Journal of Uncertain Systems
Vol.4, No.2, pp.147-160, 2010
Online at: www.jus.org.uk

Using Adaptive Nero-Fuzzy Systems to Monitor Linear Quality Profiles

M.H. Fazel Zarandi*[1], A. Alaeddini[2]

[1]*Department of Industrial Engineering, AmirKabir University of Technology, Tehran, Iran, P.o.Box:15875-13144*
[2]*Department of Industrial and Manufacturing Engineering, Wayne State University (WSU), Detroit, MI, 48202*

Received 30 November, 2008; Revised 8 July 2009

Abstract

In common Statistical Process Control (SPC) applications one or multiple quality characteristics with corresponding univariate or multivariate statistical distributions are used to represent process or product quality. However, there are several other practical situations, in which, the quality of a process or product can be characterized and summarized more effectively by a function of two or more variables. These problems are studied under the framework of quality profiles. In some applications, the profile relation can be adequately represented by the linear model, while in other applications nonlinear models are usually needed. In this paper, in order to facilitate modelling linear profile control problem, an Adaptive Neuro-Fuzzy Inference Systems (ANFIS) based scheme is investigated. The performance of the ANFIS approach is examined and compared to conventional methods.

Keywords: SPC, linear profiles, fuzzy rule-base, ANFIS

1 Introduction

There are many practical situations in which the quality of a process or product can be modelled effectively by the relationship of a quality characteristic and one or more independent variables. The quality characteristics belong to such processes or products are known as quality profiles. In many cases, the relation of quality profiles can suitably presented by a first order linear regression relation

$$y_{ij} = a_0 + a_1 x_{ij} + \varepsilon_{ij} \ , \quad x_l < x_{ij} < x_h \tag{1}$$

where, y_{ij} is the j th observation of the i th profile, a_0 and a_1 are the intercept and slope of the i th profile, x_{ij} is the independent variable for the j th observation of the i th profile, and x_l and x_h define the range of x_{ij}, where the process is characterized by the linear profile. Also, random errors ε_{ij} are assumed to be identically independent (i.i.d.) random variables with mean zero and variance σ^2. In common applications, it is assumed that $\varepsilon_{ij} \sim NID(0,\sigma^2)$, however in general ε_{ij} can follow any i.i.d random variable. Both situations are discussed in this paper (see Figure 1).

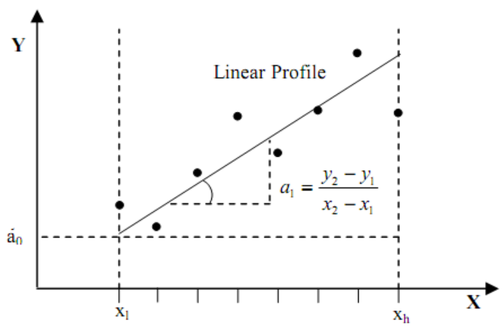

Figure 1: Schematic of a linear profile relation

* Corresponding author. Email: zarandi@aut.ac.ir(M.H. Fazel Zarandi).

Kang and Albin [8] present two examples of situations, in which product profiles are of interest. In the first example, the amount of dissolved aspartame per liter of water (as the quality characteristic) is a function of temperature. The other example was in a semiconductor manufacturing application involving calibration where the performance of a mass flow controller is characterized by a linear function. Mestek et al. [11] give similar calibration applications.

Kang and Albin [8] also propose two monitoring approaches for situations in which the quality of the product is characterized by a linear relationship. The first approach monitors profile relation parameters with a multivariate T^2 control chart and the second approach monitors average residuals between sample and reference lines with Exponential Weighted Moving Average (EWMA) and R control charts. Kim et al. [9] recommend the use of a bivariate T^2 and a univariate shewhart chart simultaneously to check the stability of the regression coefficient and variation about the regression line in phase I. They also recommend the use of the univariate control charts to monitor the intercept, the slope, and the errors of the regression line in phase II. Mahmoud and Woodall [10] propose a method based on using indicator variables in a multiple regression model to monitor a linear profile in Phase I. The use of linear functions as responses in designed experiments has also been studied by Miller [12] and Nair et al. [14]. Abbasi et al. [1] proposed a control chart based on the generalized linear test (GLT) to monitor coefficients of the linear profiles and an R-chart to monitor the error variance.

Jensen et al. [5] use linear mixed models to monitor linear profiles which accounts for any correlation structure within a profile. They show that when the data are unbalanced or missing, the linear mixed model approach is preferable to traditional methods. Zou et al. [20] propose a statistical control scheme that characterizes the quality of a process by a general linear profile for use in an industrial setting. They also introduce some enhancement features to improve the performance of the scheme. Zou et al. [21] use recursive residuals as a basis for developing a self starting control chart for monitoring linear profiles when the nominal values of the process parameters are unknown. They also show good charting performance of their method across a range of possible shifts when process parameters are unknown.

Walker and Wright [17] use additive models to represent the curves of interest in the monitoring of density profiles of particleboard. Jin and Shi [7] use wavelets to monitor "waveform signals" for diagnosis of process faults. Williams et al. [18] extend the use of the T^2 control chart to monitor the coefficients resulting from a nonlinear regression model fitted to profile data. Noorossana and Alaeddini [15] use the constrained area between the observed and baseline profile to monitor nonlinear profiles. William et al. [19] present some of the general issues involved in using control charts to monitor profiles and reviewed the SPC literature on this area. Ding et al. [3] propose two-component strategy including: a data-reduction component that projects original data into a lower dimension subspace, and a data-separation technique that can detect single and multiple shifts as well as outliers in nonlinear profiles data in phase I. Chicken et al. [2] present a semi-parametric wavelet method for monitoring changes in sequence of nonlinear profiles. Their method is used to differentiate between different radar profiles. They show their method can quickly detect a variety of changes from a given in-control profile. Zou et al. [22] propose a methodology to monitor changes in both the regression relationship and the variation of the profile online. They also provide an approach to locate the change point of the process and identify the type of change in the profile. Jensen and Birch [6] use nonlinear mixed models to monitor the nonlinear profiles to accommodate the correlated structure. Their proposed approach uses the separate nonlinear regression model fits to obtain a nonlinear mixed model fit. They also show the superior ability of their approach in detecting changes in Phase I data. Staudhammer et al. [16] propose a system of control charts that simultaneously monitors multiple lumber surfaces and specifically targets three common sawing defects. These charts can be used to monitor the slope parameter of a multiple linear regression model and the peak-to-peak waviness of observations from each board. They illustrate that these methods are better at detecting common sawing problems and identifying the causes.

In this paper we focus on designing a methodology to monitor linear profiles. For this purpose, we use Monte Carlo simulations to generate profiles dataset and Adaptive Neuro-Fuzzy Inference Systems to develop a fuzzy rule-base for monitoring linear profiles. In this regards, we propose two approaches: (i) a model-based approach, which monitors linear profiles based on their estimated parameters, and (ii) a model-free approach which controls linear profiles only based on their observations. The Schematic representation of the proposed methodology is demonstrated in Figure 2.

The remainder of the paper is organized as follows. Section 2 addresses the proposed methodology and explains the developing process of the linear profile monitoring Fuzzy Inference System (LPM-FIS). Section 3 presents the verification and validation of the proposed methodology through extensive simulation studies. Finally, the conclusions and further works are presented in Section 4.

2 Monitoring Methodology

In this section, we explain the process of the development of the proposed methodology for monitoring linear profiles in Phase II. From the SPC point of view, the process monitoring techniques may be employed for either phase I or II. In phase I, a set of observed profiles data are gathered and analyzed. Any unusual patterns in this data set indicate a lack of statistical control and lead to adjustment and fine tuning. Once all assignable causes are accounted for though, the process will be left with clean set of data, gathered under stable operating conditions and illustrative of actual performance. This set is then used to estimate the in-control distribution of the process quality characteristics.

On the other hand, Phase II data are the process readings gathered subsequently. Unlike the fixed set of phase I, they form a never-ending stream. As each new reading accrues, the SPC check is re-applied. Phase II methods require a phase I data set to have parameter estimated that can be plugged into the phase II calculations. Hence, in Phase II, it is assumed that the baseline profile parameters (A_0, A_1, σ^2) are known. In this phase, for every value of independent variable (x_i), profile observation (y_i) is collected and analyzed. The task of phase II is to decide whether the parameters of the observed profile are the same as the baseline profile or not. Quick detection of the probable shifts in parameters is the most important goal of Phase II.

2.1 FIS Development Using ANFIS

ANFIS is a class of adaptive networks that is functionally equivalent to Fuzzy Inference Systems (FIS) [4]. Using an input/output dataset, the ANFIS constructs a FIS whose membership function parameters are adjusted using training algorithms. The modelling approach consists of: First, hypothesizing an initial FIS which relates inputs to outputs. Next, collecting input/output data in a form that will be usable by ANFIS for training, and finally, using ANFIS to train the FIS model to emulate the training data presented to it by modifying the membership function parameters according to an error evaluation criterion. This study uses ANFIS as an indirect approach for developing Multiple-Inputs-Single-Outputs (MISO) first order Sugeno type fuzzy inference systems to monitor the linear profiles.

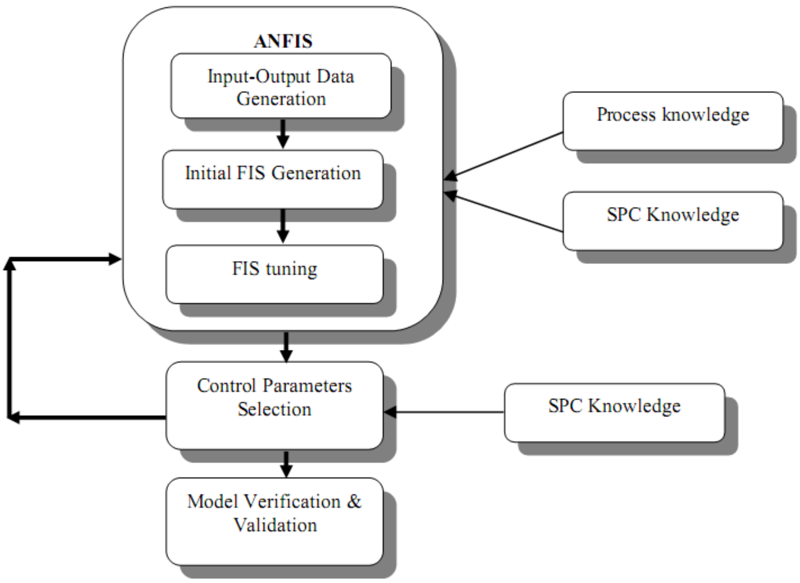

Figure 2: The steps of monitoring linear profiles using ANFIS

2.1.1 Input-Output Data Generation

The first step of the proposed methodology is generating an input-output dataset (see Figure 2). A typical input-output dataset consists of n input-output data. Each input-output data is a $1\times(k+1)$ vector including: k elements for input and one element for output (see Figures 3 and 4). Different types of input-output data are used for the model-base and model free approaches of the proposed methodology.

Each input data of the model-base approach consists of three estimated parameters of each observed profile$\left(\hat{a}_{i0}, \hat{a}_{i1}, \hat{\sigma}_i\right)$, and each output data of this approach consists of a binary value, which shows the quality state of the process:

$$\left(\hat{a}_{i0}, \hat{a}_{i1}, \hat{\sigma}_i, \theta_i\right), \quad \theta_i = \begin{cases} 0, & \text{if the profile is Under} - \text{Control} \\ 1, & \text{if the profile is Out} - \text{of} - \text{Control}. \end{cases} \tag{2}$$

Input **Output**

Input-Output Data

Input/Output Data Set

$\hat{a}_{10}$	$\hat{a}_{11}$	$\hat{\sigma}_1$	θ_1
$\hat{a}_{20}$	$\hat{a}_{21}$	$\hat{\sigma}_1$	θ_2
$\hat{a}_{10}$	$\hat{a}_{11}$	$\hat{\sigma}$	θ_n

Figure 3: Three inputs-single output dataset of the model-base approach

Moreover, each input data of the model-free approach consists of k observations of each observed profile $\left(y_{i1}, y_{i2}, ..., y_{ik}\right)$, and each output data of this approach consists of a binary value, which shows the quality state of the process:

$$\left(y_{i1}, y_{i2}, ..., y_{ik}, \theta_i\right), \quad \theta_i = \begin{cases} 0, & \text{if the profile is Under} - \text{Control} \\ 1, & \text{if the profile is Out} - \text{of} - \text{Control}. \end{cases} \tag{3}$$

Input

Input-Output Data

Input/Output Data Set

y_{11}	y_{12}	$\cdots$	y_{1k}	θ_1
y_{21}	y_{22}	$\cdots$	y_{2k}	θ_2
		...		
y_{n1}	y_{n2}	$\cdots$	y_{nk}	θ_n

Figure 4: k inputs-single output dataset of the model-free approach

Under in-control condition, the output data of both model base and model free approaches get value zero. Under this condition, the input data of the model-free approach are generated based on the baseline profile

$$y_{ij} = a_0 + a_1 x_{ij} + \varepsilon_{ij} \,. \tag{4}$$

Besides, the input data of the model-based approach are calculated based on the estimated parameters and estimated standard deviation of the generated input-out put of the baseline profile.

Under out-of-control conditions, the output data of both model-base and model-free approaches get value one. For the input data, the out-of-control state can be uniformly occurred based on different shifts in each of the profiles parameters $\left(a_0, a_1, \sigma\right)$. Hence, for generating input data under such conditions, the generated inputs should cover all possible out-of-control situations.

There are two approaches of generating out-of-control input data: 1- generating input data based on different individual shifts in the profile parameters, and, 2- generating input data based on concurrent shifts in the profile parameters. We use the first input data generation approach for the model free approach and the second input data generation approach for the model base approach.

In the first input data generation approach which is illustrated in Table 1, there are three parameters a_0, a_1, σ which can experience a shift of size λ, β, γ . In SPC it is common to design, evaluate and compare process mentoring tools for a series of specific shifts in the process parameters. Hence we generate out of control input data based on a series of specific shifts $(\lambda = \lambda_1, \lambda_2, ..., \lambda_t)$, $(\beta = \beta_1, \beta_2, ..., \beta_t)$, $(\gamma = \gamma_1, \gamma_2, ..., \gamma_t)$, where $g(0, \sigma^2)$ is the probability density function of the error.

Table 1: Individual shifts in the linear profile relation

Shift Kind	Relation
Shifts in Intercept	$y_{ij} = (a_0 + \lambda) + a_1 x_{ij} + \varepsilon_{ij}$, $\lambda = \lambda_1, \lambda_2, ..., \lambda_t$
Shifts in Slope	$y_{ij} = a_0 + (a_1 + \beta) x_{ij} + \varepsilon_{ij}$, $\beta = \beta_1, \beta_2, ..., \beta_t$
Shifts in the Variance of Errors	$y_{ij} = a_0 + a_1 x_{ij} + \varepsilon_{ij} \quad \varepsilon_{ij} \sim g(0, \sigma^2 + \gamma)$, $\gamma = \gamma_1, \gamma_2, ..., \gamma_t$

In the first input data generation approach which is illustrated in Table 2, all parameters (a_0, a_1, σ) experience a vector of shift sizes of $(\lambda_t, \beta_t, \gamma_t)$ simultaneously. Again as the SPC methods are analyzed based on a series of specific shifts in the process parameters, we generate data based on specific shifts $(\lambda = \lambda_1, \lambda_2, ..., \lambda_t$, $\beta = \beta_1, \beta_2, ..., \beta_t$, $\gamma = \gamma_1, \gamma_2, ..., \gamma_t)$.

Table 2: Concurrent shifts in the linear profile relation

Shift Kind	Relation
Concurrent Shifts	$y_{ij} = (a_0 + \lambda_t) + (a_1 + \beta_t) x_{ij} + \varepsilon_{ij} \quad \varepsilon_{ij} \sim g(0, \sigma^2 + \gamma_t)$

2.1.2 Initial FIS Generations

The second step of the proposed methodology is generating an initial FIS using the input-output dataset (See Figure 2). There are important factors which influence the size and performance of the generated FIS, including: 1- Training dataset, 2- Membership functions type, 3- Number of membership functions for each input variable, and 4- The method of generating FIS. Extensive simulation studies were conducted to evaluate the effect of above factors on the efficiency and complexity of the proposed profiles monitoring FIS and choose the best combination of them.

To identify the best number of generated data for each in- and out-of-control state, different sizes representative including: 1, 5, 10, 30, 100, 1000, and 10000 from every possible in- or out-of-control situation are examined. The results show that less than 10 representatives from different shift sizes of the in- or the out-of-control state will lead to poor results. Meanwhile the total size of the dataset for the in- or out-of-control states should keep smaller than 10,000, because based on the simulation results, larger datasets decrease the discrimination power of the resulting FIS. On the other hand, respect to simulation results, the total number of in-control representatives should not differ too much from the total number of out-of-control representatives because this phenomenon biases the inference mechanism of the proposed system. For example if there is 1000 number of in-control input-output data there should also be totally 1000 number of input-output data for all possible out-of- control situations.

In this regard, for the first numerical example of Section 3, linear profile with normal random error, which has ten possible shifts for each a_0, a_1, σ , totally 1,000 in-control data and 13,611 out-of-control data including: 300 single shift, and 13,311 concurrent shifts are used for the model-based approach. The 300 single shifts is simply the multiplication of: three input parameters, ten possible shifts, and ten number of representatives. Also 13,311 concurrent shifts is the multiplication of: 11 possible in and out of control states of parameter a_0 , 11 possible in and out of control states of parameter a_1 , 11 possible in and out of control states of parameter σ . Also, 10,000 in-control data and 30,000 single shift out-of-control data are used for the model-free approach. 30,000 out-of-control data is the multiplication of: three input parameters, ten possible shifts, and one thousand number of representatives from each out-of-control state. Meanwhile, for the second numerical example of Section 3, linear profile with non-normal error, which is formulated using model based approach, with 10 possible shifts for each a_0, a_1 and σ , totally 1,000 in-control data and 136,110 out-of-control data including: 3000 single shift, and 133,110 concurrent shifts are used.

In another series of simulation studies, different types of membership functions including: Triangular, Gaussian, and Trapezoidal Membership Functions, for the input variables and linear and constant Membership Functions for the output variables were investigated. Three measures of comparison including: 1-computational complexity, 2- number of generated rules, and 3- FIS error, were developed for membership functions evaluation. Based on the comparison measures, under the assumption of error normality, Triangular and Gaussian membership functions for the input variables, and linear membership functions for the output variables were identified more convenient for the generation of LPM-FIS. Under the assumption of error non-normality, trapezoidal membership functions for input variables, and linear membership functions for output variables showed superior performance.

Besides, with respect to the conducted simulation studies, the triangular membership functions lead to computationally simple FIS, they increase the number of rules needed to represent different process states. Controversially, moving to trapezoidal and Gaussian membership functions increases computational efforts, but they lead to lower number of rules. In this study, for the case of normal random error, Triangular membership functions are used in the model-base approach and Gaussian membership functions are employed in the model-free approach. Also for the case of non-normal random error which is modeled with model based approach, trapezoidal membership functions are used.

We also conducted a number of simulation studies to study the effect of different number of membership functions for each input variable. In these simulation studies we evaluate the numbers 1,2,3,…,10 for each input variable. The simulation studies show that the number of membership functions for each input variable affects the precision and the size of the FIS. Increasing the number of membership functions, decreases fuzziness of the FIS and increases number of rules. Based on the results of the simulation studies, 2 and 3 are the most appropriate number of membership functions for each input variable. Hence, for the numerical examples in Section 3, three membership functions are used for the model-base and model-free approached for each input variable (Figures 5 and 6).

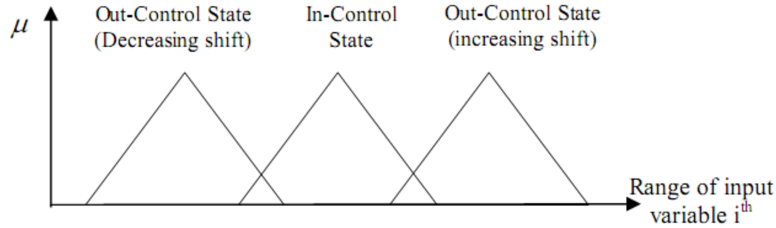

Figure 5: Typical membership functions used for each input variable of the model-base approach

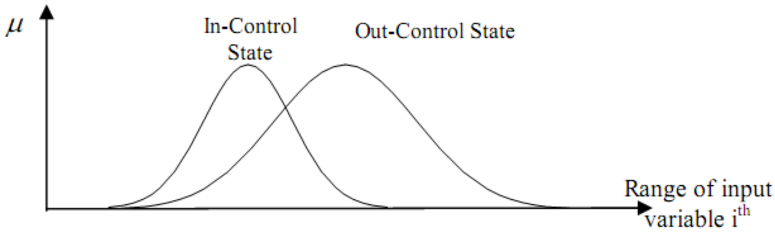

Figure 6: Typical membership functions used for each input variable of the model-free approach

In a separate series of simulation studies we assess Grid Partition and Subtractive Clustering methods employed for constructing the FIS. Besides, in these simulations the effect of different combination of: the size of training dataset, types of membership functions and the number of membership function for each input variable where studied on each of Grid Partition and Subtractive Clustering methods. As a result, Grid Partition methods generally lead to more rules but took less time for development. On the other hand Subtractive Clustering lead to less rules but took more time. In this paper, for the first example of Section 3, grid partition algorithm is followed for the model base approach and subtractive clustering is employed for the model free approach. Also for the second example grid partition algorithm is employed.

2.1.3 FIS Tuning

In this step, the parameters associated with the membership functions are changed through the learning process. Learning process is based on how well the fuzzy inference system is modelling the input/output data for a given set of parameters. There are three important factors in FIS tuning, including: 1- training dataset, 2- optimization method, and 3- Number of trainings.

The same dataset for initial FIS generation is used for tuning. Hence, for the example with normal error in section, a 14611×4 input-output dataset is used for the model-based approach FIS tuning, and a 40000×5 input-output dataset is used for the model-free approach FIS tuning. Also, for the example with non-normal error which follows model-based approach, an input-output dataset with the size of 146110×4 is employed.

The computation and adjustment of membership function parameters is facilitated by a gradient vector. Once the gradient vector is obtained, any of several optimization routines could be applied in order to adjust the parameters so as to reduce sum of the squared difference between actual and desired outputs. The adjustment can be done using either back propagation or a combination of least squares estimation and back propagation for membership function parameter estimation. In this paper, a hybrid method of squares estimation and back propagation is used for membership function parameter estimation.

All Neuro-fuzzy systems require stopping criteria for their learning processes. Different stopping criteria such as error tolerance, minimum error, training epochs, etc. can be employed for FIS learning. In this study, for the example with normal error in Section 3, on both model-base and model-free approaches 300 training epochs is used as a stopping criterion which leads to the minimum error of 0.1990 for the model-base approach and 0.2107 for the model-free approach. The resulted systems were, a 3×1 MISO system with 27 fuzzy rules with based on model base approach (See Figure 7) and a 4×1 MISO system with only 2 fuzzy rules based on the model free approach (See Figure 8).

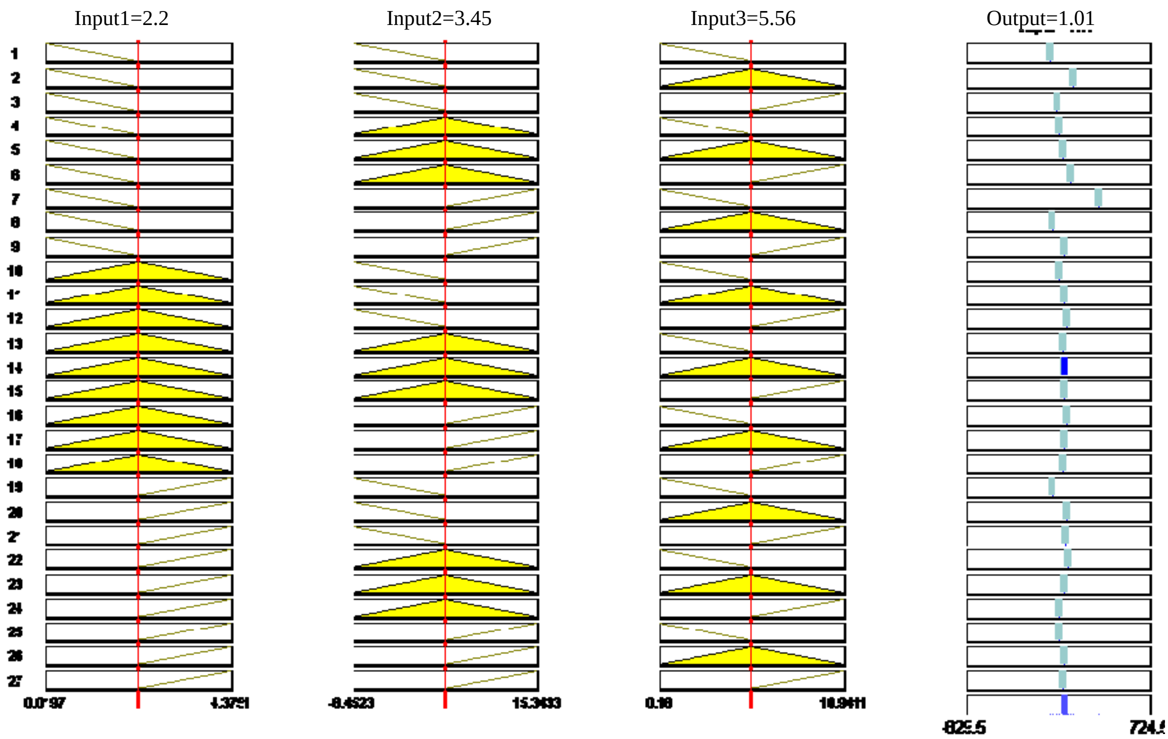

Figure 7: Graphical representation of the model-base FIS under normal error

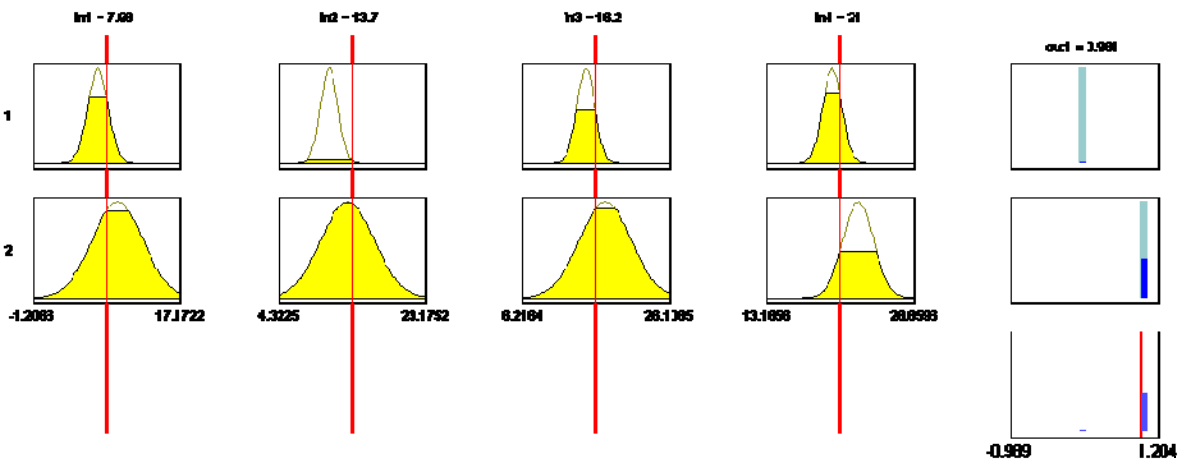

Figure 8: Graphical representation of the model-free FIS under normal error

Also for the example with non-normal error, 100 training epochs is used as a stopping criterion which leads to the minimum error of .1803. The resulted system was a 3×1 MISO system with 27 fuzzy rules (See Figure 9).

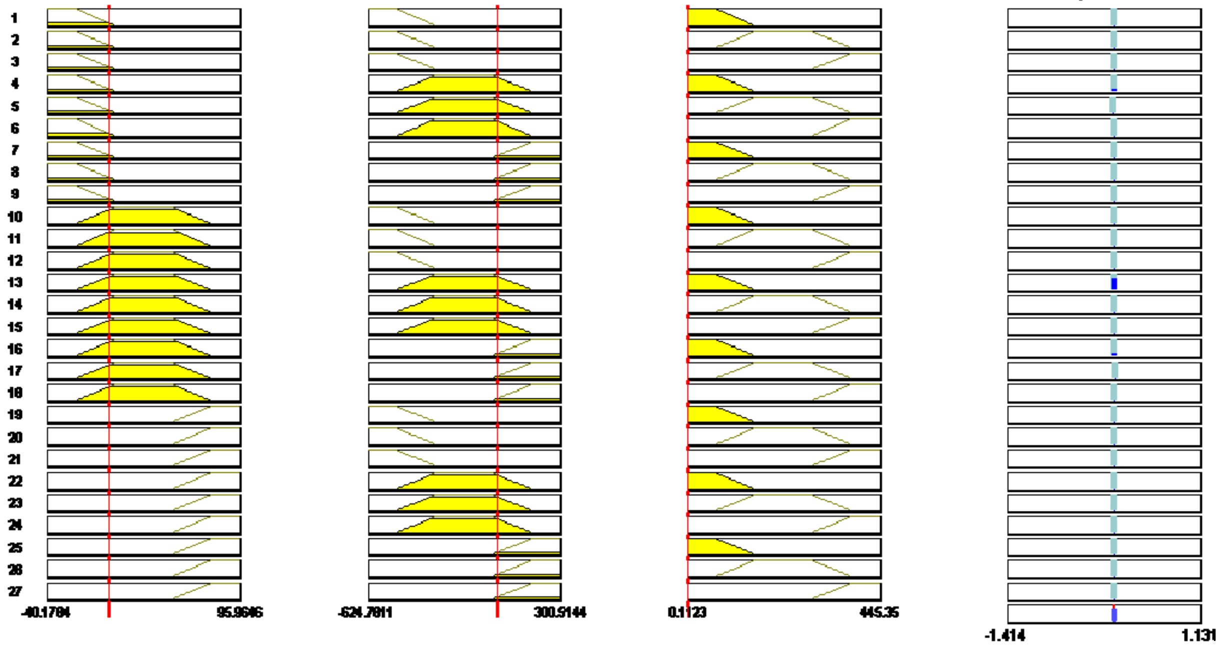

Figure 9: Graphical representation of the model-based FIS under non-normal error

2.1.4 Selecting Control Scheme Parameters

Both model-base and model-free FIS developed through above steps get vectors of inputs from a profile based process and give a spectrum value between zero and one which show the quality state of the process (Figure 10). A control limit is to be chosen to delineate between the in-control and out-of-control states. Such control limit should be determined so that if the process is in-control, nearly all of the FIS outputs fall between zero and the limit, and if the process is out-of-control, approximately all of the outputs fall between the limit and one.

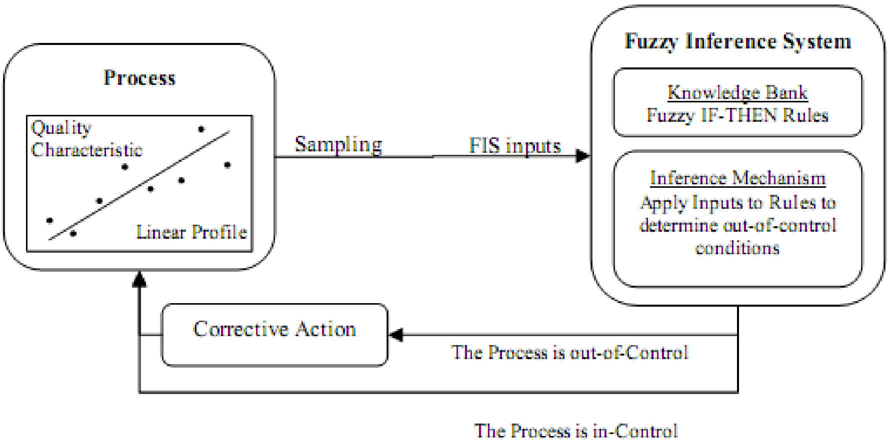

Figure 10: Monitoring mechanism LPM-FIS

In practice control limits are usually determined based on the performance (in-control performance) or equivalently type I error of the scheme. The speed with which a control scheme detects an off-target condition determines the Performance of the scheme. Average run length (ARL) has been widely used as a performance measure to compare the efficiencies of different control schemes [13]. The ARL is the average number of samples before the control chart signals an out-of-control condition. Among different SPC schemes, the in-control ARL (ARL_0) of 200 is very common for setting control limits. Hence, the control limits of the proposed approaches are set to gain the ARL_0 equal to 200.

3 Verification and Validation

Two sets of simulations studies have been conducted to evaluate the performance of the proposed approach under normal and non-normal error assumptions. For the case of normal random error two FISs have been developed using model base and model free approaches with the details discussed in Section 2. These FISs are compared with different traditional linear profiles monitoring methods including: Kang and Albin [8] T^2 control chart, Kang and Albin [8] $EWMA/R$ control chart, and Mahmould and Woodall [10] $EWMA3$ control chart. For the case of non-normal error one FIS is developed using model base approach with the conditions discussed in Section 2. This FIS is also compared with all the methods mentioned above.

Generally, in comparing two control schemes the scheme with higher ARL_0 and lower ARL_1 is better. Hence, for comparing different schemes in SPC, it is common that, first, control schemes parameters is adjusted, so that, same in-control ARL (ARL_0) is resulted for both methods. Next, out-of-control ARL (ARL_1) of two schemes is calculated for different shifts. The scheme which has a lower out-of-control ARL has a better performance.

3.1 Simulations for Profiles with Normal Error

Here, for all conducted simulations the linear profile $y_{ij} = 3 + 2x_{ij} + \varepsilon_{ij}$ with $\varepsilon_{ij} \sim NID(0,1)$ and fixed x_{ij} (2, 4, 6, 8) is considered. Also, 1,000 iterations have been specified for simulations. ARL_0 all compared schemes set to have ARL_0 equal to 200, and the smoothing constants of all approaches which includes $EWMA$ chart set equal to 0.2. All schemes were studied under different shift sizes of $\lambda = 0.2, 0.4, .., 2$ for parameter a_0, $\beta = 0.025, 0.05, .., 0.25$ for parameter a_1, and $\gamma = 1.2, 1.4, .., 3$ for parameter σ. Tables 3, 4 and 5 and Figures 11, 12 and 13 illustrated the simulation results of T^2, $EWMA/R$, $EWMA3$, model-base and model-free approaches for different shifts.

3.1.1 Shifts in Intercept

Table 3 and Figure 4 compare the ARL performance of the proposed approaches and Traditional T^2, $EWMA/R$, $EWMA3$ methods against different shifts in parameter a_0. As it can be seen the in-control ARL (the ARL of the shift size of 0) are set to be not less than 200, and then the out-of-control ARL of each methods is determined based on 10,000 iterations. For a shift size of $\lambda_t = 0.2$, the $EWMA3$ detect such a change on average after 59.10, also $EWMA/R$ after 63.50, the proposed Model-Base Approach after 76.89, the proposed Model-Free Approach after 77.266 and T^2 after 137.70 observed profile. For this shift the $EWMA3$ and $EWMA/R$ charts works better than other methods, and the proposed model-based and model free approaches stand on the second rank, also the T^2 has the lowest performance among compare methods. Same conditions is hold for shift sizes of $\lambda_t = 0.4, 0.6$. In SPC such shifts which are less than 1 called small shifts. As a consequence $EWMA3$, $EWMA/R$ performs better than other methods in small shifts. The high performance of $EWMA3$, $EWMA/R$ is that they are statistically designed for detecting small shifts.

For a shift size of $\lambda_t = 1.2$ the model free approach have an ARL of 2.761. It means that the proposed model-free approach find a shift of size 1.2 only after about 3 observations. Also ARL of the model-base approach is 2.91, the $EWMA/R$ is 3.20, $EWMA3$ is 3.10, and T^2 is 4.00. It can be seen that the proposed methods outperform all other methods. Such condition is met for all other shifts equal or more than $\lambda_t = 1.2$. In SPC such shifts which are greater than 1 called moderate and large shifts. As a consequence the propose methods performs better than other methods in large shifts.

Table 3: Shifts in a_0 for the case of normal error

ARL	**Shifts from a_0 to $a_0 + \lambda_t\sigma$**										
Method	**0**	**0.2**	**0.4**	**0.6**	**0.8**	**1**	**1.2**	**1.4**	**1.6**	**1.8**	**2**
Model-Base Approach	204.42	76.89	30.77	15.35	7.51	4.31	2.91	1.99	1.63	1.43	1.36
Model-Free Approach	204.19	77.266	31.809	14.621	7.402	4.29	2.761	1.954	1.515	1.26	1.12
EWMA/R	200.00	66.65	17.70	8.40	5.40	3.90	3.20	2.70	2.30	2.10	1.90
T2	200.00	137.70	63.50	28.00	13.20	6.90	4.00	2.60	1.80	1.50	1.90
EWMA3	200.00	59.10	16.20	7.90	5.10	3.80	3.10	2.60	2.30	2.10	1.90

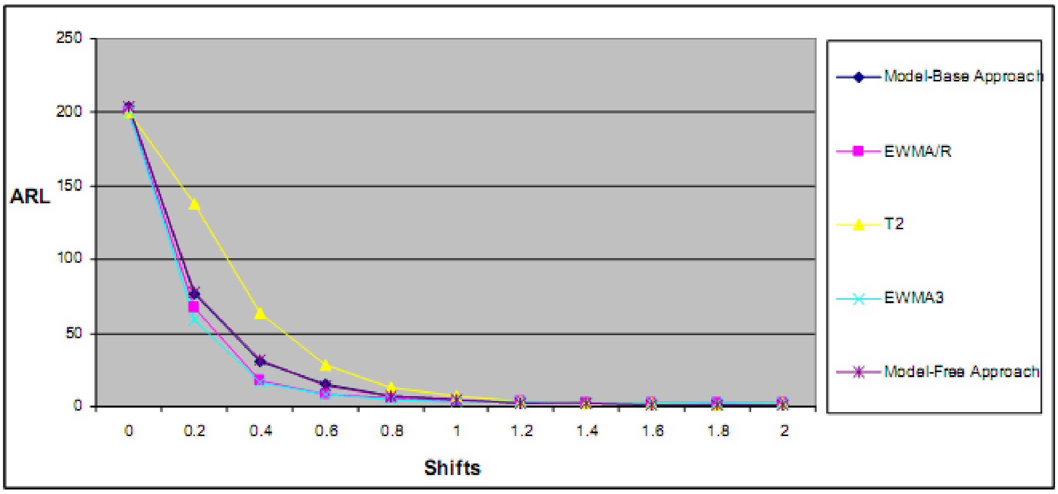

Figure 11: Shifts in a_0 for the case of normal error

3.1.2 Shifts in Slope

Table 4 and Figure 5 illustrate the ARL performances of the proposed methods and the traditional Shewhart control charts. The result of shifts in the slope of the profile is similar to shifts in the intercept. $EWMA/R$, $EWMA3$ methods have better performance than the proposed methods for small shifts; however, the proposed approaches have better performance in moderate and large shifts. Also the differences between the compared methods in this section are less than the differences in the intercept. Meanwhile, the proposed method demonstrates a clear advantage

Table 4: Shifts in a_1 for the case of normal error

ARL	Shifts from a_1 to $a_1+\beta_t\sigma$										
Method	**0**	**0.025**	**0.05**	**0.075**	**0.1**	**0.125**	**0.15**	**0.175**	**0.2**	**0.225**	**0.25**
Model-Base Approach	204.19	117.75	67.07	38.33	22.82	14.00	9.08	6.03	4.27	3.16	2.41
Model-Free Approach	204.42	103.16	55.01	32.66	18.52	11.23	7.28	4.95	3.49	2.62	2.15
EWMA/R	200.00	119.00	43.90	19.80	11.30	7.70	5.80	4.70	3.90	3.40	3.00
T2	200.00	168.00	106.50	60.70	34.50	19.90	12.30	7.80	5.20	3.70	2.70
EWMA3	200.00	101.60	36.50	17.00	10.30	7.20	5.50	4.50	3.80	3.30	2.90

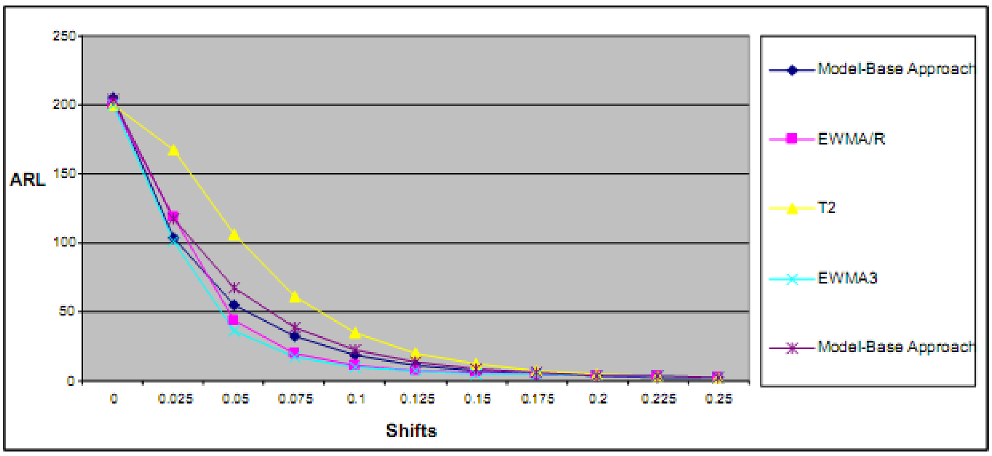

Figure 12: Shifts in a_1 for the case of normal error

3.1.3 Shifts in Standard Deviation

Table 5 and Figure 6 illustrate the ARL of the proposed methods and the traditional methods. Like other conducted simulations the ARL of all methods set to be not less than 200. For a shift of size $\gamma = 1.4$ the model-base approach has an ARL of 11.85, also the model free approach ARL is 16.37, the $EWMA/R$ ARL is 12.00, $EWMA3$ ARL is 12.70, and T^2 ARL is 14.90. It can also be seen that for any kinds of shifts in σ^2, the model-base approach has a clear advantage over all other methods. In other word the proposed model base approach is uniformly better that all other methods for different small, moderate and large shifts. Besides the model-base approach has a near performance to methods T^2, $EWMA3$ methods, but both proposed methods are totally better than $EWMA/R$ method.

Table 5: Shifts in σ for the case of normal error

ARL	**Shifts from σ to $\gamma\sigma$**										
Method	**1**	**1.2**	**1.4**	**1.6**	**1.8**	**2**	**2.2**	**2.4**	**2.6**	**2.8**	**3**
Model-Base Approach	204.19	34.75	11.85	5.89	3.75	2.70	2.11	1.80	1.57	1.45	1.35
Model-Free Approach	204.42	43.50	16.37	8.28	4.97	3.48	2.63	2.18	1.91	1.64	1.50
EWMA/R	200.00	34.30	12.00	6.10	3.90	2.90	2.30	1.90	1.70	1.50	1.40
T2	200.00	39.20	14.90	7.90	5.10	3.80	3.00	2.50	2.20	2.00	1.80
EWMA3	200.00	33.50	12.70	7.20	5.10	3.90	3.20	2.80	2.50	2.30	2.10

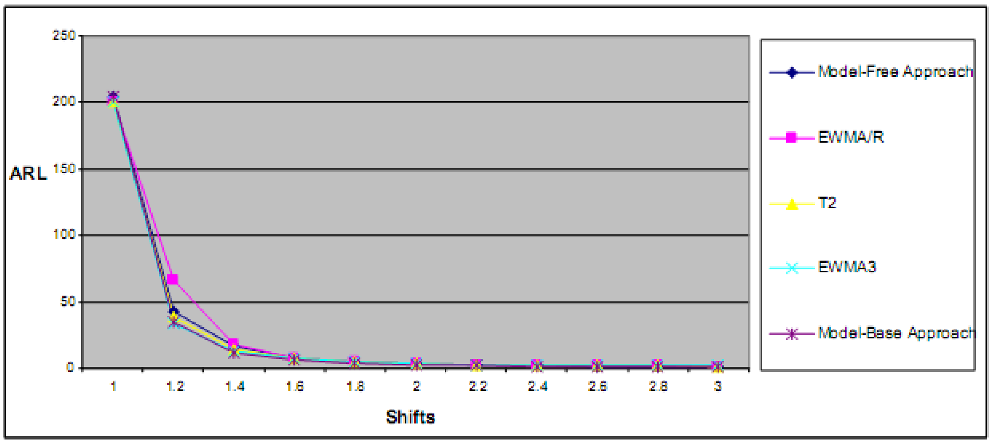

Figure 13: Shifts in σ^2 for the case of normal error

3.2 Simulations for Profiles with Non-normal Error

Here, same profile relation $y_{ij} = 3 + 2x_{ij} + \varepsilon_{ij}$ with $\varepsilon_{ij} \sim t_5$ is considered. All other assumptions are similar to the normal error case. Just for modeling the shifts in the standard deviation of the error, the relation between the degree of freedom of the t distribution and its standard error $\left(\sigma^2 = \gamma/(\gamma - 2)\right)$ is used.

3.2.1 Shifts in intercept

Table 6 and Figure 14 compare the performance of the proposed methods and T^2, $EWMA/R$ and $EWMA3$ methods. Here T^2 shows a very poor detection power but others have acceptable performance. For small shifts $EWMA/R$ and $EWMA3$ have fairly close and better performance than the proposed approach, while $EWMA3$ stands on top. However, for large shifts $\left(\lambda_i > 1.2\right)$ the proposed approach has a superior performance. As the shift size grows the difference between the proposed method and others gets clearer. One reason for such phenomenon is that *EWMA* types of chart work best for small shifts so their performance are degraded when the shift size grows.

Table 6: Shifts in a_0 for the case of non-normal error

ARL	**Shifts from a_0 to $a_0 + \lambda_t \sigma$**										
Method	**0**	**0.2**	**0.4**	**0.6**	**0.8**	**1**	**1.2**	**1.4**	**1.6**	**1.8**	**2**
T2	195.84	192.36	188.85	185.31	182.83	177.32	171.23	164.15	154.45	146.13	136.37
Proposed Approach	194.51	132.17	82.65	54.65	37.22	24.31	15.05	9.36	5.96	4.03	2.87
EWMAR	188.70	110.93	64.87	44.74	29.62	22.13	17.02	13.27	10.87	9.38	8.27
EWMA3	183.59	109.64	57.51	35.91	25.23	17.82	13.41	10.87	9.05	7.88	6.83

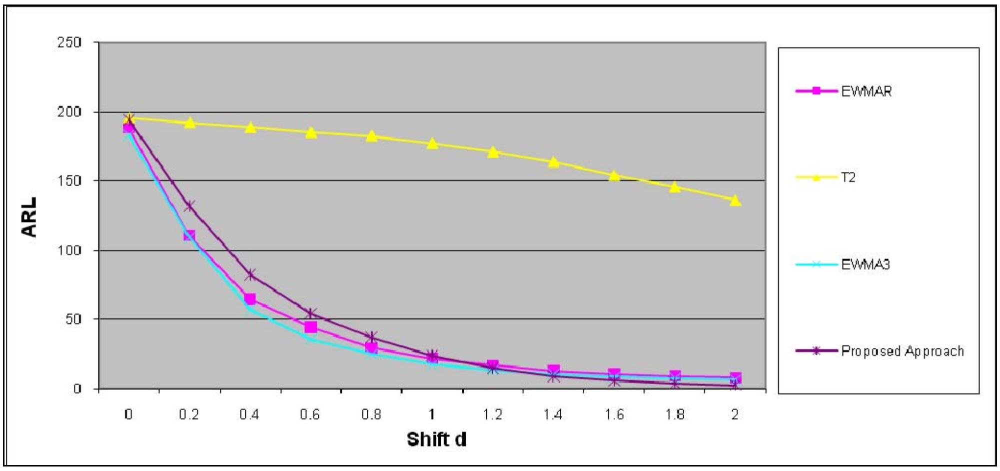

Figure 14: Shifts in a_0 for the case of non-normal error

3.2.2 Shifts in Slope

Table 7 and Figure 15 show the results of 1,000 simulation runs for the comparing methods. Except the proposed approach which has an acceptable performance all other methods performs very poor in monitoring studied shifts. Here $EWMA/R$ shows strange behavior with an increasing series of ARLs in response to the increase in a_1 parameter. Also $EWMA3$ and T^2 behave very reluctantly to different types of shift.

Table 7: Shifts in a_1 for the case of non-normal error

ARL	**Shifts from a_1 to $a_1 + \beta_t \sigma$**										
Method	**0**	**0.025**	**0.05**	**0.075**	**0.1**	**0.125**	**0.15**	**0.175**	**0.2**	**0.225**	**0.25**
T2	195.49	192.56	189.28	186.03	183.86	181.34	178.82	173.86	169.47	166.24	163.02
Proposed Approach	204.23	152.31	121.46	92.70	65.60	49.52	34.71	26.34	18.28	13.93	9.59
EWMA/R	200.74	205.96	218.54	238.11	256.31	277.37	298.92	320.84	343.93	368.32	390.06
EWMA3	200.93	201.75	202.57	203.93	201.55	199.10	196.10	195.93	194.40	192.98	188.51

3.2.3 Shifts in Standard Deviation

Finally, Table 8 and Figure 16 illustrate the performance of the comparing methods on monitoring different types of shift in the standard deviation of the error term. Here, $\gamma = 2\sigma^2/(\sigma^2 - 1)$ is used as the degree of freedom of t distribution for generating random numbers with different standard error. Here, in contrast to the results gained for

a_0 and a_1 simulation studies, all approaches have acceptable performance. Meanwhile the proposed approach along with T^2 chart perform uniformly better that the others. Next to these two methods $EWMA3$ stands in front of $EWMA/R$.

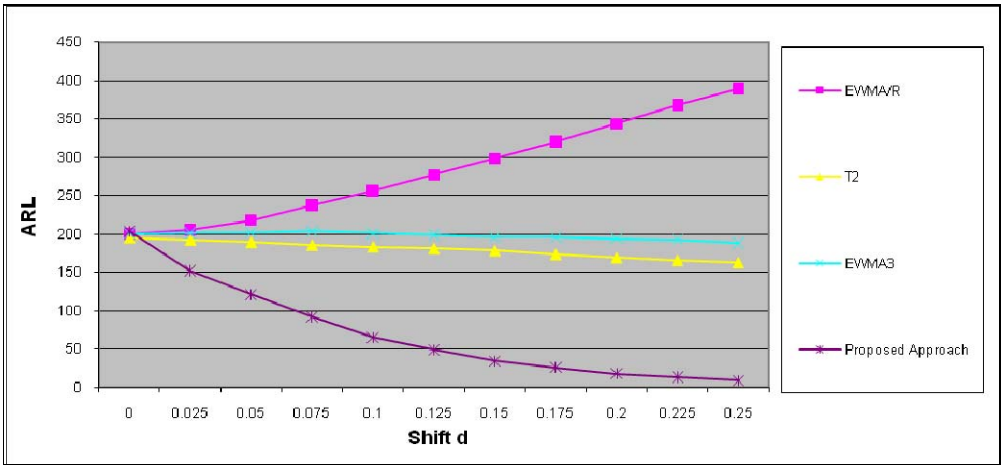

Figure 15: Shifts in a_1 for the case of non-normal error

Table 8: Shifts in σ for the case of non-normal error

ARL	Shifts from σ to $\gamma\sigma$									
Method	**1**	**1.2**	**1.4**	**1.6**	**1.8**	**2**	**2.2**	**2.4**	**2.6**	**2.8**	**3**
T2	198.55	67.69	40.80	31.18	27.80	24.26	24.27	22.00	21.09	20.08	20.27
EWMA/R	186.90	91.80	62.02	47.48	40.52	35.30	33.36	32.94	31.54	29.55	29.76
Proposed Approach	197.08	66.46	39.91	32.09	28.22	26.66	23.76	22.00	22.23	20.89	20.06
EWMA3	219.48	103.86	73.39	60.23	52.99	49.41	45.75	43.74	41.70	40.94	39.17

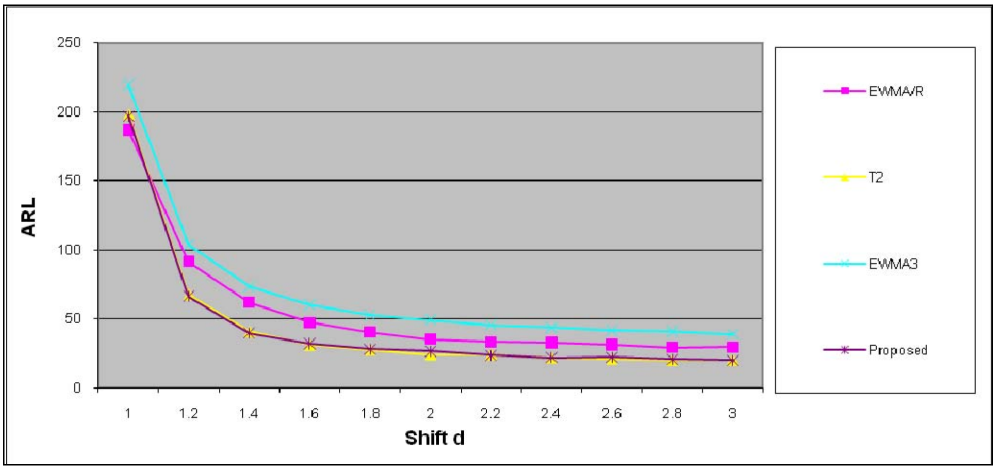

Figure 16: Shifts in σ for the case of non-normal error

4 Conclusions and Future Works

In this paper, a Fuzzy System was proposed based on Adaptive Neuro Fuzzy Systems for online monitoring of linear quality profiles considered as new remarkable issues in quality control. The performance of the proposed method was examined under both normal and non-normal errors and through different shifts and different parameters. For the normal case, the proposed method had comparable results against strong statistical methods, while for the non-normal case it worked superior to the traditional methods on average. The proposed method has two main advantageous. First, in contrary to statistical methods it does not need any strict assumptions like knowing the real value or even distribution of the quality characteristic. Second, the proposed approach is easy to construct and has simple final structures as well. Besides, The sensitivity of this method to different kind of shifts (little and/or significant shifts) can be adjusted by using different kind of membership functions and changing membership function parameters which can be seen as an area for further woks.

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