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The Possibilistic Representative Value of Type-2 Fuzzy Variable and Its Properties*

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Abstract

Type-2 (T2) fuzzy variable is an extension of an ordinary fuzzy variable. T2 fuzzy variable is defined as a measurable map from the universe to the set of real numbers, the possibility of a T2 fuzzy variable takes on a real number is a regular fuzzy variable (RFV). T2 fuzziness, which is usually used to handle linguistic uncertainties, can be described as T2 fuzzy variable. To characterize the properties of T2 fuzzy variables in some aspects, we present a scalar representative value operator for T2 fuzzy variable. Some properties of the representative value operator are discussed. For discrete T2 fuzzy variable and T2 triangular fuzzy variable, we obtain the computational formulas of the representative value.

Keywords: representative value, fuzzy possibility measure, T2 fuzzy variable, expected value, mutually independent RFVs

1 Introduction

In a fuzzy decision system, fuzziness can be described as fuzzy sets. However, fuzzy set requires crisp membership function which cannot be obtained in many situations. To overcome this difficulty, in 1975, the concept of a T2 fuzzy set as an extension of an ordinary fuzzy set was introduced by Zadeh [25]. Since then, Mizumoto and Tanaka [17] discussed what kinds of algebraic structures the grades of T2 fuzzy sets form under join, meet and negation, and showed that normal convex fuzzy grades form a distributive lattace under the join and meet; Nieminen [19] studied on the algebraic structure of T2 fuzzy sets; Dubois and Prade [1] investigated the operations in a fuzzy-valued logic, and Yager [24] applied the T2 fuzzy set to decision making. A T2 fuzzy set is characterized by a fuzzy membership function, i.e., the membership grades themselves are fuzzy sets in $[0, 1]$. A T2 fuzzy set represents the uncertainty in terms of secondary membership function and footprint of uncertainty [15]. T2 fuzziness which is usually used to handle linguistic uncertainties, can be described as T2 fuzzy sets. Now, T2 fuzzy sets have been applied successfully to handle linguistic and numerical uncertainties [4, 5, 14]. Mitchell [16] introduced a similarity measure with which to measure the similarity between two T2 fuzzy sets. He also showed that T2 fuzzy sets provide indeed a natural language for formulating classification problems in pattern recognition. In pattern recognition, the T2 fuzzy hidden Markov models [27, 28] advanced the hidden Markov models expressive power for uncertainty by T2 fuzzy set. In addition, T2 fuzzy sets have found applications in many other fields [2, 3, 7].

In 2009, Liu and Liu [13] presented the fuzzy possibility theory which is a generalization of the usual possibility theory [6, 18, 20, 22, 23, 26]. In the proposed theory, some fundamental concepts were introduced, such as fuzzy possibility measure which was defined as a set function from the ample field to a collection of regular fuzzy variables (RFVs), fuzzy possibility space (FPS), T2 fuzzy variable, T2 possibility distribution function, secondary possibility distribution function. Liu and Liu [13] also showed that FPS leads to the definition of a T2 fuzzy set on $\Re^m$, which have been called a T2 fuzzy vector. In fuzzy possibility theory, a variable-based approach is adopted to deal with type-2 fuzziness. In this paper, we will present the concept of the representative value for T2 fuzzy variable as a scalar value and discuss some properties of the representative value operator.

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The paper is organized as follows. In Section 2, we recall several required fundamental concepts. The purpose of Section 3 is to present the concept of the representative value for T2 fuzzy variable. Some properties of the representative value operator are also discussed in Section 3. Section 4 gives the computational formulas of the representative value for discrete T2 fuzzy variable. Section 5 deduces the computational formulas of the representative value for T2 triangular fuzzy variable. Finally, the main work of this paper is summarized in Section 6.

2 Preliminaries

Let Γ be a universe of discourse. An ample field $\mathcal{A}$ on Γ is a class of subsets of Γ that is closed under arbitrary unions, intersections and complement in Γ. Let ξ be a fuzzy variable which was defined on the possibility space $(\Gamma, \mathcal{A}, \mathrm{Pos})$ [23] with possibility distribution function $\mu : \Re \to [0, 1]$.

An m-ary regular fuzzy vector $\xi = (\xi_1, \xi_2, \ldots, \xi_m)$ is defined as a vector from Γ to the set $[0, 1]^m$, i.e., for any $\gamma \in \Gamma$, $\xi(\gamma) = (\xi_1(\gamma), \xi_2(\gamma), \ldots, \xi_m(\gamma)) \in [0, 1]^m$. As m=1, ξ is called a regular fuzzy variable (RFV). For example, $\xi = (r_1, r_2, r_3)$ with $0 \le r_1 < r_2 < r_3 \le 1$ is a triangular RFV. A fuzzy variable which only takes on value 0 with possibility 1 is an RFV, denoted by $\tilde{0}$. A fuzzy variable which only takes on value 1 with possibility 1 is an RFV, denoted by $\tilde{1}$.

In this paper, we denote by $\mathcal{R}([0, 1])$ as the collection of all RFVs on $[0, 1]$.

Definition 1 ([10]) *Let $\xi_i, 1 \le i \le m$ be m_i-ary regular fuzzy vectors defined on a possibility space $(\Gamma, \mathcal{A}, \mathrm{Pos})$, respectively. They are said to be mutually independent if*

$$\mathrm{Pos}\{\gamma \in \Gamma \mid \xi_1(\gamma) = t_1, \ldots, \xi_m(\gamma) = t_m\} = \min_{1\le i\le m} \mathrm{Pos}\{\gamma \in \Gamma \mid \xi_i(\gamma) = t_i\}$$

for any $t_i = (t_1^{(i)}, \ldots, t_{m_i}^{(i)}) \in [0, 1]^{m_i}$ and $i = 1, \ldots, m$.

Moreover, a family of regular fuzzy vectors $\{\xi_i, i \in I\}$ is said to be mutually independent if for each integer m, and $i_1 < i_2 < \cdots < i_m$, the regular fuzzy vectors $\xi_{i_k}, k = 1, 2, \ldots, m$ are mutually independent.

Definition 2 ([13]) *Let $\mathcal{A}$ be an ample field on the universe Γ, and $\tilde{\mathrm{Pos}} : \mathcal{A} \mapsto \mathcal{R}([0, 1])$ a set function on $\mathcal{A}$ such that $\{\tilde{\mathrm{Pos}}(A) \mid \mathcal{A} \ni A \text{ atom}\}$ is a family of mutually independent RFVs. We call $\tilde{\mathrm{Pos}}$ a fuzzy possibility measure if it satisfies the following conditions:*

Pos1) $\tilde{\mathrm{Pos}}(\emptyset) = \tilde{0}$;

Pos2) *For any subclass $\{A_i \mid i \in I\}$ of $\mathcal{A}$ (finite, countable or uncountable),*

$$\tilde{\mathrm{Pos}}\left(\bigcup_{i\in I} A_i\right) = \sup_{i\in I} \tilde{\mathrm{Pos}}(A_i).$$

Moreover, if $\mu_{\tilde{\mathrm{Pos}}(\Gamma)}(1) = 1$, then we call $\tilde{\mathrm{Pos}}$ a regular fuzzy possibility measure.

The triplet $(\Gamma, \mathcal{A}, \tilde{\mathrm{Pos}})$ is referred to as a fuzzy possibility space (FPS).

From the definition, we know that if $A \subset B$, then $\tilde{\mathrm{Pos}}(A) \vee \tilde{\mathrm{Pos}}(B) = \tilde{\mathrm{Pos}}(B)$.

If the universe Γ is a finite set, then the ample field $\mathcal{A}$ on Γ is an algebra containing a finite number of subsets of Γ. Therefore, the axiom Pos2) in Definition 2 can be replaced by

$$\tilde{\mathrm{Pos}}\left(\bigcup_{i=1}^{n} A_i\right) = \max_{1\le i\le n} \tilde{\mathrm{Pos}}(A_i)$$

for any finite subclass $\{A_i, i = 1, \ldots, n\}$ of $\mathcal{A}$.

If $\mathcal{A}$ is the power set of the universe Γ, then the atoms of $\mathcal{A}$ are all single point sets $\{\gamma\}, \gamma \in \Gamma$. Therefore, in order to define a fuzzy possibility measure on $\mathcal{A}$, it suffices to give the value of $\tilde{\mathrm{Pos}}$ at each single point set.

For the concept of atom, the interested reader may consult the reference [12].

Definition 3 ([13]) *Let* $(\Gamma, \mathcal{A}, \tilde{\text{Pos}})$ *be an FPS. A map* $\xi = (\xi_1, \xi_2, \ldots, \xi_m) : \Gamma \mapsto \Re^m$ *is called an* m*-ary T2 fuzzy vector if for any* $x = (x_1, x_2, \ldots, x_m) \in \Re^m$*, the set* $\{\gamma \in \Gamma \mid \xi(\gamma) \leq x\}$ *is an element of* $\mathcal{A}$*, i.e.,*

$$\{\gamma \in \Gamma \mid \xi(\gamma) \leq x\} = \{\gamma \in \Gamma \mid \xi_1(\gamma) \leq x_1, \ldots, \xi_m(\gamma) \leq x_m\} \in \mathcal{A}.$$

As $m = 1$*, the map* $\xi : \Gamma \mapsto \Re$ *is called a T2 fuzzy variable.*

Definition 4 ([13]) *Let* $\xi = (\xi_1, \xi_2, \ldots, \xi_m)$ *be a T2 fuzzy vector defined on an FPS* $(\Gamma, \mathcal{A}, \tilde{\text{Pos}})$*. The secondary possibility distribution function of* ξ*, denoted by* $\tilde{\mu}_\xi(x)$*, is a map* $\Re^m \mapsto \mathcal{R}[0,1]$ *such that*

$$\tilde{\mu}_\xi(x) = \tilde{\text{Pos}}\{\gamma \in \Gamma \mid \xi(\gamma) = x\}, \quad x \in \Re^m,$$

while the T2 possibility distribution function of ξ*, denoted by* $\mu_\xi(x, u)$*, is a map* $\Re^m \times J_x \mapsto [0,1]$ *such that*

$$\mu_\xi(x, u) = \text{Pos}\left\{\tilde{\mu}_\xi(x) = u\right\}, \quad (x, u) \in \Re^m \times J_x$$

where Pos *is the possibility measure induced by the distribution of* $\tilde{\mu}_\xi(x)$*, and* $J_x \subset [0,1]$ *is the support of* $\tilde{\mu}_\xi(x)$*, i.e.,* $J_x = \{u \in [0,1] \mid \mu_\xi(x, u) > 0\}$*.*

Definition 5 ([13]) *The support of a T2 fuzzy vector* ξ *is defined as*

$$supp\,\xi = \{(x, u) \in \Re^m \times [0,1] \mid \mu_\xi(x, u) > 0\}$$

where $\mu_\xi(x, u)$ *is the T2 possibility distribution function of* ξ*.*

A type-2 fuzzy variable ξ is called triangular [21] if its secondary possibility distribution function $\tilde{\mu}_\xi(x)$ is a triangular RFV

$$\left(\tfrac{x-r_1}{r_2-r_1} - \theta_l \min\{\tfrac{x-r_1}{r_2-r_1}, \tfrac{r_2-x}{r_2-r_1}\}, \tfrac{x-r_1}{r_2-r_1}, \tfrac{x-r_1}{r_2-r_1} + \theta_r \min\{\tfrac{x-r_1}{r_2-r_1}, \tfrac{r_2-x}{r_2-r_1}\}\right)$$

for $x \in [r_1, r_2]$, and

$$\left(\tfrac{r_3-x}{r_3-r_2} - \theta_l \min\{\tfrac{r_3-x}{r_3-r_2}, \tfrac{x-r_2}{r_3-r_2}\}, \tfrac{r_3-x}{r_3-r_2}, \tfrac{r_3-x}{r_3-r_2} + \theta_r \min\{\tfrac{r_3-x}{r_3-r_2}, \tfrac{x-r_2}{r_3-r_2}\}\right)$$

for $x \in [r_2, r_3]$, where $\theta_l, \theta_r \in [0,1]$ are two parameters characterizing the degree of uncertainty that ξ takes on the value x. When $x \in [r_2, \frac{r_2+r_3}{2}]$ and $u \in \left[\frac{r_3-x}{r_3-r_2}, \frac{r_3-x}{r_3-r_2} + \theta_r \min\{\frac{r_3-x}{r_3-r_2}, \frac{x-r_2}{r_3-r_2}\}\right]$, then

$$\mu_\xi(x, u) = \frac{\frac{r_3-x}{r_3-r_2} + \theta_r \frac{x-r_2}{r_3-r_2} - u}{\theta_r \frac{x-r_2}{r_3-r_2}}.$$

We denote the T2 triangular fuzzy variable ξ with the above distribution by $(\tilde{r_1}, \tilde{r_2}, \tilde{r_3}; \theta_l, \theta_r)$.

3 The Representative Value of T2 Fuzzy Variable

3.1 The Concept of Representative Value

In this section, we first define the representative value of a T2 fuzzy variable.

Definition 6 *Let* ξ *be a T2 fuzzy variable. The representative value of* ξ *is defined as*

$$R[\xi] = \int_0^{+\infty} E\left[\tilde{\text{Pos}}\{\gamma \in \Gamma | \xi(\gamma) \geq r\}\right] \mathrm{d}r - \int_{-\infty}^{0} E\left[\tilde{\text{Pos}}\{\gamma \in \Gamma | \xi(\gamma) \leq r\}\right] \mathrm{d}r. \tag{1}$$

When the two integrals are all ∞*, the representative value is not defined.*

In Definition 6, E is the expected value operator [11] of a fuzzy variable. For simplicity, we write the representative value of ξ as

$$R[\xi] = \int_0^{+\infty} E\left[\tilde{\text{Pos}}\{\xi \geq r\}\right] \mathrm{d}r - \int_{-\infty}^{0} E\left[\tilde{\text{Pos}}\{\xi \leq r\}\right] \mathrm{d}r. \tag{2}$$

It is easy to know that $R[\xi] = \int_0^{+\infty} E\left[\tilde{\text{Pos}}\{\gamma \in \Gamma | \xi(\gamma) \geq r\}\right] \mathrm{d}r$ when $\tilde{\text{Pos}}\{\xi \leq 0\} = \tilde{0}$. And if $\tilde{\text{Pos}}\{\xi \geq 0\} = \tilde{0}$, then $R[\xi] = -\int_{-\infty}^{0} E\left[\tilde{\text{Pos}}\{\gamma \in \Gamma | \xi(\gamma) \leq r\}\right] \mathrm{d}r$.

Based on the representative value of a T2 fuzzy variable, we define the representative value of a T2 fuzzy vector as follows.

Definition 7 *Let $\xi = (\xi_1, \xi_2, \ldots, \xi_m)$ be a T2 fuzzy vector defined on an FPS $(\Gamma, \mathcal{A}, \tilde{\text{Pos}})$. Then the representative value of ξ is defined as*

$$R[\xi] = (R[\xi_1], R[\xi_2], \ldots, R[\xi_m]).$$

In the following, we provide one example to illustrate how to calculate the representative value of a T2 fuzzy variable.

Example 1: Let $\Gamma = [-10, 10]$, and $\mathcal{A} = \mathcal{P}(\Gamma)$. Define a set function $\tilde{\text{Pos}} : \mathcal{P}(\Gamma) \mapsto \mathcal{R}([0,1])$ as follows

$$\tilde{\text{Pos}}\{\gamma\} = \left(1 - \tfrac{|\gamma|}{10}, 1 - \tfrac{|\gamma|}{15}, 1 - \tfrac{|\gamma|}{20}\right), \quad \gamma \in [-10, 10]$$

and

$$\tilde{\text{Pos}}(A) = \sup_{\gamma \in A} \tilde{\text{Pos}}\{\gamma\}$$

for any $A \in \mathcal{P}(\Gamma)$, where $\{(1 - \frac{|\gamma|}{10}, 1 - \frac{|\gamma|}{15}, 1 - \frac{|\gamma|}{20}), \gamma \in [-10, 0) \cup (0, 10]\}$ is supposed to be a family of mutually independent triangular RFVs. Then $\tilde{\text{Pos}}$ is a fuzzy possibility measure, and the triplet $(\Gamma, \mathcal{P}(\Gamma), \tilde{\text{Pos}})$ is an FPS. Define a function $\xi : \Gamma \to \Re$ as follows

$$\xi(\gamma) = \gamma.$$

Then ξ is a T2 fuzzy variable. The support of ξ is showed in Figure 1.

Since $\tilde{\text{Pos}}\{\gamma_1\} \vee \tilde{\text{Pos}}\{\gamma_2\} = \tilde{\text{Pos}}\{\gamma_1\}$ for $\gamma_1, \gamma_2 \in \Gamma$ such that $|\gamma_1| < |\gamma_2|$, we have

$$\sup_{-10 \leq \gamma \leq t \leq 0} \tilde{\text{Pos}}\{\gamma\} = \tilde{\text{Pos}}\{t\}$$

and

$$\sup_{0 \leq t \leq \gamma \leq 10} \tilde{\text{Pos}}\{\gamma\} = \tilde{\text{Pos}}\{t\}.$$

Moreover, we obtain the representative value of ξ as follows

$$\begin{aligned} R[\xi] &= \int_0^{+\infty} E\left[\tilde{\text{Pos}}\{\xi \geq t\}\right] \mathrm{d}t - \int_{-\infty}^{0} E\left[\tilde{\text{Pos}}\{\xi \leq t\}\right] \mathrm{d}t \\ &= \int_0^{10} E\left[\tilde{\text{Pos}}\{\xi \geq t\}\right] \mathrm{d}t - \int_{-10}^{0} E\left[\tilde{\text{Pos}}\{\xi \leq t\}\right] \mathrm{d}t \\ &= \int_0^{10} E\left[\tilde{\text{Pos}}\{t\}\right] \mathrm{d}t - \int_{-10}^{0} E\left[\tilde{\text{Pos}}\{t\}\right] \mathrm{d}t \end{aligned}$$

According to the definition of the expectation [11] of fuzzy variables, if $\xi = (r_1, r_2, r_3)$ is a triangular RFV, then we have $E[\xi] = \frac{r_1 + 2r_2 + r_3}{4}$. Therefore,

$$E[\tilde{\text{Pos}}\{t\}] = \; 1 - \tfrac{17}{240}|t| \;, \qquad t \in [-10, 0) \cup (0, 10].$$

As a consequence of calculation, we have $R[\xi] = 0$.

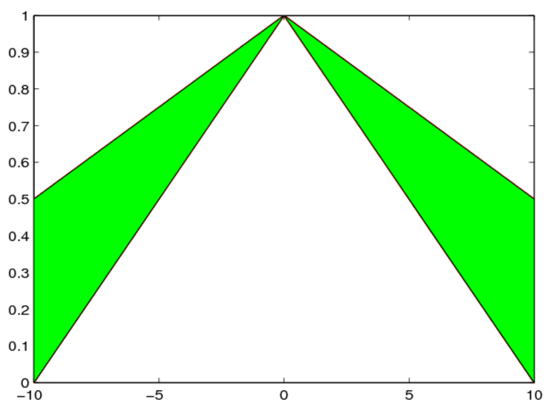

Figure 1: The support of the T2 fuzzy variable ξ defined in Example 1

3.2 The Properties of Representative Value

For the representative value operator of a T2 fuzzy variable, we deduce the following results.

Theorem 1 *Let ξ be a T2 fuzzy variable which only takes on value c with possibility $\tilde{1}$. Then $R[\xi] = c$.*

Proof: For $c \geq 0$, one has

$$R[\xi] = \int_0^c E\left[\tilde{\mathrm{Pos}}\{\gamma \in \Gamma | \xi(\gamma) \geq r\}\right] \mathrm{d}r = \int_0^c E[\tilde{1}]\mathrm{d}r = c.$$

Similarly, we have

$$R[\xi] = -\int_c^0 E\left[\tilde{\mathrm{Pos}}\{\gamma \in \Gamma | \xi(\gamma) \leq r\}\right] \mathrm{d}r = -\int_c^0 E[\tilde{1}]\mathrm{d}r = c$$

for $c < 0$. The proof of the theorem is complete.

Theorem 2 *Let ξ be a T2 fuzzy variable. If $R[\xi]$ exists, then $R[a\xi] = aR[\xi]$ for any real number a.*

Proof: For $a = 0$, we have $R[a\xi] = aR[\xi] = 0$. For $a < 0$, we obtain

$$\begin{aligned}
&\int_0^{+\infty} E\left[\tilde{\mathrm{Pos}}\{\gamma \in \Gamma | (a\xi)(\gamma) \geq r\}\right] \mathrm{d}r - \int_{-\infty}^0 E\left[\tilde{\mathrm{Pos}}\{\gamma \in \Gamma | (a\xi)(\gamma) \leq r\}\right] \mathrm{d}r \\
&= a\int_0^{+\infty} E\left[\tilde{\mathrm{Pos}}\{\gamma \in \Gamma | \xi(\gamma) \leq \tfrac{r}{a}\}\right] \mathrm{d}\left(\tfrac{r}{a}\right) - a\int_{-\infty}^0 E\left[\tilde{\mathrm{Pos}}\{\gamma \in \Gamma | \xi(\gamma) \geq \tfrac{r}{a}\}\right] \mathrm{d}\left(\tfrac{r}{a}\right) \\
&= a\int_0^{-\infty} E\left[\tilde{\mathrm{Pos}}\{\gamma \in \Gamma | \xi(\gamma) \leq t\}\right] \mathrm{d}t - a\int_{+\infty}^0 E\left[\tilde{\mathrm{Pos}}\{\gamma \in \Gamma | \xi(\gamma) \geq t\}\right] \mathrm{d}t \\
&= a\int_0^{+\infty} E\left[\tilde{\mathrm{Pos}}\{\gamma \in \Gamma | \xi(\gamma) \geq t\}\right] \mathrm{d}t - a\int_{-\infty}^0 E\left[\tilde{\mathrm{Pos}}\{\gamma \in \Gamma | \xi(\gamma) \leq t\}\right] \mathrm{d}t.
\end{aligned}$$

As a consequence, by Definition 6, $R[a\xi] = aR[\xi]$ for any real number $a < 0$.

Similarly, in the case of $a > 0$, we have $R[a\xi] = aR[\xi]$, too.

Therefore, for any real number a, we have $R[a\xi] = aR[\xi]$. The proof of the theorem is complete.

Theorem 3 *Let ξ be a T2 fuzzy variable defined on an FPS $(\Gamma, \mathcal{A}, \tilde{\mathrm{Pos}})$, $E[\tilde{\mathrm{Pos}}(\Gamma)] = M$, and $a > 0$ be a real number. If $R[\xi]$ exists, then*

1) *$R[\xi + b] = R[\xi] + bM$ for $\tilde{\mathrm{Pos}}\{\xi < a\} = \tilde{0}$ and any real number $b > -a$;*

2) *$R[\xi + b] = R[\xi] + bM$ for $\tilde{\mathrm{Pos}}\{\xi > -a\} = \tilde{0}$ and any real number $b < a$.*

Proof: 1) From $\tilde{\text{Pos}}\{\xi < a\} = \tilde{0}$ and $b > -a$, we have $\tilde{\text{Pos}}\{\xi < -b\} = \tilde{\text{Pos}}\{\xi < 0\} = \tilde{\text{Pos}}\{\xi + b < 0\} = \tilde{0}$. So, the representative value of $\xi + b$ is

$$\begin{aligned} R[\xi + b] &= \int_0^{+\infty} E\left[\tilde{\text{Pos}}\{\gamma \in \Gamma | (\xi + b)(\gamma) \geq r\}\right] \mathrm{d}r \\ &= \int_0^{+\infty} E\left[\tilde{\text{Pos}}\{\gamma \in \Gamma | \xi(\gamma) \geq r - b\}\right] \mathrm{d}r \\ &= \int_{-b}^{+\infty} E\left[\tilde{\text{Pos}}\{\gamma \in \Gamma | \xi(\gamma) \geq t\}\right] \mathrm{d}t \\ &= \int_0^{+\infty} E\left[\tilde{\text{Pos}}\{\gamma \in \Gamma | \xi(\gamma) \geq t\}\right] \mathrm{d}t + \int_{-b}^{0} E\left[\tilde{\text{Pos}}\{\gamma \in \Gamma | \xi(\gamma) \geq t\}\right] \mathrm{d}t \\ &= R[\xi] + bM. \end{aligned}$$

2) From $\tilde{\text{Pos}}\{\xi > -a\} = \tilde{0}$ and $b < a$, we have $\tilde{\text{Pos}}\{\xi > -b\} = \tilde{\text{Pos}}\{\xi > 0\} = \tilde{\text{Pos}}\{\xi + b > 0\} = \tilde{0}$. So, we obtain the representative value of $\xi + b$ as follows

$$\begin{aligned} R[\xi + b] &= -\int_{-\infty}^{0} E\left[\tilde{\text{Pos}}\{\gamma \in \Gamma | (\xi + b)(\gamma) \leq r\}\right] \mathrm{d}r \\ &= -\int_{-\infty}^{0} E\left[\tilde{\text{Pos}}\{\gamma \in \Gamma | \xi(\gamma) \leq r - b\}\right] \mathrm{d}r \\ &= -\int_{-\infty}^{-b} E\left[\tilde{\text{Pos}}\{\gamma \in \Gamma | \xi(\gamma) \leq t\}\right] \mathrm{d}t \\ &= -\int_{-\infty}^{0} E\left[\tilde{\text{Pos}}\{\gamma \in \Gamma | \xi(\gamma) \leq t\}\right] \mathrm{d}t - \int_{0}^{-b} E\left[\tilde{\text{Pos}}\{\gamma \in \Gamma | \xi(\gamma) \leq t\}\right] \mathrm{d}t \\ &= R[\xi] + bM. \end{aligned}$$

The proof of the theorem is complete.

Theorem 4 *Let ξ be a T2 fuzzy variable, $f : \Re \to \Re$ and $g : \Re \to \Re$ be real-valued continuous functions. If $f \leq g$, then we have $R[f(\xi)] \leq R[g(\xi)]$.*

Proof: Since $f \leq g$, we have that $f(\xi(\gamma)) \leq g(\xi(\gamma))$ for any $\gamma \in \Gamma$. So, we can say that

$$\{\gamma \in \Gamma | f(\xi(\gamma)) \geq r\} \subseteq \{\gamma \in \Gamma | g(\xi(\gamma)) \geq r\}, \tag{3}$$

$$\{\gamma \in \Gamma | g(\xi(\gamma)) \leq r\} \subseteq \{\gamma \in \Gamma | f(\xi(\gamma)) \leq r\}. \tag{4}$$

From (3), we have

$$\tilde{\text{Pos}}\{\gamma \in \Gamma | f(\xi(\gamma)) \geq r\} \vee \tilde{\text{Pos}}\{\gamma \in \Gamma | g(\xi(\gamma)) \geq r\} = \tilde{\text{Pos}}\{\gamma \in \Gamma | g(\xi(\gamma)) \geq r\}.$$

Moreover, it is easy to know

$$E\left[\tilde{\text{Pos}}\{\gamma \in \Gamma | f(\xi(\gamma)) \geq r\}\right] \leq E\left[\tilde{\text{Pos}}\{\gamma \in \Gamma | g(\xi(\gamma)) \geq r\}\right].$$

In the same way, from (4), we have

$$E\left[\tilde{\text{Pos}}\{\gamma \in \Gamma | g(\xi(\gamma)) \leq r\}\right] \leq E\left[\tilde{\text{Pos}}\{\gamma \in \Gamma | f(\xi(\gamma)) \leq r\}\right].$$

Therefore, we have

$$\begin{aligned} &\int_0^{+\infty} E\left[\tilde{\text{Pos}}\{\gamma \in \Gamma | f(\xi(\gamma)) \geq r\}\right] \mathrm{d}r - \int_{-\infty}^{0} E\left[\tilde{\text{Pos}}\{\gamma \in \Gamma | f(\xi(\gamma)) \leq r\}\right] \mathrm{d}r \\ &\leq \int_0^{+\infty} E\left[\tilde{\text{Pos}}\{\gamma \in \Gamma | g(\xi(\gamma)) \geq r\}\right] \mathrm{d}r - \int_{-\infty}^{0} E\left[\tilde{\text{Pos}}\{\gamma \in \Gamma | g(\xi(\gamma)) \leq r\}\right] \mathrm{d}r. \end{aligned}$$

By Definition 6, we know that the inequation $R[f(\xi)] \leq R[g(\xi)]$ is true. The proof of the theorem is complete.

Theorem 5 *Let ξ be a T2 fuzzy variable, $f : \Re \to \Re$ be real-valued continuous function. If $R[f(\xi)]$ exists, then we have $|R[f(\xi)]| \le R[|f(\xi)|]$.*

Proof: If $R[f(\xi)] \ge 0$, with Theorem 4, we have

$$|R[f(\xi)]| = R[f(\xi)] \le R[|f(\xi)|].$$

If $R[f(\boldsymbol{\xi})] < 0$, then by Theorems 2 and 4, one has

$$|R[f(\xi)]| = -R[f(\xi)] = R[-f(\xi)] \le R[|f(\xi)|].$$

So, we can say that

$$|R[f(\xi)]| \le R[|f(\xi)|].$$

The proof of the theorem is complete.

4 The Representative Value of Discrete T2 Fuzzy Variable

Let

$$\tilde{\mu}_\xi(x) = \begin{cases} \eta_1, & \text{if } x = a_1 \\ \cdots & \\ \eta_i, & \text{if } x = a_i \\ \cdots & \\ \eta_n, & \text{if } x = a_n, \end{cases}$$

i.e., ξ takes on crisp value $a_i (i = 1, 2, \ldots, n)$ with possibility η_i. Suppose that $\{\eta_i,\ i = 1, 2, \cdots, n\}$ is a family of mutually independent RFVs. Then ξ is a discrete T2 fuzzy variable. Without loss of generality, we assume that $a_1 \le a_2 \le \cdots \le a_n$. By Definition 6, we have

Theorem 6 *For the above discrete T2 fuzzy variable ξ,*

1) *if $a_1 \ge 0$, then*

$$R[\xi] = \sum_{t=1}^{n} (a_t - a_{t-1}) p_t \tag{5}$$

where $a_0 = 0,\ p_t = E\left[\max_{t \le j \le n} \eta_j\right],\ t = 1, \ldots, n;$

2) *if $a_n \le 0$, then*

$$R[\xi] = \sum_{t=1}^{n} (a_{t+1} - a_t) p_t \tag{6}$$

where $a_{n+1} = 0,\ p_t = -E\left[\max_{1 \le j \le t} \eta_j\right],\ t = 1, \ldots, n;$

3) *if $a_i \le 0 \le a_{i+1}, i = 1, 2, \ldots, n-1$, then*

$$R[\xi] = a_{i+1} E\left[\max_{i+1 \le j \le n} \eta_j\right] + a_i E\left[\max_{1 \le j \le i} \eta_j\right] + \sum_{t=2, t \ne i+1}^{n} (a_t - a_{t-1}) p_t \tag{7}$$

where

$$p_t = -E\left[\max_{1 \le j \le t-1} \eta_j\right],\ t = 2, \ldots, i, \quad p_t = E\left[\max_{t \le j \le n} \eta_j\right],\ t = i+2, \ldots, n.$$

Proof: We only prove 3). The rest conclusions can be proved similarly.

Since $a_i \le 0 \le a_{i+1}, i = 1, 2, \ldots, n-1$, one has

$$\begin{aligned}
R[\xi] &= \int_0^{+\infty} E\left[\tilde{\text{Pos}}\{\xi \ge r\}\right] \mathrm{d}r - \int_{-\infty}^{0} E\left[\tilde{\text{Pos}}\{\xi \le r\}\right] \mathrm{d}r \\
&= \int_0^{a_{i+1}} E\left[\tilde{\text{Pos}}\{\xi \ge r\}\right] \mathrm{d}r + \int_{a_{i+1}}^{a_{i+2}} E\left[\tilde{\text{Pos}}\{\xi \ge r\}\right] \mathrm{d}r + \cdots + \int_{a_{n-1}}^{a_n} E\left[\tilde{\text{Pos}}\{\xi \ge r\}\right] \mathrm{d}r \\
&\quad - \int_{a_1}^{a_2} E\left[\tilde{\text{Pos}}\{\xi \le r\}\right] \mathrm{d}r - \cdots - \int_{a_{i-1}}^{a_i} E\left[\tilde{\text{Pos}}\{\xi \le r\}\right] \mathrm{d}r - \int_{a_i}^{0} E\left[\tilde{\text{Pos}}\{\xi \le r\}\right] \mathrm{d}r \\
&= \int_0^{a_{i+1}} E\left[\max_{i+1\le j\le n} \eta_j\right] \mathrm{d}r + \int_{a_{i+1}}^{a_{i+2}} E\left[\max_{i+2\le j\le n} \eta_j\right] \mathrm{d}r + \cdots + \int_{a_{n-1}}^{a_n} E\left[\max_{n\le j\le n} \eta_j\right] \mathrm{d}r \\
&\quad - \int_{a_1}^{a_2} E\left[\max_{1\le j\le 1} \eta_j\right] \mathrm{d}r - \cdots - \int_{a_{i-1}}^{a_i} E\left[\max_{1\le j\le i-1} \eta_j\right] \mathrm{d}r - \int_{a_i}^{0} E\left[\max_{1\le j\le i} \eta_j\right] \mathrm{d}r \\
&= a_{i+1} E\left[\max_{i+1\le j\le n} \eta_j\right] + a_i E\left[\max_{1\le j\le i} \eta_j\right] + \textstyle\sum_{t=2, t\ne i+1}^{n} (a_t - a_{t-1}) p_t
\end{aligned}$$

where

$$p_t = -E\left[\max_{1\le j\le t-1} \eta_j\right], \ t = 2, \ldots, i, \quad p_t = E\left[\max_{t\le j\le n} \eta_j\right], \ t = i+2, \ldots, n.$$

The proof of 3) is complete.

It is easy to know that Theorem 6 can be described as the following theorem.

Theorem 7 *For the above discrete T2 fuzzy variable ξ,*

1) *if $a_1 \ge 0$, then*

$$R[\xi] = \sum_{t=1}^{n} a_t p_t \tag{8}$$

where

$$p_t = E\left[\max_{t\le j\le n} \eta_j\right] - E\left[\max_{t+1\le j\le n} \eta_j\right], \ t = 1, 2, \ldots, n-1, \quad p_n = E\left[\eta_n\right];$$

2) *if $a_n \le 0$, then*

$$R[\xi] = \sum_{t=1}^{n} a_t p_t \tag{9}$$

where

$$p_1 = E\left[\eta_1\right], \quad p_t = E\left[\max_{1\le j\le t} \eta_j\right] - E\left[\max_{1\le j\le t-1} \eta_j\right], \ t = 2, \ldots, n;$$

3) *if $a_i \le 0 \le a_{i+1}, i = 1, 2, \ldots, n-1$, then*

$$R[\xi] = \sum_{t=1}^{n} a_t p_t \tag{10}$$

where

$$\begin{aligned}
&p_1 = E\left[\eta_1\right], \quad p_n = E\left[\eta_n\right], \\
&p_t = E\left[\max_{1\le j\le t} \eta_j\right] - E\left[\max_{1\le j\le t-1} \eta_j\right], \ t = 2, \ldots, i, \\
&p_t = E\left[\max_{t\le j\le n} \eta_j\right] - E\left[\max_{t+1\le j\le n} \eta_j\right], \ t = i+1, \ldots, n-1.
\end{aligned}$$

In the following, we provide one example to illustrate Theorems 6 and 7.

Example 2: Let $\Gamma = \{\gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5\}$, and $\mathcal{A} = \mathcal{P}(\Gamma)$. Define a set function $\tilde{\text{Pos}} : \mathcal{P}(\Gamma) \mapsto \mathcal{R}([0,1])$ as follows

$$\tilde{\text{Pos}}\{\gamma_i\} = \left(0, \tfrac{i}{10}, \tfrac{i}{5}\right), \quad i = 1, \ldots, 5$$

and

$$\tilde{\text{Pos}}(A) = \max_{\gamma\in A} \tilde{\text{Pos}}\{\gamma\}$$

for any subset A of Γ, where $\left(0, \frac{i}{10}, \frac{i}{5}\right), i = 1, \ldots, 5$ are supposed to be mutually independent triangular RFVs. Then, $\tilde{\text{Pos}}$ is a fuzzy possibility measure on $\mathcal{P}(\Gamma)$, and $(\Gamma, \mathcal{P}(\Gamma), \tilde{\text{Pos}})$ is an FPS. Define a function $\xi : \Gamma \to \Re$ as follows

$$\xi(\gamma_i) = 2i.$$

Then ξ is a T2 fuzzy variable. Since

$$\max_{1\le j\le 5} \tilde{\text{Pos}}\{\gamma_j\} = \max_{2\le j\le 5} \tilde{\text{Pos}}\{\gamma_j\} = \max_{3\le j\le 5} \tilde{\text{Pos}}\{\gamma_j\} = \max_{4\le j\le 5} \tilde{\text{Pos}}\{\gamma_j\} = \tilde{\text{Pos}}\{\gamma_5\},$$

we have

$$E\left[\max_{1\le j\le 5} \tilde{\text{Pos}}\{\gamma_j\}\right] = E\left[\max_{2\le j\le 5} \tilde{\text{Pos}}\{\gamma_j\}\right] = E\left[\max_{3\le j\le 5} \tilde{\text{Pos}}\{\gamma_j\}\right] = E\left[\max_{4\le j\le 5} \tilde{\text{Pos}}\{\gamma_j\}\right] = E\left[\tilde{\text{Pos}}\{\gamma_5\}\right] = 0.5.$$

By (8), we have

$$R[\xi] = 2 \times p_1 + 4 \times p_2 + 6 \times p_3 + 8 \times p_4 + 10 \times p_5 = 5$$

where $p_1 = p_2 = p_3 = p_4 = 0,\ p_5 = 0.5$. Also, by (5), we have

$$R[\xi] = (2-0) \times p_1 + (4-2) \times p_2 + (6-4) \times p_3 + (8-6) \times p_4 + (10-8) \times p_5 = 5$$

where $p_1 = p_2 = p_3 = p_4 = p_5 = 0.5$.

5 The Representative Value of T2 Triangular Fuzzy Variable

In this section, we deduce the computational formula of the representative value for T2 triangular fuzzy variable.

Theorem 8 *Let $\xi = (\tilde{r_1}, \tilde{r_2}, \tilde{r_3}; \theta_l, \theta_r)$ be a T2 triangular fuzzy variable. The secondary possibility distribution function of ξ is $\tilde{\mu}_\xi(x), x \in \Re$, $\{\tilde{\mu}_\xi(x), x \in [r_1, r_3]\}$ is supposed to be a family of mutually independent RFVs.*

1) *If $r_1 \ge 0$, then*

$$R[\xi] = \tfrac{r_2+r_3}{2} + \tfrac{(\theta_r-\theta_l)(r_3-r_2)}{16}; \tag{11}$$

2) *If $r_1 \le 0 \le \frac{r_1+r_2}{2}$, then*

$$R[\xi] = \tfrac{r_2+r_3}{2} - \tfrac{r_1^2}{2(r_2-r_1)} + \tfrac{(\theta_r-\theta_l)(r_3-r_2)}{16} - \tfrac{(\theta_r-\theta_l)r_1^2}{8(r_2-r_1)}; \tag{12}$$

3) *If $\frac{r_1+r_2}{2} \le 0 \le r_2$, then*

$$R[\xi] = \tfrac{r_1+r_2+r_3}{2} - \tfrac{r_1 r_2}{2(r_2-r_1)} + \tfrac{(\theta_r-\theta_l)(r_3+r_1)}{16} + \tfrac{(\theta_r-\theta_l)r_1 r_2}{8(r_2-r_1)}; \tag{13}$$

4) *If $r_2 \le 0 \le \frac{r_3+r_2}{2}$, then*

$$R[\xi] = \tfrac{r_1+r_2+r_3}{2} + \tfrac{r_2 r_3}{2(r_3-r_2)} + \tfrac{(\theta_r-\theta_l)(r_3+r_1)}{16} - \tfrac{(\theta_r-\theta_l)r_2 r_3}{8(r_3-r_2)}; \tag{14}$$

5) *If $\frac{r_2+r_3}{2} \le 0 \le r_3$, then*

$$R[\xi] = \tfrac{r_1+r_2}{2} + \tfrac{r_3^2}{2(r_3-r_2)} - \tfrac{(\theta_r-\theta_l)(r_2-r_1)}{16} + \tfrac{(\theta_r-\theta_l)r_3^2}{8(r_3-r_2)}; \tag{15}$$

6) *If $r_3 \le 0$, then*

$$R[\xi] = \tfrac{r_1+r_2}{2} - \tfrac{(\theta_r-\theta_l)(r_2-r_1)}{16}. \tag{16}$$

Proof: We only prove 5). The rest can be proved similarly.

Note that the secondary possibility distribution $\tilde{\mu}_\xi(x)$ of ξ is the following triangular RFV

$$\left(\tfrac{x-r_1}{r_2-r_1} - \theta_l \min\{\tfrac{x-r_1}{r_2-r_1}, \tfrac{r_2-x}{r_2-r_1}\}, \tfrac{x-r_1}{r_2-r_1}, \tfrac{x-r_1}{r_2-r_1} + \theta_r \min\{\tfrac{x-r_1}{r_2-r_1}, \tfrac{r_2-x}{r_2-r_1}\}\right)$$

for $x \in [r_1, r_2]$, and

$$\left(\tfrac{r_3-x}{r_3-r_2} - \theta_l \min\{\tfrac{r_3-x}{r_3-r_2}, \tfrac{x-r_2}{r_3-r_2}\}, \tfrac{r_3-x}{r_3-r_2}, \tfrac{r_3-x}{r_3-r_2} + \theta_r \min\{\tfrac{r_3-x}{r_3-r_2}, \tfrac{x-r_2}{r_3-r_2}\}\right)$$

for $x \in [r_2, r_3]$. Therefore, we have

$$E[\tilde{\mu}_\xi(x)] = \begin{cases} \frac{(4+\theta_r-\theta_l)(x-r_1)}{4(r_2-r_1)}, & if\ x \in [r_1, \frac{r_1+r_2}{2}] \\ \frac{(4-\theta_r+\theta_l)x+(\theta_r-\theta_l)r_2-4r_1}{4(r_2-r_1)}, & if\ x \in [\frac{r_1+r_2}{2}, r_2] \\ \frac{(-4+\theta_r-\theta_l)x+4r_3-(\theta_r-\theta_l)r_2}{4(r_3-r_2)}, & if\ x \in [r_2, \frac{r_2+r_3}{2}] \\ \frac{(4+\theta_r-\theta_l)(r_3-x)}{4(r_3-r_2)}, & if\ x \in [\frac{r_2+r_3}{2}, r_3]. \end{cases}$$

By the definition of $\tilde{\mu}_\xi(x)$ and Extension Principle of Zadeh, we have

$$\tilde{\mu}_\xi(x_1) \vee \tilde{\mu}_\xi(x_2) = \tilde{\mu}_\xi(x_2), \quad x_1 \le x_2 \le r_2$$

and

$$\tilde{\mu}_\xi(x_1) \vee \tilde{\mu}_\xi(x_2) = \tilde{\mu}_\xi(x_1), \quad r_2 \le x_1 \le x_2.$$

Therefore, we know that

$$\begin{aligned} R[\xi] &= \int_0^{+\infty} E\left[\tilde{\text{Pos}}\{\gamma \in \Gamma | \xi(\gamma) \ge r\}\right] \mathrm{d}r - \int_{-\infty}^0 E\left[\tilde{\text{Pos}}\{\gamma \in \Gamma | \xi(\gamma) \le r\}\right] \mathrm{d}r \\ &= \int_0^{r_3} E\left[\tilde{\text{Pos}}\{\gamma \in \Gamma | \xi(\gamma) \ge r\}\right] \mathrm{d}r - \int_{r_1}^{\frac{r_1+r_2}{2}} E\left[\tilde{\text{Pos}}\{\gamma \in \Gamma | \xi(\gamma) \le r\}\right] \mathrm{d}r \\ &\quad - \int_{\frac{r_1+r_2}{2}}^{r_2} E\left[\tilde{\text{Pos}}\{\gamma \in \Gamma | \xi(\gamma) \le r\}\right] \mathrm{d}r - \int_{r_2}^0 E\left[\tilde{\text{Pos}}\{\gamma \in \Gamma | \xi(\gamma) \le r\}\right] \mathrm{d}r \\ &= \int_0^{r_3} E\left[\tilde{\text{Pos}}\{\gamma \in \Gamma | \xi(\gamma) = r\}\right] \mathrm{d}r - \int_{r_1}^{\frac{r_1+r_2}{2}} E\left[\tilde{\text{Pos}}\{\gamma \in \Gamma | \xi(\gamma) = r\}\right] \mathrm{d}r \\ &\quad - \int_{\frac{r_1+r_2}{2}}^{r_2} E\left[\tilde{\text{Pos}}\{\gamma \in \Gamma | \xi(\gamma) = r\}\right] \mathrm{d}r - \int_{r_2}^0 E\left[\tilde{\text{Pos}}\{\gamma \in \Gamma | \xi(\gamma) = r_2\}\right] \mathrm{d}r \\ &= \int_0^{r_3} \frac{(4+\theta_r-\theta_l)(r_3-x)}{4(r_3-r_2)} \mathrm{d}r - \int_{r_1}^{\frac{r_1+r_2}{2}} \frac{(4+\theta_r-\theta_l)(x-r_1)}{4(r_2-r_1)} \mathrm{d}r \\ &\quad - \int_{\frac{r_1+r_2}{2}}^{r_2} \frac{(4-\theta_r+\theta_l)x+(\theta_r-\theta_l)r_2-4r_1}{4(r_2-r_1)} \mathrm{d}r - \int_{r_2}^0 1\mathrm{d}r \\ &= \frac{r_1+r_2}{2} + \frac{r_3^2}{2(r_3-r_2)} - \frac{(\theta_r-\theta_l)(r_2-r_1)}{16} + \frac{(\theta_r-\theta_l)r_3^2}{8(r_3-r_2)}. \end{aligned}$$

The proof of assertion 5) is complete.

Example 3: Let $\xi = (\tilde{2}, \tilde{3}, \tilde{4}; 0.6, 0.8)$ be a T2 triangular fuzzy variable. The support of ξ is showed in Figure 2. The secondary possibility distribution function of ξ is $\tilde{\mu}_\xi(x), x \in \Re$, $\{\tilde{\mu}_\xi(x), x \in [2,4]\}$ is supposed to be a family of mutually independent RFVs. Then, by (11), we have

$$R[\xi] = \tfrac{r_2+r_3}{2} + \tfrac{(\theta_r-\theta_l)(r_3-r_2)}{16} = 3.5125.$$

Let $\eta = -\xi = (\tilde{-4}, \tilde{-3}, \tilde{-2}; 0.6, 0.8)$. By (16), we have

$$R[\eta] = \tfrac{r_1+r_2}{2} - \tfrac{(\theta_r-\theta_l)(r_2-r_1)}{16} = -3.5125.$$

By Theorem 2, we also have

$$R[\eta] = R[-\xi] = -R[\xi] = -3.5125.$$

Let $\zeta_1 = \eta - 3 = (\widetilde{-7}, \widetilde{-6}, \widetilde{-5}; 0.6, 0.8)$. By (16), we have

$$R[\zeta_1] = \frac{r_1+r_2}{2} - \frac{(\theta_r-\theta_l)(r_2-r_1)}{16} = -6.5125.$$

Since $E[\tilde{\mu}_\eta(-3)] = E[\tilde{1}] = 1$, by Theorem 3, we also have

$$R[\zeta_1] = R[\eta] - 3 = -6.5125.$$

Let $\zeta_2 = \xi - 2 = (\tilde{0}, \tilde{1}, \tilde{2}; 0.6, 0.8)$. By (11), we have

$$R[\zeta_2] = \frac{r_2+r_3}{2} + \frac{(\theta_r-\theta_l)(r_3-r_2)}{16} = 1.5125.$$

By Theorem 3, we also have

$$R[\zeta_2] = R[\xi] - 2 = 1.5125.$$

Let $\xi_1 = \xi - 3 = (\widetilde{-1}, \tilde{0}, \tilde{1}; 0.6, 0.8)$. By (14), we have

$$R[\xi_1] = \frac{r_1+r_2+r_3}{2} + \frac{r_2 r_3}{2(r_3-r_2)} + \frac{(\theta_r-\theta_l)(r_3+r_1)}{16} - \frac{(\theta_r-\theta_l)r_2 r_3}{8(r_3-r_2)} = 0.$$

But $R[\xi] - 3 = 0.5125$. That is to say, $R[\xi - 3] \neq R[\xi] - 3$. So, in general, $R[\xi - c] \neq R[\xi] - c$.

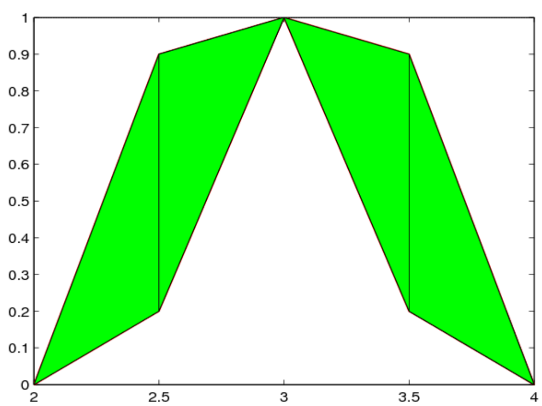

Figure 2: The support of the T2 fuzzy variable ξ defined in Example 3

Usually, the analytical expression of the possibility distribution of $\tilde{\mathrm{Pos}}(A)$ can not be obtained easily due to the complexity of the possibility distributions of $\tilde{\mathrm{Pos}}\{\gamma\}, \gamma \in \Gamma$. So, in general, the representative value of a T2 fuzzy variable can not be obtained easily. In this case, we may employ the approximation scheme developed in [8] to estimate $E\left[\tilde{\mathrm{Pos}}\{\gamma \in \Gamma | \xi(\gamma) \geq r\}\right]$ and $E\left[\tilde{\mathrm{Pos}}\{\gamma \in \Gamma | \xi(\gamma) \leq r\}\right]$ when the possibility distributions of $\tilde{\mathrm{Pos}}\{\gamma \in \Gamma | \xi(\gamma) \geq r\}$ and $\tilde{\mathrm{Pos}}\{\gamma \in \Gamma | \xi(\gamma) \leq r\}$ have infinite supports. The convergence of the approximation method has been discussed in [9].

6 Conclusions

In this paper, we introduced the concept of the representative value for T2 fuzzy variable. Based on the representative value of a T2 fuzzy variable, we also defined the representative value for T2 fuzzy vector. Then we discussed some properties of the representative value operator for T2 fuzzy variable. For discrete T2 fuzzy variable, we obtained two kinds of computational formula of the representative value. Moreover, for T2 triangular fuzzy variables, we deduced the computational formula of the representative value.

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