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A New Multi-objective Approach in Distribution Centers Location Problem in Fuzzy Environment

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Abstract

As has been mentioned in the literature, distribution centers location problem (DCLP) is a multiobjective problem. Cost (or distance) minimization is the primary objective in this area. While significance of each relation between a distribution center (DC) and a customer/retailer is different, it has not been considered seriously. By adapting available facility and new facility concepts of the multifacility location problem with costumer/retailer and a DC respectively, a utility function as the second objective based on the importance has been used. These importances are in decision maker (DM) s' mind which should be quantified to be of use for modeling purpose. We propose a method for the uncapacitated single stage facility location problem (UFSLP) in which a fuzzy AHP method is used to achieving these importances. So we present a multiobjective model in which minimizing total cost is first objective and maximizing the utility function is the second one. LP-metric method is used to solve our multiobjective model. Finally, a numerical example is expressed for illustration of the proposed method.

Keywords: distribution centers location problem, multi-objective decision making (MODM), fuzzy AHP.

1 Introduction

Distribution is a key driver of the overall profitability of a firm because it directly impacts both the supply chain cost and the customer experience [5]. Locating the distribution centers (DC) in some alternative places and allocating customers/retailers is one of the most important problems in supply chain management and in distribution network designing. Actually distribution refers to the steps taken to move and store a product from the supplier stage to a customer stage in the supply chain. It is categorized as location problem where a location problem is a spatial resource allocation. In the general location paradigm, one or more service facilities ("servers") serve a spatially distributed set of demands ("customer") [2]. In DCLP each DC plays a role of the facility or server. Location science research investigates where to physically locate a set of facilities (resources) so as to minimize the cost of satisfying a set of demands subject to certain constraints. So distribution centers location problem is concerned with how to select distribution centers from the potential set so that the total relevant cost is minimized [17]. Although minimizing the costs is an important objective in DCLP, Chopra has mentioned that good distribution can be used for achieving a variety of supply chain objectives ranging from low cost to high responsiveness [5]. This view influences DCLP and makes a challenge to discuss more about new objectives on location problems in distribution networks designing.

Because of the interaction between shortest path problem (SPP) and DCLP and also strong body of literature in multiobjective SPP, first, we study their relation and important multiobjective models in SPP. Our goal is not to review all models that have been presented in multiobjective SPP, but just we mean to show the importance of designing multiobjective shortest path models. To do this we should develop new objectives regarding DMs' opinions to interact by DMs. By adapting that important approach, we prove our point of view in making a new multiobjective

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model in DCLP is necessary for considering DMs' opinions in decision making process and also it is a significant issue.

Shortest path problem (SPP) is related to DCLP in a way that the SPP aims to find the shortest path to satisfy some objectives and the DCLP aims to locate and connects customers/retailers to DCs to satisfy some objectives. Obviously, both of them try to connect nodes with considering one or more objectives. The original SPP has been a single objective problem, with the objective being the minimization of total distance or travel time. However, due to the multiobjective nature of many transportation planning problems, there has been an increasing interest in multiobjective path problems since 1975 in this area [7]. Considering the strong literature of the multiobjective SPP and weak attention to multiobjective modeling in DCLP, we develop a new multiobjective model with a new objective to regard DMs' opinion. In this paper, our goal is to consider the objectives which are related to decision makers' opinion in our model. This sort of objectives is qualitative and we present a method to quantify and insert them in the uncapacitated single stage facility location problem (UFSLP). UFSLP is reviewed overall by Klose and Drexl [12]. A rough classification of discrete facility location models can be given as follows: (a) single- vs. multistage models, (b) uncapacitated vs. capacitated models, (c) multiple- vs. single-sourcing, (d) single- vs. multi-product models, (e) static vs. dynamic models, and, last but not least, (f) models without and with routing options included. UFSLP is the simplest model in this sort of location problem. Models without capacity constraints do not restrict demand allocation. If capacity constraints for the potential sites have to be obeyed demand has to be allocated carefully. In the latter case we have to examine whether single-sourcing or multiple-sourcing is essential. Single-stage models focus on distribution systems covering only one stage explicitly. In multi-stage models the flow of goods comprising several hierarchical stages has to be examined [12]. So in UFSLP it is supposed that there is not any upper bound for each DC's capacity. UFSLP solely considers the tradeoff between fixed operating and variable delivery cost [12]. In this paper we extend the UFSLP which is formulated with Mixed-integer programming approach by adding a utility function.

Klose and Drexl [12] have been mentioned while there is a large body of literature on single-objective facility location problems; the work which has been carried out on multi-objective discrete location problems seems to be very limited and is a topic of current research. A number of multi-objective formulations and objectives to be considered in location problems are described in [7]. ReVelle [14] considers a two-objective maximum covering location problem and proposes to weight objectives in order to preserve the "integer-friendly" problem structure. Heller et al. [10] discuss the use of a p-median model and simulation for locating emergency medical service facilities in case of multiple objectives. ReVelle and Laporte [15] describe two alternative formulations for a bicriteria plant location problem, where one objective is to minimize cost and the second objective is to maximize the demand that can be served by a plant within a certain time limit; in order to solve the problem, they also propose the use of a weighting method. Fernandez and Puerto [8] investigate the multi-objective uncapacitated facility location problem; they develop a dynamic programming method as well as an enumerative approach in order to determine the set of pareto-optimal solutions and supported pareto-optimal solutions, respectively [12]. This paper aims to present a method to combine the DMs' experiences and their needs and the designers' knowledge to detect the best solution. This combination permits the DM to include additional criteria, values and personal judgments in the decision making process. We concern two objectives for our proposed model, one considers the location costs and the other one is a utility function which is designed to consider DMs' opinions.

1.1 Quantifying DMS' Opinions

DMs most of the times think qualitative which commonly contains some ambiguity. But for modeling uses we need some clear and qualitative values which present DMs' opinions exactly. A hardship in converting qualitative opinions of DMs to quantitative ones is to be of use to involve as an objective in our proposed model. One way to express qualitative variables for modeling purpose is to use linguistic variables. To use DMs' view points, in this paper we have used Fuzzy AHP method which uses pairwise comparison to express DMs' qualitative view points and uses linguistic variables to eliminate the inherent ambiguities in personal judgments. Linguistic variables are useful in expressing situations which are too complex or too ill-defined to be reasonably described in conventional quantitative expressions [19]. In the other hand, Analytical Hierarchy Process (AHP) is one of the most applicable techniques for applying decision maker's opinion [16]. Fuzzy AHP method has been developed to consider linguistic variable to empower AHP method for situations where ambiguity plays an important role.

Multiple objective problems can consider a variety of aspects of a problem by applying more than one objective, simultaneously. In location selection problems that deal with ranking alternatives and selecting the best one, where relations between alternatives are considered negligible, Multiple Attribute Decision Making (MADM) techniques such as AHP and TOPSIS could be sufficient, but when selection of more than one alternative is desirable, ranking methods are not responsive [6] for relations between alternatives is important in selecting more than one alternative which is generally assumed negligible by these methods. In this paper, we add a new objective as a utility function to the uncapacitated single stage facility location problem (UFSLP) which considers the importance of each relation between a facility and a customer/retailer in DMs' mind.

Bashiri and Hosseininezhad [1] introduced a new fuzzy AHP approach to obtain weights of relations between old facilities and new facilities. By their approach they considered and covered all the DMs' opinions in multifacility location problem. To measure the importance of relations in DCLP, we inspire from the multifacility location problem where each DC is assumed as a new facility and each customer/retailer as an existing facility. Generally, in DCLP, the importance of each relation is measured by distance between its related DC and destination customer/retailer. It undoubtedly can't cover all the DMs' opinions. The DMs' opinions maybe consists of strategic dimensions, political parameters, reliability, accessibility, risk, and etc. of each relation between a DC and a customer/retailer. Depending on the case, some of these criteria can be more significant than the other ones from the DMs' point of view. We derive a similar approach as the one presented by Bashiri and Hosseininezhad to obtaining the utility of each relation. In this paper, we develop a method for locating DCs and allocating customer/retailer considering DMs' opinions within a multiobjective model with the aforementioned objective functions. The first objective considers location costs. Adapting multifacility location concepts, we use a utility function as the second objective in which some criteria are used to assess the utility of each relation. Fuzzy AHP method is used for this purpose. We develop an algorithm with 8 steps. This algorithm, firstly, makes a model based on a single objective DCLP model. Every single objective DCLP model can be considered here and it gives flexibility to the algorithm. Klose and Drexl [12] have presented a review for different single objective DCLP models. This algorithm tries to identify some important criteria to regard DMs' opinions. In next steps, using pairwise comparison, the algorithm obtains weights of the criteria and utility of each candidate location for the customers/retailers according to these criteria. It makes a multi-objective model and therefore it has to be solved using an especial method for this class of problem.

This paper is organized as follows. In Sections 2 and 3 we briefly describe fuzzy theory and fuzzy AHP, respectively. In Section 4 we express the problem. In the next section which is consists of our proposed algorithm and its steps, solving methodology is discussed. For illustrating the proposed algorithm, a numerical example is solved in Section 6 and its result is discussed in Section 7. Finally Section 8 draws the conclusions.

2 Fuzzy Set Theory

In the following, we briefly review some basic definitions of fuzzy sets [3, 11, 13, 18, 20]. These basic definitions and notations below will be used throughout the paper unless otherwise stated. A fuzzy set $\tilde{A}$ in a universe of discourse X is characterized by a membership function $\mu_{\tilde{A}}(x)$ which associates with each element x in X a real number in the interval [0, 1]. The function value $\mu_{\tilde{A}}(x)$ is termed the grade of membership of x in $\tilde{A}$ [18].

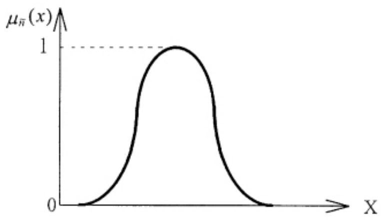

Figure 1: A fuzzy number $\tilde{n}$

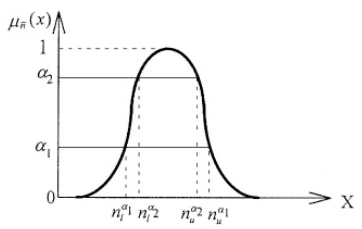

Figure 2: Fuzzy number $\tilde{n}$ with α -cuts

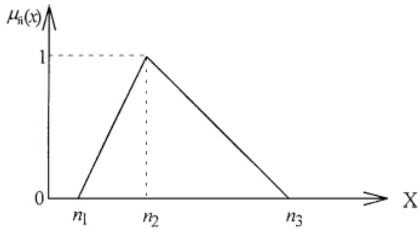

Figure 3: A triangular fuzzy number $\tilde{n}$

Definition 2.1 A fuzzy set $\tilde{A}$ of the universe of discourse X is convex if and only if for all x_1, x_2 in X,

$$\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \text{Min } (\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)) \quad \lambda \in [0,1].$$

Definition 2.2 A fuzzy set $\tilde{A}$ of the universe of discourse X is called a normal fuzzy set implying that

$$\exists x_i \in X : \mu_{\tilde{A}}(x_i) = 1.$$

A fuzzy number $\tilde{n}$ is a fuzzy subset in the universe of discourse X whose membership function is both convex and normal (see Figure 1).

Definition 2.3 The α -cut of a fuzzy number $\tilde{n}$ is defined as

$$\tilde{n}^{\alpha} = \{x_i \mid \mu_{\tilde{n}}(x_i) \geq \alpha, x_i \in X\}; \quad \alpha \in [0,1].$$

$\tilde{n}^{\alpha}$ is a non-empty bounded closed interval contained in *X* and it can be denoted by $\tilde{n}^{\alpha} = [n_1^{\alpha}, n_u^{\alpha}]$, $n_1^{\alpha}, n_u^{\alpha}$ are the lower and upper bounds of the closed interval, respectively [11, 20].

Figure 2 shows a fuzzy number $\tilde{n}$ with α -cuts, where

$$\tilde{n}^{\alpha_1} = [n_1^{\alpha_1}, n_u^{\alpha_1}], \tilde{n}^{\alpha_2} = [n_1^{\alpha_2}, n_u^{\alpha_2}].$$

From Figure 2, we can see that if $\alpha_1 \geq \alpha_2$, then $n_1^{\alpha_2} \geq n_1^{\alpha_1}$ and $n_u^{\alpha_1} \geq n_u^{\alpha_2}$.

A triangular fuzzy number $\tilde{n}$ can be defined by a triplet (n_1, n_2, n_3) shown in Figure 3. The membership function $\mu_{\tilde{n}}(x)$ is defined as [11]

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x < n_1 \\ \dfrac{x - n_1}{n_2 - n_1}, & n_1 \leq x \leq n_2 \\ \dfrac{x - n_3}{n_2 - n_3}, & n_2 \leq x \leq n_3 \\ 0, & x > n_3. \end{cases}$$

Definition 2.4([3,13]) If $\tilde{n}$ is a fuzzy number and $n_1^{\alpha} > 0$ for $\alpha \in [0,1]$, then $\tilde{n}$ is called a positive fuzzy number.

Give any two positive fuzzy numbers $\tilde{m}, \tilde{n}$ and a positive real number *r*, the α -cut of two fuzzy numbers are $\tilde{m}^{\alpha} = [m_1^{\alpha}, m_u^{\alpha}], \tilde{n}^{\alpha} = [n_1^{\alpha}, n_u^{\alpha}], \alpha \in [0,1]$, respectively.

According to the interval of confidence [11], some main operations of positive fuzzy numbers $\tilde{m}, \tilde{n}$ can be expressed as follows

$$\left(\tilde{m}(+)\tilde{n}\right)^{\alpha} = [m_1^{\alpha} + n_1^{\alpha}, m_u^{\alpha} + n_u^{\alpha}],$$

$$\left(\tilde{m}(-)\tilde{n}\right)^{\alpha} = [m_1^{\alpha} - n_u^{\alpha}, m_u^{\alpha} - n_1^{\alpha}],$$

$$\left(\tilde{m}(\cdot)\tilde{n}\right)^{\alpha} = [m_1^{\alpha} \cdot n_1^{\alpha}, m_u^{\alpha} \cdot n_u^{\alpha}],$$

$$\left(\tilde{m}(:)\tilde{n}\right)^{\alpha} = [m_1^{\alpha} : n_u^{\alpha}, m_u^{\alpha} : n_1^{\alpha}],$$

$$\left(\tilde{m}^{\alpha}\right)^{-1} = \left[\frac{1}{m_u^{\alpha}}, \frac{1}{m_1^{\alpha}}\right],$$

$$\left(\tilde{m}(\cdot)r\right)^{\alpha} = [m_1^{\alpha} \cdot r, m_u^{\alpha} \cdot r],$$

$$\left(\tilde{m}(:)r\right)^{\alpha} = \left[\frac{m_1^{\alpha}}{r,}, \frac{m_u^{\alpha}}{r,}\right].$$

Definition 2.5([3]) $\tilde{A}$ is called a fuzzy matrix, if at least an entry in $\tilde{A}$ is a fuzzy number.

Definition 2.6([13]) If $\tilde{n}$ is a fuzzy number and $n_1^{\alpha} > 0, n_u^{\alpha} \leq 1$ for $\alpha \in [0,1]$, then $\tilde{n}$ is called a normalized positive fuzzy number.

Definition 2.7 A linguistic variable is a variable whose values are linguistic terms.

Linguistic variable is useful in expressing situations which are too complex or too ill-defined to be reasonably described in conventional quantitative expressions [19]. These linguistic values can also be represented by fuzzy numbers.

3 Fuzzy AHP

Analytical Hierarchy Process (AHP) is one of the most applicable techniques for applying decision maker's opinion [16]. But since there is uncertainty in decisions, DMs ask to express their opinions with linguistics data. Chang [4] proposed a fuzzy AHP method, namely extent analysis, which converts linguistic variables to triangular fuzzy numbers as follows.

If $\tilde{M}_1 = (l_1, m_1, u_1)$ and $\tilde{M}_2 = (l_2, m_2, u_2)$ represent two triangular fuzzy numbers (Figure 4),

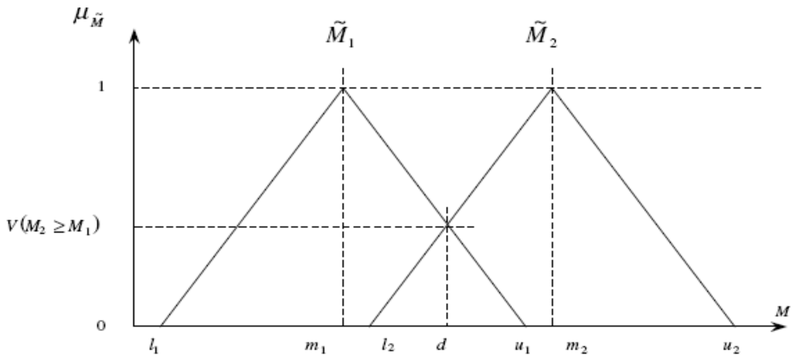

Figure 4: Comparision of two trianglar fuzzy numbers

where d is the ordinate of the highest intersection point d between two membership function, the value of $\tilde{M}_k$ relate to row *k* is calculated as follows

$$\tilde{M}_k = \sum_{j=1}^{m} M_{kj} \left[\sum_{i=1}^{m} \sum_{j=1}^{m} M_{ij} \right]^{-1}.$$

$\tilde{M}_{ij}$ is the element in row *i* and column *j*.

The degree of possibility of $\tilde{M}_2 \geq \tilde{M}_1$ is defined as

$$V(\tilde{M}_2 \geq \tilde{M}_1) = hgt(\tilde{M}_1 \cap \tilde{M}_2) = \begin{cases} 1, & \text{if} \quad m_2 \geq m_1 \\ 0, & \text{if} \quad l_1 \geq u_2 \\ \dfrac{l_1 - u_1}{(m_2 - u_2) - (m_1 - l_1)}, & \text{otherwise.} \end{cases}$$

In this method, for each matrix $j = 1, \ldots, n$,

$$W_i = \text{Min}(V(\tilde{M}_i \geq \tilde{M}_k), k = 1, \ldots, m, k \neq i).$$

Via normalization, the normalized weight vectors are

$$w_i^* = \frac{w_i}{\sum w_i}, \quad i = 1, \ldots, m.$$

So the DMs are asked to use their pair wise comparisons base on Table 1.

Table 1: Linguistic scales for and importance

Linguistic scale for importance	*Triangular fuzzy scale*	*Triangular fuzzy reciprocal scale*
Just equal	(1, 1, 1)	(1, 1, 1)
Equally important (EI)	(1/2, 1, 3/2)	(2/3, 1, 2)
Weakly more important (WMI)	(1, 3/2, 2)	(1/2, 2/3, 1)
Strongly more important (SMI)	(3/2, 2, 5/2)	(2/5, 1/2, 2/3)
Very strongly more important (VSMI)	(2, 5/2, 3)	(1/3, 2/5, 1/2)
Absolutely more important (AMI)	(5/2, 3, 7/2)	(2/7, 1/3, 2/5)

4 Problem Description

There are some customers/retailers which should be served by some DCs. In the other hand, there are some candidate locations which these DCs can be located in. Many locating pattern are possible and some objectives can be defined to evaluate these locating patterns. Now we aim to find the best pattern to optimize these objective functions.

Suppose that *n*, *m* and *p* is the numbers of customers/retailers, locations and distribution centers, respectively. The problem is to find the best location to establish these *p* DCs to get the best value of our defined objective. The problem scheme is shown illustratively in Figure 5.

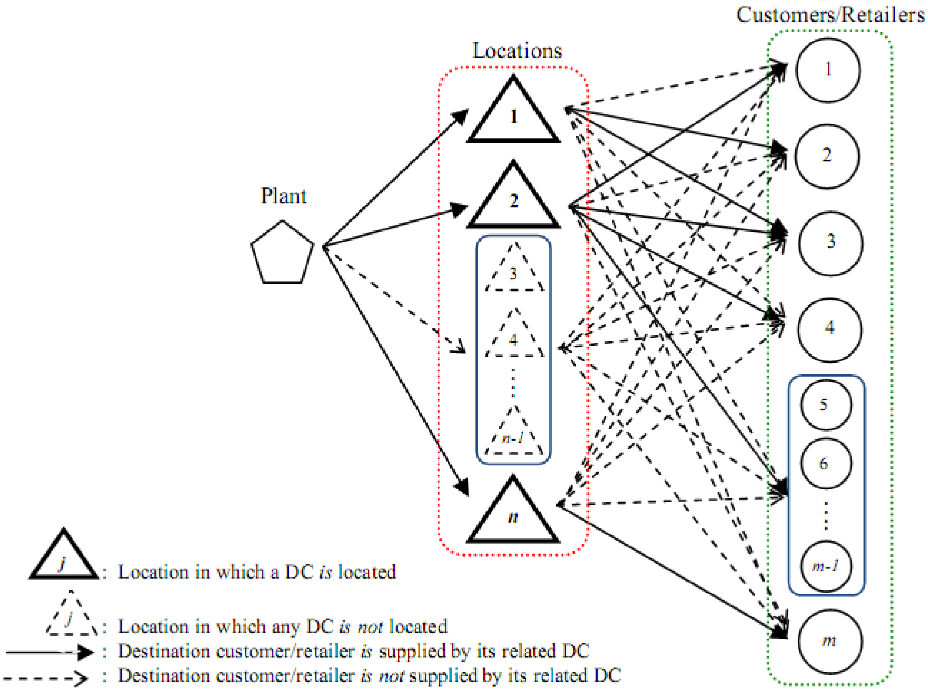

Figure 5: The illustrative figure

In Figure 5, a distribution network is shown. It consists of one plant, *n* locations and *m* customers/retailers. Each arc shows part of its destination customer/retailer demand which is provided by its related DC. The plant can provide more than one DC and also each DC can serve more than one customer/retailer. Moreover, each customer/retailer can be served by more than one DC. Because of our approach in modeling which is mixed integer programming it is defensible.

In this illustrative figure, we represent one of the possible patterns in which the plant provides three DCs that they are located in location 1, 2 and *n* (it means *p*=3). The first DC which is located in location 1 is supplying customer/retailer 2 and 3; the second DC which is located in location 2 is providing customer/retailer 1, 3 and 4 to $m-1$; the third DC which is located in location *n* is providing only customer/retailer *m*.

4.1 Assumption

The capacity of DCs is unlimited and there is not any relation or transportation between DCs and also there is not any relation or transportation between customers/retailers. We suppose that demand of customers/retailers is initially known, deterministic and constant. And total transportation costs between ith costumer/retailer and jth location are initially known, deterministic and constant. Moreover, sum of the cost of locating DC at location j and the cost of transporting products from plant to this location which is considered as a single parameter, is initially known, deterministic and constant.

4.2 Input Parameters

α_{ij} :	Utility of jth location for ith customer/retailer.
n :	Number of customers/retailers.
m :	Number of candidate locations.
$I \in \{1,...,n\}$:	Index for customer/retailer.
$J \in \{1,...,m\}$:	Index for candidate location.
f_j :	Sum of locating cost and transportation cost between jth DC and the plant.
c_{ij} :	Total transportation cost between ith costumer/retailer and jth candidate location.
C_k :	kth criterion for k=1,2,…,K.
w^{C_k} :	Weight of kth criterion for k=1,2,…,K.
$\alpha_{ij}^{C_k}$:	Utility of jth location for ith customer/retailer with respect to kth criterion.

4.3 Decision Variables

X_{ij} : The percentage of i th customer/retailer demand which is supplied by j th DC (range between 0 and 1).

$$y_j = \begin{cases} 1, & \text{if DC is located at location } j \\ 0, & \text{otherwise.} \end{cases}$$

4.4 Initial Mathematical Formulation

As has been mentioned, our model is a multiobjective model. Its formulation, without considering the second objective, is similar to UFSLP formulation which is one of the famous single objective formulations in the distribution network design literature. We explained it briefly in the introduction section. In that section we discussed UFSLP and its important points. We postpone our multiobjective modeling after introducing some significant discussions and our proposed algorithm.

Formulation of UFSLP is as follows

$$\min \sum_{i\in I}\sum_{j\in J} c_{ij} \times X_{ij} + \sum_{j\in J} f_j \times y_j \tag{1}$$

$$\text{s.t.} \quad \sum_{j\in J} X_{ij} = 1, \quad \forall i \in I \tag{2}$$

$$X_{ij} - y_j \le 0, \quad \forall i \in I, j \in J \tag{3}$$

$$0 \le X_{ij} \le 1, \quad \forall i \in I, j \in J \tag{4}$$

$$y_j \in \{0,1\}, \quad \forall j \in J. \tag{5}$$

Equ. (1) is the objective function which considers the costs of transportation and locating DCs. Constraint (2) ensures that demand is satisfied, constraint (3) ensures that demands of customers/retailers can be supplied by a DC at a location if and only if that DC is located at that location. Constraint (4) implies that decision variable X, as is defined, range between 0 and 1 and finally constraint (5) means that decision variable y, as is defined, is a binary variable.

5 Solving Methodology

5.1 Proposed Algorithm

Overall this algorithm consists of 8 steps. Firstly, we make a model based on a single objective DCLP model. Then it tries to identify some important criteria to regard DMs' opinions. In next steps, using pairwise comparison, the algorithm obtains weights of the criteria and utility of each candidate location for the customers/retailers according to these criteria. Based on these weights a utility function can be formed. Finally a multiobjective model is formulated which should be solved by an especial method for this class of problem. Since this is a multiobjective, mixed integer programming model in which the objective functions are completely inconsistence, we use LP-metric method which is one of the famous MCDM methods in solving multiobjective problems with inconsistent objective functions to solve it.

In LP-metric method multi-objective problem is solved regarding to each objective function separately and then a single objective is formulated which aims to minimize the summation of normalized difference between each objective and the optimal values of them. For our proposed model assume that 2 objective functions are named as OB_1, OB_2. According to LP-metric method, the proposed model should be solved for every one of these two objective functions separately.

Assume that the optimal values for these 2 problems are OB_1*, OB_2*. Now, LP-Metric objective function can be formulated as follows

$$\min\left[w_1.\frac{OB_1-OB_1^*}{OB_1^*}+w_2.\frac{OB_2^*-OB_2}{OB_2^*}\right]. \tag{6}$$

where w_1 and w_2 describe the weight of each objective function from the DMs' point of view. w_1 and w_2 determine the preference of one objective function in comparison with the other one from DMs' point of view.

Using this LP-metric objective function and considering the proposed model's constraints, we have a single objective mixed integer programming model, which can be solved by linear programming solvers. We used lingo 8.0 software to solve our proposed model.

5.2 Algorithm Steps

In this section, the proposed algorithm is expressed thoroughly. It is designed in 8 steps.

Step 1: Formulation of the multi distribution center location selection problem

In this paper we use the UFSLP which formulated with Mixed-integer programming approach by adding a utility function as a model to be basic model. This step is one of the strength points of the algorithm because of its flexibility in presenting different model.

Step 2: Identification of criteria for determining location utility

For decision making about the problem, it needs to identify criteria, suppose DMs determine k criteria. Let C_k shows kth criterion.

Step 3: Obtaining weights of criteria

Let w^{C_k} be weight of kth criterion. We use extent analysis method to obtain weights.

Step 4: Obtaining utility of each candidate location for the customers/retailers with respect to criteria

Let $\alpha_{ij}^{C_k}$ be utility of jth location for ith customer/retailer with respect to kth criterion (Table 2).

Table 2: Utility with respect to the criteria

Criterion / Utility	***C₁***	***C₂***	. . .	***C_q***
$\alpha_{ij}^{C_k}$	$\alpha_{ij}^{C_1}$	$\alpha_{ij}^{C_2}$	. . .	$\alpha_{ij}^{C_q}$

Step 5: Obtaining utility of locations for each customer/retailer

Let α_{ij} be utility of jth location for ith customer/retailer, so use SAW method for obtaining α_{ij}. Then

$$\alpha_{ij}=\sum_{k=1}^{q} w^{C_k}\alpha_{ij}^{C_k} .$$

Step 6: Construction of utility objective function

Utility function for the problem express as

$$UtilityFunction=\sum_{i=1}^{n}\sum_{j=1}^{m}\alpha_{ij}X_{ij} .$$

Step 7: Formulation of multi objective multi distribution center location selection problem

After construction of utility function formulate multi objective multi distribution center location selection problem as follows

$$
\begin{aligned}
\min \quad & \sum_{i\in I}\sum_{j\in J} c_{ij}\times X_{ij} + \sum_{j\in J} f_j \times y_j \\
\max \quad & \sum_{i\in I}\sum_{j\in J} \alpha_{ij}\times X_{ij} \\
\text{s.t.} \quad & \sum_{j\in J} X_{ij} = 1, \quad \forall i \in I \\
& X_{ij} - y_j \le 0, \quad \forall i\in I, j \in J \\
& 0 \le X_{ij} \le 1, \quad \forall i\in I, j\in J \\
& y_j \in \{0,1\}, \quad \forall j \in J\ .
\end{aligned}
$$

Step 8: Solving multi objective multi distribution center location selection problem

In this step, the problem is solved so that the number of DCs and the amount of assigned for each customer/retailer by DCs is determined.

Figure 6 shows these steps graphically.

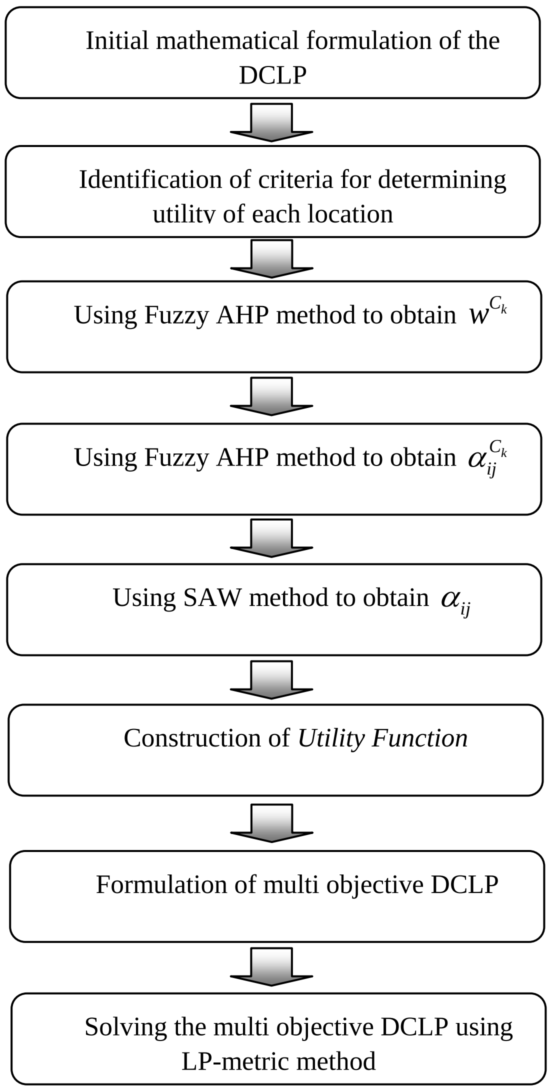

Figure 6: The algorithm of the proposed method

6 A Numerical Example for the Proposed Algorithm

In this section our proposed algorithm is exemplified. Let's assume there are 5 DCs and 12 customers as shown in Figure 7.

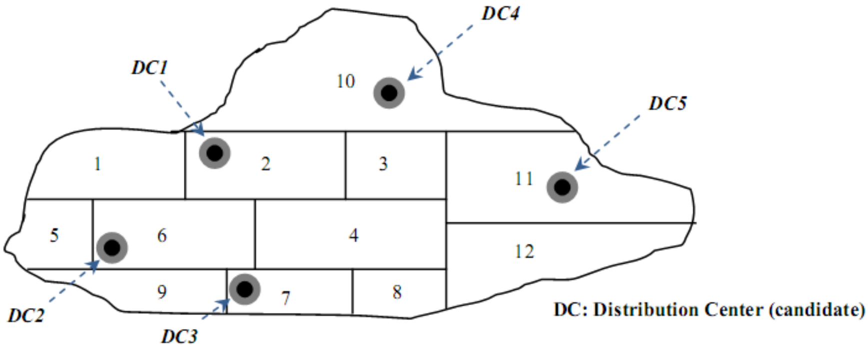

Figure 7: City map

Cost of locating each facility in location j is 100 and unit cost for each demand point is 0.3.
If distribution total cost be unit costs multiplied by distance, then C_{ij}'s are shown in Table 3.

Table 3: Distribution total cost

		Demand Point (Customer/Retailer)											
DC Candidates	***c_{ij}***	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
	1	0.3	0	0.3	0.6	0.6	0.3	0.6	0.9	0.9	0.6	0.9	1.2
	2	0.6	0.3	0.6	0.9	0.3	0	0.3	0.6	0.6	0.3	0.6	0.9
	3	0.9	0.6	0.3	0.6	0.6	0.3	0	0.3	0.9	0.6	0.3	0.6
	4	0.9	0.6	0.9	1.2	0.6	0.3	0.6	0.9	0.3	0	0.3	0.6
	5	1.2	0.9	0.6	0.9	0.9	0.6	0.3	0.6	0.6	0.3	0	0.3

The problem is to find the best location to establish *p* DCs to get the best value of our defined objective.

Step1: Based on (1-5) the first formulation of described problem for this example is as follows

$$\min \sum_{i=1}^{12}\sum_{j=1}^{5} c_{ij} X_{ij} + \sum_{j=1}^{5} f_j \times y_j$$

$$\text{s.t.} \quad \sum_{j=1}^{5} X_{ij} = 1, \quad \forall i = 1,2,\ldots,12$$

$$X_{ij} - y_j \le 0, \quad \forall i = 1,2,\ldots 12, \forall j = 1,2,\ldots,5$$

$$0 \le X_{ij} \le 1, \quad \forall i = 1,2,\ldots 12, \forall j = 1,2,\ldots,5$$

$$y_j \in \{0,1\}, \quad \forall j = 1,2,\ldots,5.$$

Step2: After discussion about the problem three criteria are identified as Political Policy (PP), Tax Freedom (TF) and Environment Condition (EC) for decision making.

Step3: To obtain weights of criteria, we use DMs' opinion showed in following matrix.

	PP	***TF***	***EC***
PP	-	*WMI*	*SMI*
TF		-	*EI*
EC			-

After applying fuzzy AHP introduced, $w^{C_1} = 0.50$, $w^{C_2} = 0.25$, $w^{C_3} = 0.25$.

Step4: Now calculate utility $\alpha_{ij}^{C_k}$ of *j*th location for *i*th customer/retailer with respect to *k*th criterion. For example use following matrix to calculate utility $\alpha_{1j}^{C_1}$ of *j*th location for 1th customer/retailer with respect to 1th criterion based on DMs' opinions:

	DC1	***DC2***	***DC3***	***DC4***	***DC5***
DC1	-	*SMI*	*WMI*	*EI*	*WMI*
DC2		-	*WMI*	*SMI*	*VSMI*
DC3			-	*WMI*	*SMI*
DC4				-	*EI*
DC5					-

After applying fuzzy AHP introduced, $\alpha_{1j}^{C_1} = (0.29, 0.31, 0.24, 0.03, 0.13)$.

After taking DMs' opinions we have:

	C_1	***C_2***	***C_3***
$\alpha_{1j}^{C_k}$	$(0.29, 0.31, 0.24, 0.03, 0.13)$	$(0.32, 0.15, 0.18, 0.20, 0.15)$	$(0.34, 0.26, 0.13, 0.11, 0.16)$

Step5: In this step, based on all of the achieved utilities from last step (each $\alpha_{ij}^{C_k}$), the utility of locations for each customer/retailer is obtained.

$$\alpha_{1j} = 0.5\times(0.29, 0.31, 0.24, 0.03, 0.13) + 0.25\times(0.32, 0.15, 0.18, 0.20, 0.15) + 0.25\times(0.34, 0.26, 0.13, 0.11, 0.16)$$
$$= (0.31, 0.26, 0.20, 0.09, 0.14)$$

Following matrix shows α_{ij}, $i = 1,...,12$, $j = 1,...,5$.

α_{ij}	***DC1***	***DC2***	***DC3***	***DC4***	***DC5***
Customer1	*0.31*	*0.26*	*0.20*	*0.09*	*0.14*
Customer2	*0.30*	*0.34*	*0.16*	*0.08*	*0.12*
Customer3	*0.16*	*0.29*	*0.05*	*0.25*	*0.25*
Customer4	*0.24*	*0.10*	*0.26*	*0.21*	*0.19*
Customer5	*0.08*	*0.17*	*0.33*	*0.32*	*0.10*
Customer6	*0.22*	*0.28*	*0.16*	*0.13*	*0.21*
Customer7	*0.33*	*0.35*	*0.17*	*0.14*	*0.01*
Customer8	*0.34*	*0.24*	*0.09*	*0.22*	*0.11*
Customer9	*0.24*	*0.24*	*0.09*	*0.22*	*0.21*
Customer10	*0.18*	*0.19*	*0.28*	*0.23*	*0.22*
Customer11	*0.17*	*0.16*	*0.13*	*0.26*	*0.28*
Customer12	*0.27*	*0.15*	*0.22*	*0.13*	*0.23*

Step6: This step considers construction of the utility function as a new objective in DCLP area.

$$UtilityFunction = \sum_{i=1}^{12}\sum_{j=1}^{5}\alpha_{ij}X_{ij} = 0.31X_{11} + 0.26X_{12} + ... + 0.23X_{125}.$$

Step7: The multiobjective model for this example is as follows

$$\min \sum_{i=1}^{12}\sum_{j=1}^{5} c_{ij}X_{ij} + \sum_{j=1}^{5} f_j \times y_j$$

$$\max \sum_{i=1}^{12}\sum_{j=1}^{5}\alpha_{ij}X_{ij} = 0.31X_{11} + 0.26X_{12} + ... + 0.23X_{12}$$

$$\text{s.t.} \quad \sum_{j=1}^{5} X_{ij} = 1, \quad \forall i = 1, 2, ..., 15$$

$$X_{ij} - y_j \le 0, \quad \forall i = 1, 2, ...15, \; j = 1, 2, ..., 5$$

$$0 \le X_{ij} \le 1, \quad \forall i = 1, 2, ...15, \; j = 1, 2, ..., 5.$$

Step8: After solving the multiobjective DCLP, the optimum solution based on the solving method is obtained as below:

Table 4: Results

Locating variables		***Allocated demand***
y_1	1	$X_{11}= X_{21}= X_{51}= X_{71}= X_{81}= X_{91}= X_{10,1}= X_{11,1}= 1$
y_2	0	---
y_3	0	---
y_4	1	$X_{34}= X_{44}= X_{64}= X_{12,4}= 1$
y_5	0	---

Value of cost objective function = 6.2
Value of utility objective function = 3.26

In the next section these achieved results were analyzed.
Selected DCs are shown in Figure 8.

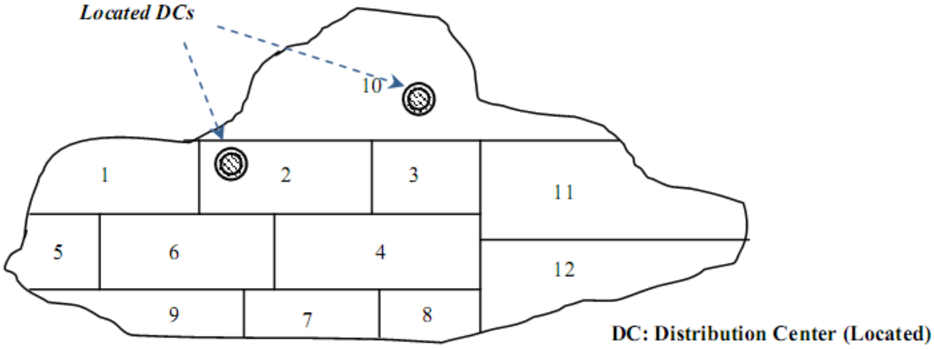

Figure 8: Located DCs in the city

According to this pattern, it is obvious that there are some better solutions for this example if merely costs are considered. But since this is the result of considering costs and DMs' opinions simultaneously, it can be completely defensible.

7 Result Analysis

To emphasize the importance of considering two objective functions simultaneously as introduced in our proposed model, three models are defined for the numerical example as follows

1） Model1 considers merely objective 1
2） Model2 considers merely objective 2
3） LP-Metric Model:

The objective function of this model is calculated by the formulation (6), in which

OB_1=First objective function of the model, OB_1^*=Best objective value found for Model1,
OB_2=Second objective function of the model, OB_2^*=Best objective value found for Model2.

All of the constraints are common for all of these models and weights of objective functions (w_1, w_2) are considered equal to 1. Then these models are solved for the numerical example and the value obtained for each objective function is used to compare the performance of models.

The results of first objective function for these models in the numerical example are shown in Figure 9.

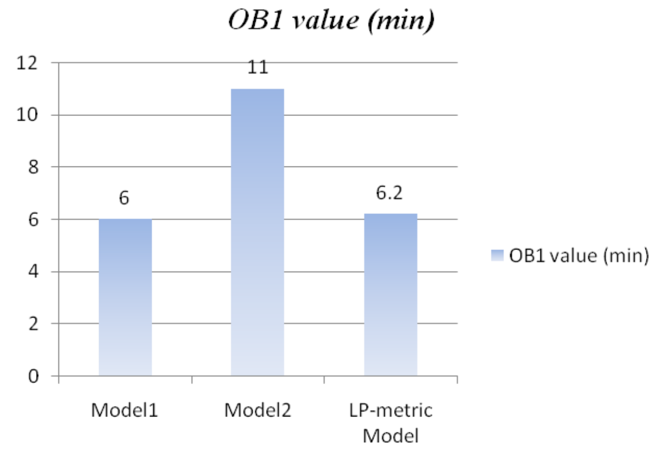

Figure 9: The results for min of the first objective function of the numerical example

The results of second objective function for these models in the numerical example are shown in Figure 10.

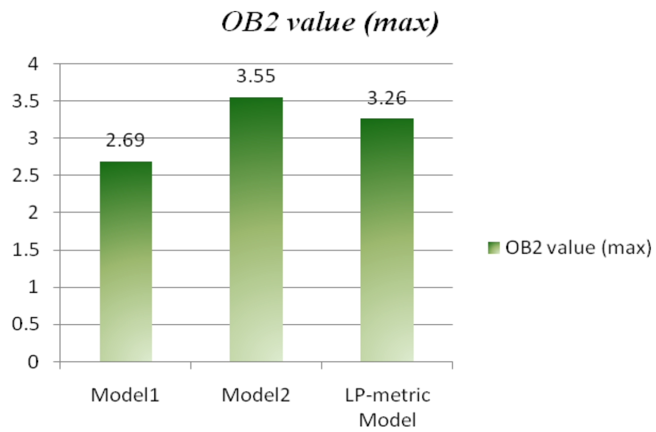

Figure 10: The results for max of the second objective function of the numerical example

The best result for the first objective function is obtained via model1 but in this case the worst result for the second objective function is obtained here. On the other hand, the best results for the second objective function is obtained via model2. But in this case the worst results for the first objective function is obtained here. Obviously considering merely one objective may sacrifice the other. Comparison of results shows that LP-metric model make a tradeoff between this two objective functions.

8 Conclusion

Distribution center location problem (DCLP) is multiobjective in nature and single objective models could not be sufficient. In the other hand, it is very important to consider decision makers' opinion in this sort of location problems. In such a challenge, presenting new objectives based on quantifying DMs' opinions to make a multiobjective model is a significant issue. In this paper we inspired an idea from multifacility location problem and added a utility function to the UFSLP model. Based on this new objective we proposed a multiobjective model which is entering qualitative parameters in model, considering DMs' opinions. Fuzzy AHP as a known strong method in quantifying DMs' opinions is used here.

A numerical example is used to illustrate problem environment. After the multiobjective model is formed using our algorithm for this numerical example, LP-metric method is employed to solve it. The result analysis shows that considering merely one objective may sacrifice the other. It means that adding the utility function as a new objective function may give solutions which are elementally different from the solutions obtained considering solely cost related issues, neglecting DMs' opinions.

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