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# Adaptive Control for Anticipated Function Projective Synchronization of 2D Discrete-time Chaotic Systems with Uncertain Parameters

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**Abstract**

In this paper, adaptive control for anticipated function projective synchronization of 2D discrete-time chaotic systems with uncertain parameters is discussed both theoretically and numerically. On the basis of the chaotic controlling methods: the backstepping design approach and active control method, adaptive control for anticipated function projective synchronization scheme is developed to realize the synchronization of 2D discrete-time chaotic systems with uncertain parameters. Numerical simulations are used to verify the effectiveness of the scheme.


**Keywords:** projective synchronization, discrete-time chaotic system, backstepping design, adaptive control

## 1 Introduction

Synchronizing and controlling chaotic dynamical systems have recently attracted a great deal of attention since the early work on the synchronizing of chaos by Pecora and Carroll [24] and on the controlling of chaos by Ott et al. [22] was published in 1990. Up to now, several synchronization schemes have been proposed due to their many potential applications in secure communication, biological systems, chemical reactions, information processing, social science, and many other fields [5].

A significant result is the discovery of different synchronization phenomena [1-3, 6-10, 15-16, 18, 20-21, 23, 25-26, 28-33], such as complete synchronization [20, 23], anti-synchronization [10, 33], projective synchronization [21, 28], lag synchronization [15, 16], anticipated synchronization [7], phase synchronization [2, 25], generalized synchronization [8, 32], and Q-S synchronization [29, 31]. In the realizing of the synchronization, an adaptive controller is usually used while the chaotic systems to be synchronized are uncertain. Based on backstepping design [3, 9, 26, 30] and the function synchronization method [3], adaptive control and function projective synchronization proposed by us [11-13, 30]. In this paper, we will find that adaptive control for anticipated function projective synchronization (AAFPS) of 2D discrete-time chaotic systems with uncertain parameters can alternatively occur under certain conditions.

The rest of this paper is organized as follows. In Section 2, based on the previous work [24, 28, 25, 29, 18-19], we give the definition of adaptive control for anticipated function projective synchronization in 2D discrete-time chaotic systems with uncertain parameters, and consequently only one controller is obtained. Then, in Section 3, numerical simulations are given for illustration. Finally, some concluding remarks are given in Section 4.

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## 2 Adaptive Control for Anticipated Function Projective Synchronization of 2D Discrete-Time Chaotic Systems with Uncertain Parameters

In this section, based on the previous synchronization [1, 6, 18] and adaptive control and function projective synchronization proposed by us [11-13, 30], we will extend to synchronize 2D discrete-time chaotic systems with uncertain parameters up to a scaling function matrix.

**Definition 1** *For two discrete-time dynamical systems*

$$(i)\ x(k+1) = F(x(k))$$

*and*

$$(ii)\ y(k+1) = G(y(k)) + u(x(k), y(k))$$

*where* $(x(k), y(k)) \in R^{m+m}, k \in Z/Z^-$, *and* $u(x(k), y(k)) \in R^m$, *let*

$$\begin{aligned}(iii)\ E(k) &= (E_1(k), E_2(k), \cdots, E_m(k)) \\ &= (x_1(k+\tau) - f_1(x(k+\tau))y_1(k), x_2(k+\tau) - f_2(x(k+\tau))y_2(k), ..., \\ &\qquad x_m(k+\tau) - f_m(x(k+\tau))y_m(k))\end{aligned}$$

*or*

$$(x_1(k+\tau) - y_1(k), x_2(k+\tau) - y_2(k), \cdots, x_m(k+\tau) - f_m(x(k+\tau))y_m(k))$$

*or*

$$(x_1(k+\tau) - f_1(x(k+\tau))y_1(k), x_2(k+\tau) - y_2(k), ..., x_m(k+\tau) - y_m(k))$$

*be boundary vector functions, if there exists proper controllers* $U(x(k+\tau), y(k)) = (u_1(x(k+\tau), y(k)), u_2(x(k+\tau), y(k)), ..., u_m(x(k+\tau), y(k)))^T$ *such that* $\lim_{k\to\infty}(E(k)) = 0$, *we say that there exist adaptive control for anticipated function projective synchronization between the systems (i) and (ii), and we call* $f_m$ *a "scaling function matrix".*

Consider the drive system in the form of

$$x(k+1) = A_1x(k) + h_1(x(k)), \tag{2.1}$$

assume that the response system is as follows:

$$y(k+1) = A_2y(k) + h_2(y(k)) + U \tag{2.2}$$

where $(x(k), y(k)) \in R^{m+m}$, $A_1$, $A_2$ are $m \times m$ constant matrixes, $h_1, h_2 : R^m \to R^m$ are nonlinear function vectors, and $U$ is a controller to be determined later.

**Theorem 1** *For an invertible diagonal function matrix* $f$, *adaptive control for anticipated function projective synchronization with uncertain parameters between the two systems (2.1) and (2.2) will occur, if the following conditions are satisfied:*

*(i)* $U = f^{-1}h_1(x(k+\tau)) + (f^{-1}A_1f - A_2)y(k) + f^{-1}B(x(k+\tau) - fy(k)) - h_2(y(k))$ *where* $B \in R^{m\times m}$.

*(ii) The real parts of all the eigenvalues of* $(A_1 - B)$ *are negative.*

**Proof:** From $E(k) = x(k+\tau) - fy(k)$ in definition of AAFPS, one can get

$$\begin{aligned}E(k+1) &= x(k+1+\tau) - fy(k+1) \\ &= A_1x(k+\tau) + h_1(x(k+\tau)) - f(A_2y(k) + h_2(y(k)) + U) \\ &= A_1x(k+\tau) + h_1(x(k+\tau)) - fA_2y(k) - fh_2(y(k)) - h_1(x(k+\tau)) \\ &\quad -A_1fy(k) + fA_2y(k) - B(x(k+\tau) - fy(k)) + fh_2(y(k)) \\ &= (A_1 - B)e.\end{aligned}$$

Regards with the Lyapnov stability theory and for a feasible control, the feedback $B$ must be selected such that all the eigenvalues of $(A_1 - B)$ have negative real parts. Thus, if the controllability matrix $(A_1 - B)$ is in full rank, the system $E(k+1)$ is asymptotically stable at the origin, which implies (2.1) and (2.2) are in the state of adaptive control for anticipated function projective synchronization.

It is necessary to point out that the scaling function matrix $f$ also has no effect on the eigenvalues of $(A_1 - B)$ like the modified projective synchronization. Thus one can adjust the scaling matrix arbitrarily during control without worrying about the control robustness. The AAFPS is more general: when $f_1 = f_2 = \cdots = f_m = 1, \tau = 0$, $f_1 = f_2 = \cdots = f_m = \alpha, \tau = 0$ and $f_1 = \alpha_1, f_2 = \alpha_2, \cdots, f_m = \alpha_m, \tau = 0$, the complete synchronization, the projective synchronization and the modified projective synchronization will appear, respectively.

# 3 AAFPS of Lorenz Discrete-Time System with Uncertain Parameters

Consider Lorenz discrete-time system [26]

$$\begin{cases} x_1(k+1) = (1+\alpha\beta)x_1(k) - \beta x_1(k)x_2(k), \\ x_2(k+1) = (1-\beta)x_2(k) + \beta x_1(k)^2 \end{cases} \tag{3.1}$$

and Lorenz system with controllers $u(x, y)$

$$\begin{cases} y_1(k+1) = (1+a(k)b(k))y_1(k) - b(k)y_1(k)y_2(k) + u_1(x,y), \\ y_2(k+1) = (1-b(k))y_2(k) + b(k)y_1(k)^2 + u_2(x,y) \end{cases} \tag{3.2}$$

as the drive system and response system, respectively.

In the following, we would like to realize the AAFPS of Lorenz discrete-time system with uncertain parameters by backstepping design method.

**I :** Let the error states be

$$\begin{aligned} E_1(k) &= x_1(k+\tau) - y_1(k), \\ E_2(k) &= x_2(k+\tau) - \big[1 + \frac{1}{4}\cos(x_1(k+\tau))^2\big]y_2(k), \\ E_3(k) &= a(k) - \alpha, \\ E_4(k) &= b(k) - \beta. \end{aligned}$$

Then from (3.1) and (3.2), we have the discrete-time error dynamical system

$$\begin{cases} E_1(k+1) = x_1(k+1+\tau) - (1+a(k)b(k))y_1(k) + b(k)y_1y_2(k) - u_1(x,y), \\ E_2(k+1) = x_2(k+1+\tau) - (1+\frac{1}{4}\cos(x_1(k+1+\tau)^2) \\ \qquad\qquad ((1-b(k))y_2(k) + b(k)y_1(k)^2 + u_2(x,y)). \end{cases} \tag{3.3}$$

Based on the backstepping design and the improved ideas in Refs. [14, 27], we give a systematic and constructive algorithm to derive the controllers $u(x, y)$ step by step such that systems (3.1) and (3.2) are synchronized together.

**Step 1.** Let the first partial Lyapunov function be $L_1(k) = |E_1(k)|$ and the second error variable be

$$E_2(k) = E_1(k+1) - \delta_{11}E_1(k) \tag{3.4}$$

where $\delta_{11} \in R$. Then we have the derivative of $L_1(k)$

$$\Delta L_1(k) = |E_1(k+1)| - |E_1(k)| \le (|\delta_{11}| - 1)|E_1(k)| + |E_2(k)|. \tag{3.5}$$

**Step 2.** Let the second partial Lyapunov function candidate be $L_2(k) = L_1(k) + c_1|E_2(k)|$ and the third error variable be

$$E_3(k) = E_2(k+1) - \delta_{21}E_1(k) - \delta_{22}E_2(k) \tag{3.6}$$

where $c_1 > 1, \delta_{21}, \delta_{22} \in R$. Therefore, from (3.4) and (3.6) we have the derivative $L_2(k)$

$$\begin{aligned}\Delta L_2(k) &= L_2(k+1) - L_2(k)\\ &\leq (c_1|\delta_{21}| + |\delta_{11}| - 1)|E_1(k)| + (c_1|\delta_{22}| + 1 - c_1)|E_2(k)| + c_1|E_3(k)|.\end{aligned} \tag{3.7}$$

**Step 3.** Let the third partial Lyapunov function candidate be $L_3(k) = L_2(k) + c_2|E_3(k)|$ and the fourth error state be

$$E_4(k) = E_3(k+1) - \delta_{31}E_1(k) - \delta_{32}E_2(k) - \delta_{33}E_3(k) \tag{3.8}$$

where $c_2 > c_1 > 1, \delta_{31}, \delta_{32}, \delta_{33} \in R$. Therefore, from (3.6) and (3.8) we have the derivative $L_3(k)$

$$\begin{aligned}\Delta L_3(k) &= L_3(k+1) - L_3(k)\\ &\leq (c_2|\delta_{31}| + c_1|\delta_{21}| + |\delta_{11}| - 1)|E_1(k)| + (c_2|\delta_{32}| + c_1(|\delta_{22}| - 1) + 1)|E_2(k)|\\ &\quad +(c_2|\delta_{33}| + c_1 - c_2)|E_3(k)| + c_2|E_4(k)|.\end{aligned} \tag{3.9}$$

**Step 4.** Let the fourth partial Lyapunov function candidate be $L_4(k) = L_3(k) + c_3|E_4(k)|$ and the fourth error state be

$$E_4(k+1) - \delta_{41}E_1(k) - \delta_{42}E_2(k) - \delta_{43}E_3(k) - \delta_{44}E_4(k) = 0 \tag{3.10}$$

where $c_3 > c_2 > c_1 > 1, \delta_{41}, \delta_{42}, \delta_{43}, \delta_{44} \in R$. Therefore, from (3.8) and (3.10) we have the derivative $L_4(k)$

$$\begin{aligned}\Delta L_4(k) &= L_4(k+1) - L_4(k)\\ &\leq (c_3|\delta_{41}| + c_2|\delta_{31}| + c_1|\delta_{21}| + |\delta_{11}| - 1)|E_1(k)| + (c_3|\delta_{42}| + c_2|\delta_{32}|\\ &\quad +c_1(|\delta_{22}| - 1) + 1)|E_2(k)| + (c_3|\delta_{43}| + c_2|\delta_{33}| + c_1 - c_2)|E_3(k)|\\ &\quad +(c_3|\delta_{44}| + c_2 - c_3)|E_4(k)|.\end{aligned} \tag{3.11}$$

From (3.11), we know that the right-hand side of (3.11) is negative definite, if the parameters $c_i (i = 1, 2, 3, 4)$ and $\delta_{ij} (1 \leq j \leq i \leq 4)$ satisfy

$$\begin{aligned}&c_1|\delta_{21}| + c_2|\delta_{31}| + c_3|\delta_{41}| + |\delta_{11}| < 1,\\ &c_1|\delta_{22}| + c_2|\delta_{32}| + c_3|\delta_{42}| < c_1 - 1,\\ &c_2|\delta_{33}| + c_3|\delta_{43}| < c_2 - c_1,\\ &|\delta_{44}| < \tfrac{c_3 - c_2}{c_3}.\end{aligned} \tag{3.12}$$

That $\Delta L(k)$ is negative definite denotes the resulting close-loop discrete-time system

$$\begin{pmatrix} E_1(k+1)\\ E_2(k+1)\\ E_3(k+1)\\ E_4(k+1)\end{pmatrix} = \begin{pmatrix} \delta_{11} & 1 & 0 & 0\\ \delta_{21} & \delta_{22} & 1 & 0\\ \delta_{31} & \delta_{32} & \delta_{33} & 0\\ \delta_{41} & \delta_{42} & \delta_{43} & \delta_{44}\end{pmatrix}\begin{pmatrix} E_1(k)\\ E_2(k)\\ E_3(k)\\ E_4(k)\end{pmatrix}. \tag{3.13}$$

With the aid of symbolic computation, from the above equations (3.4), (3.6), (3.8) and (3.10) we obtained the controllers

$$\begin{cases} u_1(x,y) = & x_1(k+1+\tau) - y_1(k) - y_1(k)a(k)b(k) + b(k)y_1(k)y_2(k)\\ & -\delta_{11}x_1(k+\tau) + \delta_{11}y_1(k) - x_2(k+\tau)\\ & +y_2(k) + \frac{1}{4}y_2(k)\cos(x_1(k+\tau))^2,\\ u_2(x,y) = & -\frac{1}{4+\cos(x_1(k+1+\tau))^2}\big[-4x_2(k+1+\tau) + 4y_2(k) - 4y_2(k)b(k)\\ & +4y_1^2(k)b(k) + \cos(x_1(k+1+\tau))^2y_2(k) - \cos(x_1(k+1+\tau))^2y_2(k)b(k)\\ & +\cos(x_1(k+1+\tau))^2y_1(k)^2b(k) + 4\delta_{21}x_1(k+\tau) - 4\delta_{21}y_1(k)\\ & +4\delta_{22}x_2(k+\tau) - 4\delta_{22}y_2(k) - \delta_{22}y_2(k)\cos(x_1(k+\tau))^2\big]\end{cases} \tag{3.14}$$

and

$$\begin{cases} a(k+1) = & \alpha + \delta_{31}x_1(k+\tau) - \delta_{31}y_1(k) + \delta_{32}x_2(k+\tau) - \delta_{32}y_2(k))\\ & -\frac{1}{4}\delta_{32}y_2(k)\cos(x_1(k+\tau))^2 + \delta_{33}a(k) - \delta_{33}\alpha + b(k) - \beta,\\ b(k+1) = & \beta + \delta_{41}x_1(k+\tau) - \delta_{41}y_1(k) + \delta_{42}x_2(k+\tau) - \delta_{42}y_2(k))\\ & -\frac{1}{4}\delta_{42}y_2(k)\cos(x_1(k+\tau))^2 + \delta_{43}a(k) - \delta_{43}\alpha + \delta_{44}b(k) - \delta_{44}\beta.\end{cases} \tag{3.15}$$

In the following we use numerical simulations to verify the effectiveness of the above-mentioned controllers. The parameters are chosen as $\alpha = -0.1, \beta = -1.7, \delta_{11} = 0.3, \delta_{21} = 0.02, \delta_{22} = 0.4, \delta_{31} = 0.05, \delta_{32} = 0.1, \delta_{33} = -0.2, \delta_{41} = 0.01, \delta_{42} = 0.02, \delta_{43} = 0.03, \delta_{44} = 0.04, c_1 = 2, c_2 = 3, c_3 = 5$ and the initial values $[x_1(0) = 0.1, x_2(0) = 0.2], [y_1(0) = 0.5, y_2(0) = 0.2]$, and $a(0) = 0.1, b(0) = 0.1$, and the figures of synchronization errors are displayed in Figure 1 (a)-(b), and simulations of the two parameters $a(k), b(k)$ are displayed in Figure 2 (A)-(B). Finally the attractors after being synchronized with controllers are displayed in Figure 3.

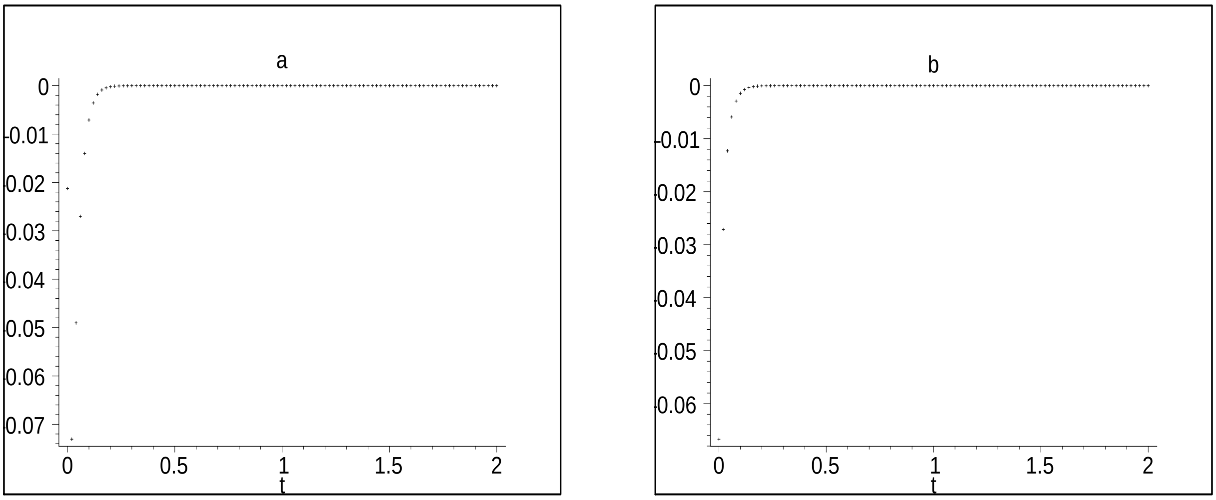


Figure 1: The orbits of the error states: (a) $E_1(k) = x_1(k+\tau) - y_1(k), \tau = 1$, (b) $E_2(k) = x_2(k+\tau) - \big[1 + \frac{1}{4}\cos(x_1(k+\tau))^2\big] y_2(k), \tau = 1$

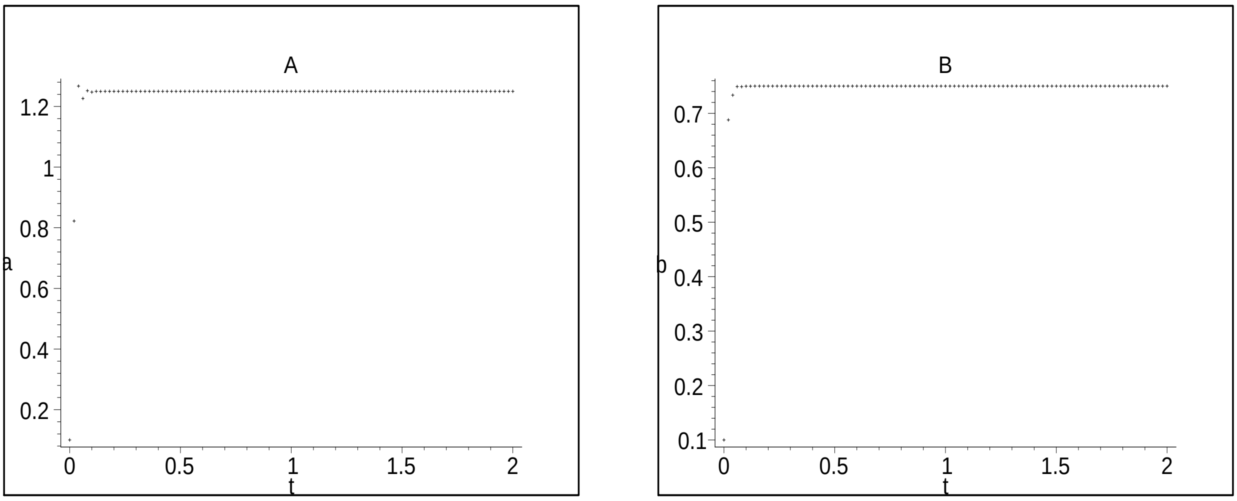


Figure 2: The orbits of uncertain parameters

**II :** Let the error states be

$$
\begin{aligned}
E_1(k) &= x_1(k+\tau) - \big[1 + 6\mathrm{sech}(x_1(k+\tau))^2\big] y_1(k),\\
E_2(k) &= x_2(k+\tau) - y_2(k),\\
E_3(k) &= a(k) - \alpha,\\
E_4(k) &= b(k) - \beta.
\end{aligned}
$$

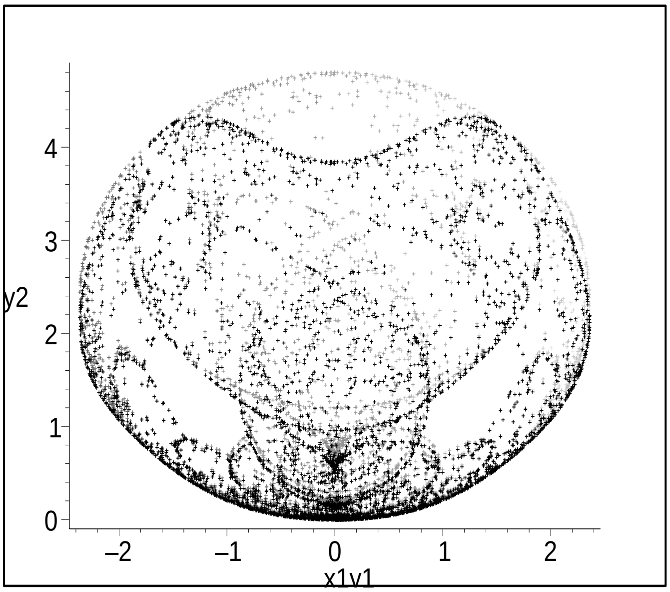


Figure 3: The two attractors after being synchronized with $(f_1(x), f_2(x)) = (1, 1 + \frac{1}{4}\cos(x_1(k+\tau))^2)$: $\tau = 1$, the dark one is the response system with the controllers, and the other is the drive system

Similarly, from (3.1) and (3.2), we have the discrete-time error dynamical system

$$\begin{cases} E_1(k+1) = x_1(k+1+\tau) - \big[1 + 6\mathrm{sech}(x_1(k+1+\tau)^2)\big] \\ \qquad\qquad \big[(1+a(k)b(k))y_1(k) - b(k)y_1y_2(k) + u_1(x,y)\big], \\ E_2(k+1) = x_2(k+1+\tau) - \big[(1-b(k))y_2(k) - b(k)y_1(k)^2 - u_2(x,y)\big]. \end{cases} \tag{3.16}$$

Repeat the process in **I**, we can get the controllers

$$\begin{cases} u_1(x,y) = & -\frac{1}{1+6\mathrm{sech}(x_1(k+1+\tau))^2}\big[-x_1(k+1+\tau) + y_1(k) + y_1(k)a(k)b(k) \\ & -b(k)y_1(k)y_2(k) + 6\mathrm{sech}(x_1(k+1+\tau))^2 y_1(k) + \delta_{11}x_1(k+\tau) \\ & +6\mathrm{sech}(x_1(k+\tau))^2 y_1(k)a(k)b(k) - 6\mathrm{sech}(x_1(k+\tau))^2 y_1(k)y_2(k)b(k) \\ & -\delta_{11}y_1(k) - 6\delta_{11}\mathrm{sech}(x_1(k+\tau))^2 y_1(k) - x_2(k+\tau) - y_2(k)\big], \\ u_2(x,y) = & x_2(k+1+\tau) - y_2(k) + y_2(k)b(k) - y_1(k)^2 b(k) \\ & -\delta_{21}x_1(k+\tau) + \delta_{21}y_1(k) + 6\delta_{21}y_1(k)\mathrm{sech}(x_1(k+\tau))^2 \\ & -\delta_{22}x_2(k+\tau) - \delta_{22}y_2(k)\big] \end{cases} \tag{3.17}$$

and

$$\begin{cases} a(k+1) = & \alpha + \delta_{31}x_1(k+\tau) - \delta_{31}y_1(k) - 6\delta_{31}y_1(k)\mathrm{sech}(x_1(k+\tau))^2 \\ & +\delta_{32}x_2(k+\tau) - \delta_{32}y_2(k) + \delta_{33}a(k) - \delta_{33}\alpha + b(k) - \beta, \\ b(k+1) = & \beta + \delta_{41}x_1(k+\tau) - \delta_{41}y_1(k) - 6\delta_{41}y_1(k)\mathrm{sech}(x_1(k+\tau))^2 \\ & +\delta_{42}x_2(k+\tau) - \delta_{42}y_2(k) + \delta_{43}a(k) - \delta_{43}\alpha + \delta_{44}b(k) - \delta_{44}\beta. \end{cases} \tag{3.18}$$

Take the same values of $[\alpha, \beta, \delta_{11}, \delta_{21}, \delta_{22}, \delta_{31}, \delta_{32}, \delta_{33}, \delta_{41}, \delta_{42}, \delta_{43}, \delta_{44}, c_1, c_2, c_3]$ and the same initial values, we also use numerical simulations to verify the effectiveness of the above-mentioned controllers. The figures of synchronization errors are displayed in Figure 4 (a)-(b), and simulations of the two parameters $a(k), b(k)$ are displayed in Figure 5 (A)-(B). Finally the attractors after being synchronized with controllers are displayed in Figure 6.

**III :** Let the error states be

$$\begin{aligned} E_1(k) &= x_1(k+\tau) - 3y_1(k), \\ E_2(k) &= x_2(k+\tau) - \big[1 + \tanh(x_1(k+\tau))^2\big]y_2(k), \\ E_3(k) &= a(k) - \alpha, \\ E_4(k) &= b(k) - \beta. \end{aligned}$$

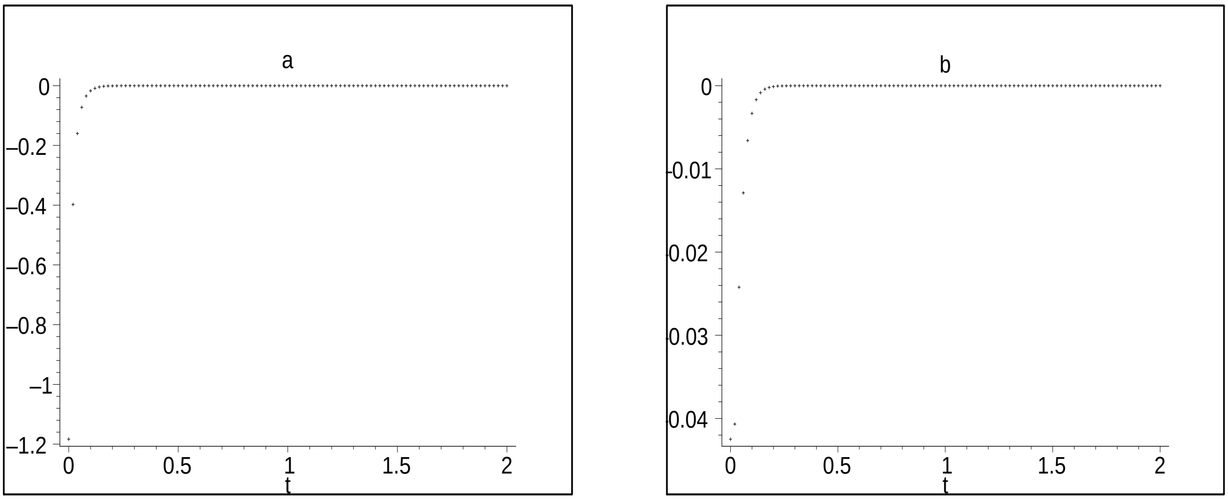


Figure 4: The orbits of the error states: (a) $E_1(k) = x_1(k+\tau) - \big[1 + 6\mathrm{sech}(x_1(k+\tau))^2\big]y_1(k), \tau = 1$, (b) $E_2(k) = x_2(k+\tau) - y_2(k), \tau = 1$

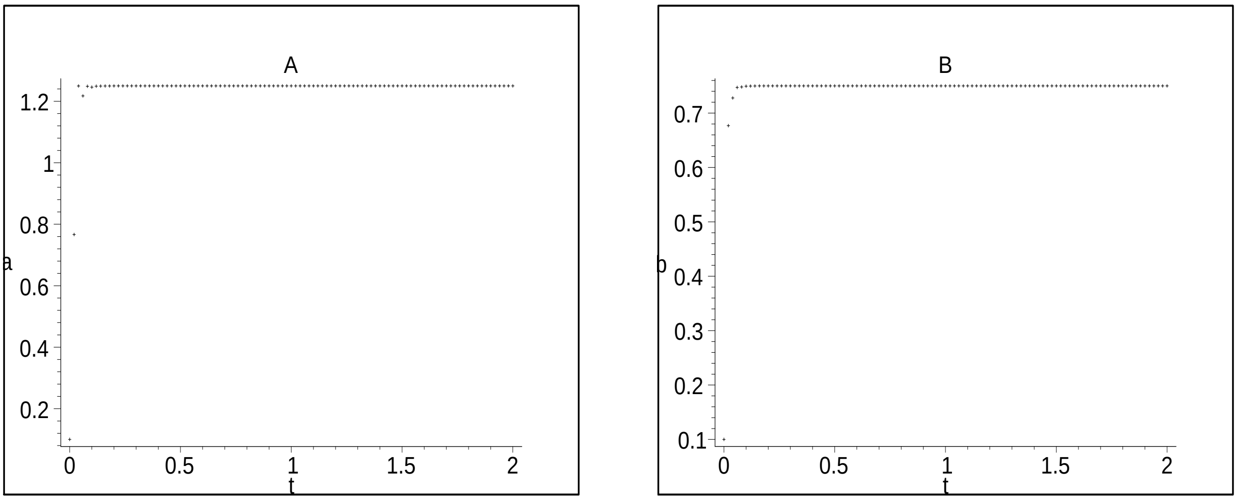


Figure 5: The orbits of uncertain parameters

Similarly, from (3.1) and (3.2), we have the discrete-time error dynamical system

$$\begin{cases} E_1(k+1) = x_1(k+1+\tau) - 3\big[1 + a(k)b(k)\big]y_1(k) + 3b(k)y_1y_2(k) - 3u_1(x,y), \\ E_2(k+1) = x_2(k+1+\tau) - \big[1 + \tanh(x_2(k+1+\tau)^2\big] \\ \qquad\qquad \big[(1-b(k))y_2(k) + b(k)y_1(k)^2 + u_2(x,y)\big]. \end{cases} \tag{3.19}$$

Repeat the process in **I**, we can get the attractors

$$\begin{cases} u_1(x,y) = & \frac{1}{3}x_1(k+1+\tau) - y_1(k) - y_1(k)a(k)b(k) + b(k)y_1(k)y_2(k) \\ & -\frac{1}{3}\delta_{11}x_1(k+\tau) + \delta_{11}y_1(k) - \frac{1}{3}x_2(k+\tau) \\ & +\frac{1}{3}y_2(k) + \frac{1}{3}y_2(k)\tanh(x_2(k+\tau))^2, \\ u_2(x,y) = & -\frac{1}{1+\tanh(x_2(k+1+\tau))^2}\big[ - x_2(k+1+\tau) + y_2(k) - y_2(k)b(k) \\ & +y_1^2(k)b(k) - \tanh(x_2(k+1+\tau))^2y_2(k)b(k) + \tanh(x_2(k+1+\tau))^2y_1^2(k)b(k) \\ & +\tanh(x_2(k+1+\tau))^2y_2(k)^2 + \delta_{21}x_1(k+\tau) - 3\delta_{21}y_1(k) \\ & +\delta_{22}x_2(k+\tau) - \delta_{22}y_2(k) - \delta_{22}y_2(k)\tanh(x_2(k+\tau))^2\big] \end{cases} \tag{3.20}$$

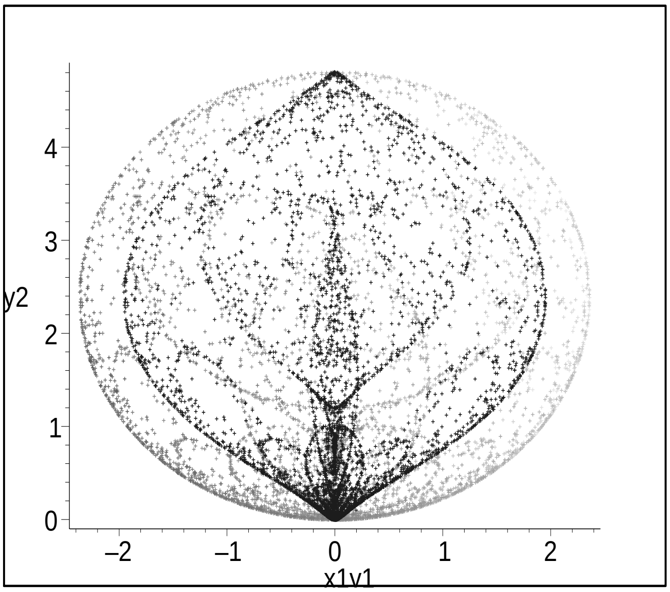


Figure 6: The two attractors after being synchronized with $(f_1(x), f_2(x)) = (1+6\text{sech}(x_1(k+\tau))^2, 1)$: $\tau = 1$, the dark one is the response system with the controllers, and the other is the drive system

and

$$\begin{cases} a(k+1) = & \alpha + \delta_{31}x_1(k+\tau) - 3\delta_{31}y_1(k) + \delta_{32}x_2(k+\tau) - \delta_{32}y_2(k)) \\ & -\delta_{32}y_2(k)\tanh(x_1(k+\tau))^2 + \delta_{33}a(k) - \delta_{33}\alpha + b(k) - \beta, \\ b(k+1) = & \beta + \delta_{41}x_1(k+\tau) - 3\delta_{41}y_1(k) + \delta_{42}x_2(k+\tau) - \delta_{42}y_2(k)) \\ & -\delta_{42}y_2(k)\tanh(x_2(k+\tau))^2 + \delta_{43}a(k) - \delta_{43}\alpha + \delta_{44}b(k) - \delta_{44}\beta. \end{cases} \tag{3.21}$$

$[\alpha, \beta, \delta_{11}, \delta_{21}, \delta_{22}, \delta_{31}, \delta_{32}, \delta_{33}, \delta_{41}, \delta_{42}, \delta_{43}, \delta_{44}, c_1, c_2, c_3]$ and the same initial values, we also use numerical simulations to verify the effectiveness of the above-mentioned controllers, and the figures of synchronization errors are displayed in Figure 7 (a)-(b), and simulations of the two parameters $a(k), b(k)$ are displayed in Figure 8 (A)-(B). Finally the attractors after being synchronized with controllers are displayed in Figure 9.

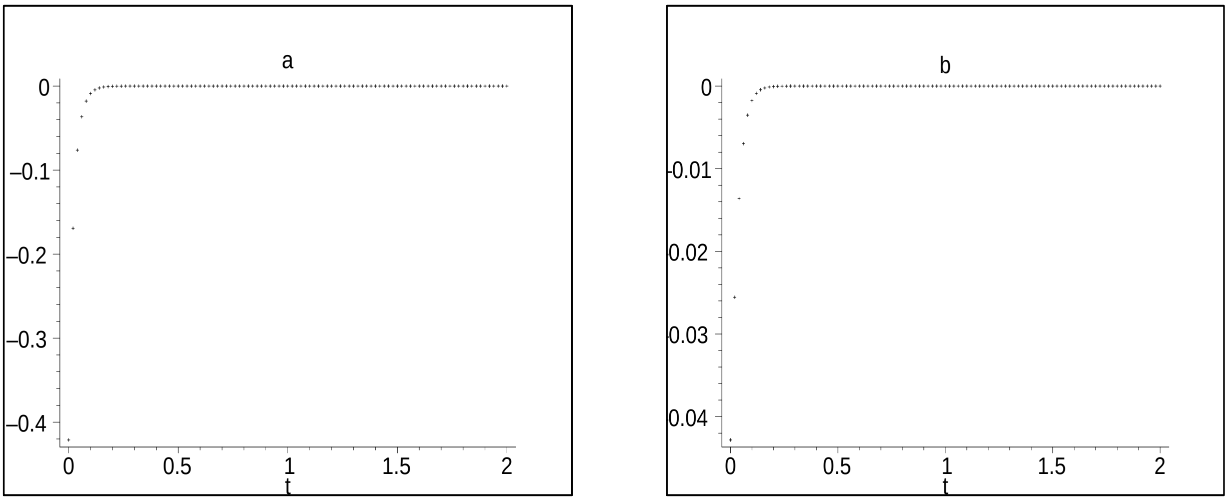


Figure 7: The orbits of the error states: (a) $E_1(k) = x_1(k+\tau) - 3y_1(k), \tau = 1$, (b) $E_2(k) = x_2(k+\tau) - \big[1 + \tanh(x_1(k+\tau))^2\big]y_2(k), \tau = 1$

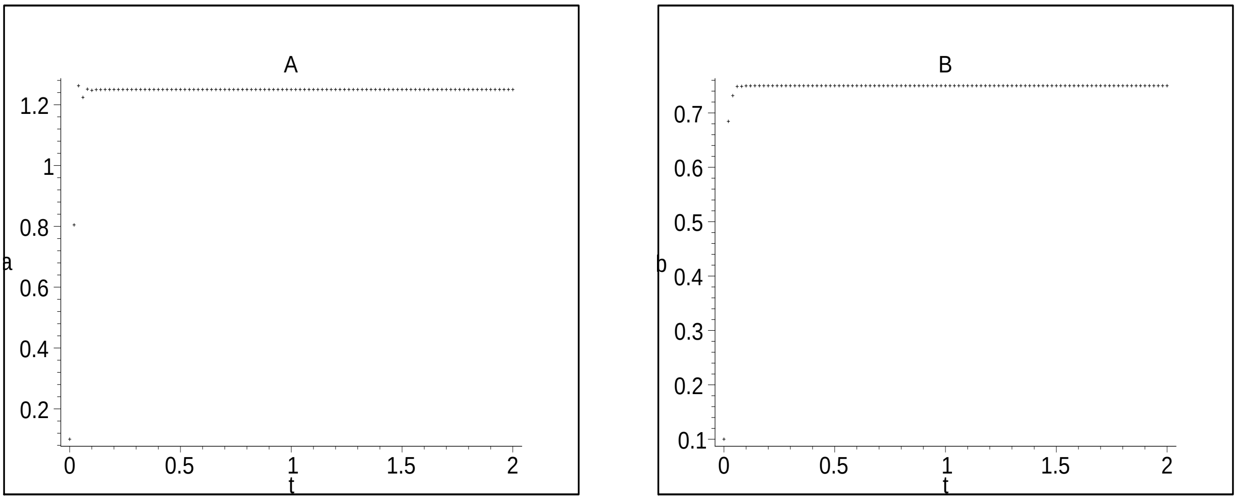


Figure 8: The orbits of uncertain parameters

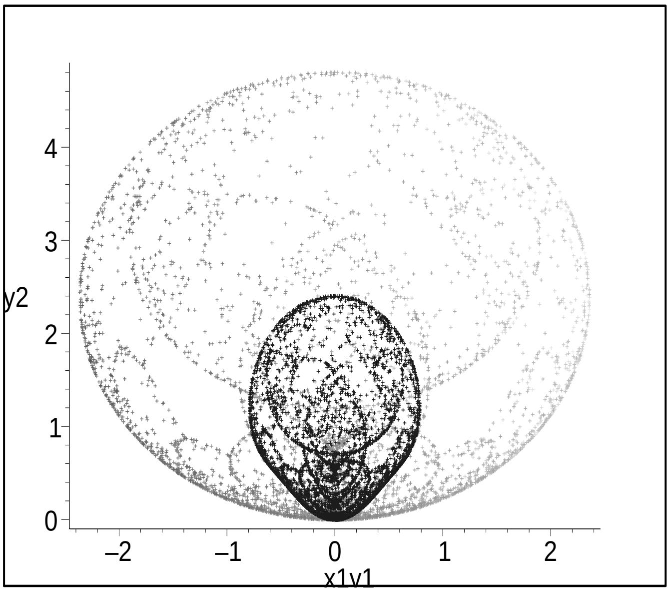


Figure 9: The two attractors after being synchronized with $(f_1(x), f_2(x)) = (3, 1 + \tanh(x_1(k + \tau))^2)$: $\tau = 1$, the dark one is the response system with the controllers, and the other is the drive system

# 4 Summary and Conclusions

In this paper, we define adaptive control for anticipated function projective synchronization in discrete-time dynamical systems. And then backstepping control method is proposed for achieving adaptive control for anticipated function projective synchronization in a general class of discrete-time Lorenz chaotic systems. This control method allows us to arbitrarily amplify or reduce the scale of the dynamics of the respond system through a control. Numerical simulations are used to verify the effectiveness of the proposed scheme.

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# References


[1] Artstein, Z., Stabilization with relaxed controls, *Nonlinear Anal.*, vol.7, no.11, pp.1163–1173, 1983.

[2] Boccaletti, S., J. Kurths, G. Osipov, D.L. Valladares, and C.S. Zhou, The synchronization of chaotic systems, *Phys. Rep.*, vol.366, pp.1–7, 2002.

[3] Chen, Y., and X. Li, Function projective synchronization between two different chaotic systems, *Z. Naturforsch. A*, vol.62, pp.22–29, 2007.

[4] Chen, G., and T. Ueta, Yet another chaotic attractor, *Int. J. Bifur. Chaos*, vol.9, pp.1465–1466, 1999.

[5] Chen, G.R, and X.H. Yu, *Chaos Control: Theory and Applications*, Springer, Berlin, 2003.

[6] Fujisaka, H., and T. Yamada, Stability theory of synchronized motion in coupled-oscillator systems, *Prog. Theor. Phys.*, vol.69, pp.32–47, 1983.

[7] Hu, J., S.H. Chen, and L. Chen, Adaptive control for anti-synchronization of Chua's chaotic system, *Phys. Lett. A*, vol.339, pp.455–460, 2005.

[8] Kocarev, L., and U. Parlitz, Generalized synchronization, predictability and equivalence of unidirectionally coupled dynamical systems, *Phys. Rev. Lett.*, vol.76, pp.1816–1819, 1996.

[9] Krstic, M., I. Kanellakopoulos, and P.V. Kokotovic, *Nonlinear and Adaptive Control Design*, Wiley, New York, 1995.

[10] Li, G.H., An observer-based anti-synchronization, *Chaos, Solitons and Fractals*, vol.29, pp.495–498, 2005.

[11] Li, Y., Y. Chen, and B. Li, Adaptive control and function projective synchronization in 2D discrete-time chaotic systems, *Commun. Theor. Phys.*, vol.51, pp.270–278, 2009.

[12] Li, Y., Y. Chen, and B. Li, Anticipated function synchronization with unknown parameters in discrete-time chaotic (hyperchaotic) systems, *Int. J. Mod. Phys. C*, vol.20, pp.597–608, 2009.

[13] Li, Y., and B. Li, Chaos control and projective synchronization of a chaotic Chen-Lee system, *Chin. J. Phys.*, vol.47, pp.270–278, 2009.

[14] Li, G.H., Modified projective synchronization of chaotic system, *Chaos, Solitons and Fractals*, vol.32, pp.1786–1792, 2007.

[15] Li, C.D., and X.F. Liao, Lag synchronization of Rossler system and Chua circuit via a scalar signal, *Phys. Lett. A*, vol.329, pp.301–308, 2004.

[16] Li, C.D., and X.F. Liao, Impulsive stabilization of delayed neural networks with and without uncertainty, *Chaos, Solitons and Fractals*, vol.22, pp.857–859, 2004.

[17] Li, Y., B. Li, and Y. Chen, Generalized cascade synchronization of discrete-time Henon-like map, *The 9th International Conference for Young Computer Scientists*, pp.2908–2913, 2008.

[18] Lorenz, E.N., and J. Atoms, Deterministic nonperiodic flow, *Sci.*, vol.20, pp.130–141, 1963.

[19] Lu, J., X.Q, Wu, and J.H. L*ü*, Synchronization of a unified chaotic system and the application in secure communication, *Phys. Lett. A*, vol.305, pp.365–370, 2002.

[20] L*ü*, J.H., T.S. Zhou, S.C. Zhang, Chaos synchronization between linearly coupled chaotic systems, *Chaos, Solitons and Fractals*, vol.14, pp.529–541, 2002.

[21] Mainieri, R., and J. Rehacek, Projective synchronization in three-dimensional chaotic systems, *Phys. Rev. Lett.*, vol.82, pp.3042–5, 1999.

[22] Ott, E., C. Grebogi, and J.A. Yorke, Controlling chaos, *Phys. Rev. Lett.*, vol.64, pp.1196–1199, 1990.

[23] Park, J.H., Adaptive controller design for modified projective synchronization of Genesio-Tesi chaotic system with uncertain parameters, *Chaos Solitons Fractals*, vol.34, pp.1154–1159, 2007.

[24] Pecora, L.M., and T.L. Carroll, Synchronization in chaotic systems, *Phys. Rev. Lett.*, vol.64, pp.821–824, 1990.

[25] Rosenblum, M.G., Phase synchronization of chaotic oscillators, *Phys. Rev. Lett.*, vol.76, pp.1804–1807, 1996.

[26] Wang, X.Y., *Chaos in Complex Nonlinear Systems*, Publishing House of Electronics Industry, Beijing, 2003.

[27] Wang, H., Z.Z. Han, Q.Y. Xie, and W. Zhang, Finite-time synchronization of uncertain unified chaotic systems based on CLF, *Nonlinear Anal.*, RWA, doi: 10. 1016/j.nonrwa.2008. 08. 010.

[28] Xu, D., Control of projective synchronization in chaotic systems, *Phys. Rev. E*, vol.63, no.2, pp.027201–05, 2001.

[29] Yan, Z.Y., Q-S synchronization in 3D Hénon-like map and generalized Hénon map via a scalar controller, *Phys. Lett. A*, vol.342, pp.309–315, 2005.

[30] Yan, Z.Y., Q-S (complete or anticipated) synchronization backstepping scheme in a class of discrete-time chaotic (hyperchaotic) systems: A symbolic-numeric computation approach, *Chaos*, vol.16, pp.013119–25, 2006.

[31] Yan, Z.Y., Chaos Q-S synchronization between Rösler system and the new unified chaotic system, *Phys. Lett. A*, vol.334, pp.406–412, 2005.

[32] Yang, X.S., Concepts of synchronization in dynamical systems, *Phys. Lett. A*, vol.260, pp.340–344, 1999.

[33] Yau, H.T., Synchronization and anti-synchronization coexist in two-degree-of-freedom dissipative gyroscope with nonlinear inputs, *Non. Anal.*, RWA, vol.9, pp.2253–2261, 2008.