{y}_L$ and $\hat{y}_U$ lines. The means of α_j's and c_j's $(j \neq 0)$ are calculated as follow:

$$\alpha_j = \frac{1}{2}(\alpha_{jL} + \alpha_{jU}) \quad j = 1, \ldots, k, \tag{9}$$

$$c_j = \frac{1}{2}(c_{jL} + c_{jU}) \quad j = 1, \ldots, k, \tag{10}$$

where α_{jL} and c_{jL} are the estimated values for $\hat{y}_L$, and α_{jU} and c_{jU} are the estimated values for $\hat{y}_U$. In the cases (b) and (c), it is not necessary that the slops of $\hat{y}_L$ and $\hat{y}_U$ lines to be equal. Hence, some of the estimated α_j's and c_j's $(j \neq 0)$ are used.

4 Results

To illustrate the capability of the proposed model, here three examples are solved. The first two examples are with outliers and the last example is without outliers.

Example 1: Table 1 lists the numerical values used by Chen [3]. In this example, three cases A, B and C with one outlier point are considered.

Table 1: Outliers with constant, increasing and decreasing spread

x	(y_i,e_i)		
	A: Constant spread	B: Increasing spread	C: Decreasing spread
1	(8.0,1.8)	(11,2)	(11,12)
2	(6.4,2.2)	(13,2)	(13,12)
3	(9.5,2.6)	(21,4)	(21,10)
4	(13.5,2.6)	(29,4)	(24,10)
5	(13.0,2.4)	(29,6)	(31,8)
6	(15.2,2.3)	(34,6)	(34,8)
7	(17.0,2.2)	(45,15)[a]	(42,4)
8	(19.3,4.8)[a]	(44,8)	(44,15)[a]
9	(20.1,1.9)	(48,12)	(51,2)
10	(24.3,2.0)	(54,12)	(54,2)

[a] Indicates the outlier.

In all cases, to find $\hat{y}_L$ and $\hat{y}_U$, the models (5) and (6) are applied with H=0. The results are shown in Table 2. Note that the slops of $\hat{y}_L$ and $\hat{y}_U$ lines are modified by using of equations (7) and (8). As shown, the value of $\sum_{i=1}^{n}\sum_{j=0}^{k} c_j x_{ij}$ in the proposed model is smaller than in comparison with Tanaka et al. [20] and Chen [3] models. The value of $\sum_{i=1}^{n}\sum_{j=0}^{k} c_j x_{ij}$ in the proposed model is obtained from combination of models (5) and (6) as follow:

$$\sum_{i=1}^{n}\sum_{j=0}^{k} c_j x_{ij} = \left\{ \left(\sum_{i=1}^{n}\sum_{j=0}^{k} c_j x_{ij} \right)_{j \in S_1} + \left(\sum_{i=1}^{n}\sum_{j=0}^{k} c_j x_{ij} \right)_{j \in S_2} \right\} \tag{11}$$

where S_1 and S_2 show c_j's obtained from models (5) and (6), respectively. Figures 2, 3 and 4 show the results of above models graphically.

Table 2: Comparison between Tanaka et al. [20], Chen [3] and proposed model

	Model	$\sum_{i=1}^{n}\sum_{j=0}^{k} c_j x_{ij}$	**Results**
Case (A)	Tanaka	44.45703	$y=(4.43,3.67)+(1.86,0.14)x$
	Chen	32.75	$y=(4.75,4.55)+(1.85,-0.15)x$
	Proposed model	23	$\hat{y}_L=(2.65-1.15)+(1.85)x$ [a] $\hat{y}_U=(5.55+1.15)+(1.85)x$ [a]
Case (B)	Tanaka	123.75	$y=(4.51,0.65)+(5.70,2.13)x$
	Chen	95.0	$y=(5.76,0.90)+(4.95,1.38)x$
	Proposed model	69.68	$\hat{y}_L=(5.81-0.48)+(4.19-0.52)x$ $\hat{y}_U=(6.70+0.42)+(5.29+0.58)x$
Case (C)	Tanaka	144.0469	$y=(4.76,13.10)+(4.90,0.24)x$
	Chen	89.375	$y=(5,16.5)+(4.875,-1.375)x$
	Proposed model	80	$\hat{y}_L=(-1.14-3.8)+(5.4)x$ $\hat{y}_U=(11.4+4.2)+(4.6)x$

[a] The means of α_{1L},α_{1U} and c_{1L},c_{1U} are used, respectively.

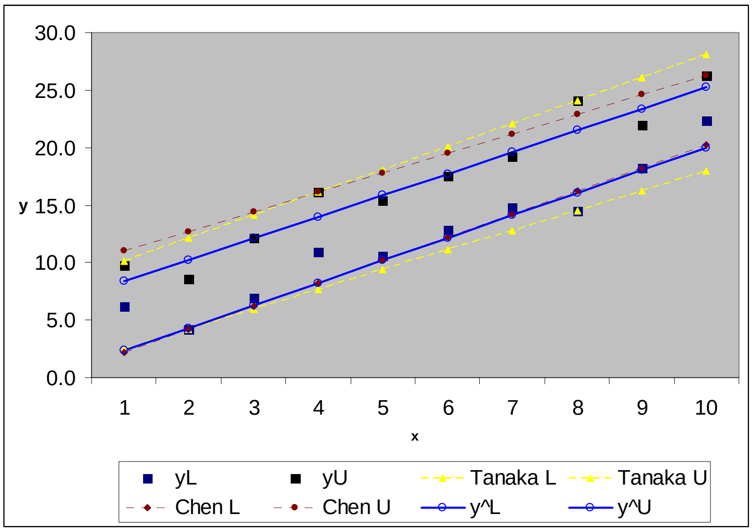

Figure 2: Case A - Comparison between Chen [3], Tanaka et al. [20] and proposed model

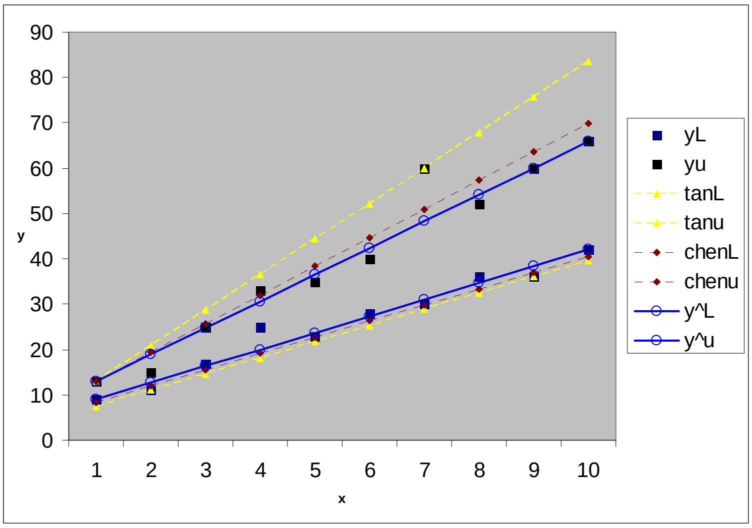

Figure 3: Case B - Comparison between Chen [3], Tanaka et al. [20] and proposed model

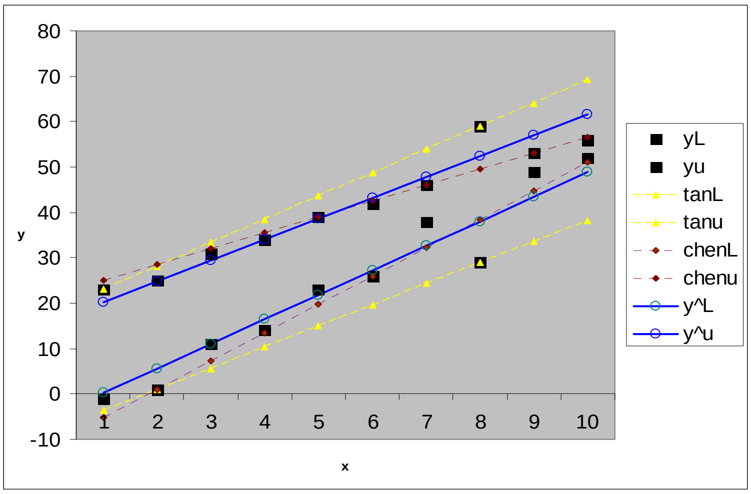

Figure 4: Case C - Comparison between Chen [3], Tanaka et al. [20] and proposed model

Example 2: This example has also the outlier problem. The difference between this example and previous example is the existence of the point with small e_i and large y_i. These data were used by Nasrabadi et al. [13]. In this case, the values of α_1's need to be modified. The results of Nasrabadi et al. [13] model and proposed model are shown in Table 3 and Figure 5. As shown, the value of $\sum_{i=1}^{n}\sum_{j=0}^{k} c_j x_{ij}$ for the proposed model is smaller than the Nasrabadi et al. [13] model.

Example 3: This example has no outliers and used by Kim and Bishu [10] to illustrate how the proposed method performs. We compare the results of our method with methods in literature. To evaluate the performance of a fuzzy regression model, Kim and Bishu [10] used the ratio of the difference between the membership values to the observed membership values as follows:

$$error_i = \frac{\int S_{\tilde{y}_i \cup S_{\hat{y}_i}} \,|\, \hat{y}_i(t) - y_i(t) \,|\, dt}{\int S_{\tilde{y}_i} y_i(t) dt} \tag{12}$$

where $S_{\tilde{\hat{y}}_i}$ and $S_{\tilde{y}_i}$ are the support of $\hat{y}_i$ and y_i, respectively. To compare the performance of the FLR models, Eq. (12) is applied to calculate the errors in estimation the observed responses. The data given for this example was used by Tanaka et al. [20]. The data and results are shown in Table 4. By solving models (5) and (6) (at H=0) and modification the values a_1's and c_1's, two below fuzzy bands are calculated:

$$\hat{y}_L = (5.55, 1.2) \oplus (1.325, 0)x, \tag{13}$$
$$\hat{y}_L = (7.20, 1.2) \oplus (1.325, 0)x. \tag{14}$$

Table 3: Numerical data and comparison of Nasrabadi et al. [13] and proposed model

x	**(y_i,e_i)**		
	Observed data	**Nasrabadi model**	**Proposed model**
1	(6.4,2.2)	(5.66,9.82)	(4.82,8.57)
2	(8.0,1.8)	(7.21,11.61)	(6.38,10.27)
3	(16.5,2.6)[a]	(8.76,13.4)	(7.94,11.97)
4	(11.5,2.6)	(10.31,15.19)	(9.50,13.67)
5	(13.0,2.4)	(11.86,16.98)	(11.06,15.37)
$\sum_{i=1}^{n}\sum_{j=0}^{k} c_j x_{ij}$	-	16.97	11.09

[a] Indicates the outlier.

Table 4: Comparison between different methods

i	x_i	**(y_i,e_i)**	Errors in estimation					
			Tanaka et al.	Diamond	Savic-Pedrycz	Kim-Bishu	Modarres et al.	Proposed method
1	1	(8.0,1.8)	1.86	1.23	1.54	1.22	1.35	0.32
2	2	(6.4,2.2)	1.30	1.39	1.52	1.38	1.27	1.62
3	3	(9.5,2.6)	0.58	0.42	0.7	0.4	0.23	0.59
4	4	(13.5,2.6)	0.86	1.09	1.16	1.12	1.25	1.13
5	5	(13.0,2.4)	1.0	0.40	0.86	0.36	0.13	0.16
Total error			5.6	4.53	5.78	4.48	4.23	3.82

The right half of Table 4 shows the errors of the five observations for the different methods. The total error of the proposed method is 3.82 which obviously better than the other total errors.

5 Conclusion

There are two approaches in Fuzzy linear regression: linear programming and least squares. In this paper, a simple model based on first approach is presented for computing of fuzzy linear regression. This model is based on Goal Programming. Since the existence of outliers in the data set causes incorrect results, the ability of proposed model is less sensitive to outliers. Furthermore, unlike previous models, it is not necessary to select any parameters beforehand. In this model, the upper and lower fuzzy bands are computed by two linear goal programming model, separately.

Several examples are solved by using the proposed model with and without outliers and the results are compared with previous models. The proposed model results illustrate that this model has the goodness fit depend both on the observation and fuzzy bands.

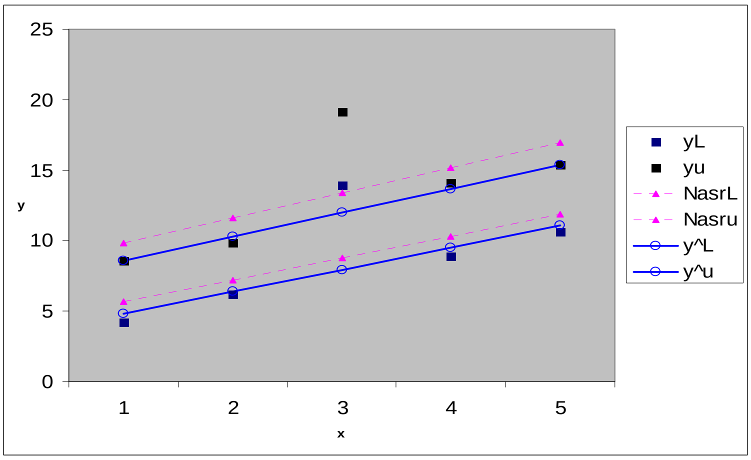

Figure 5: Comparison between Nasrabadi et al. [13] model and proposed model

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