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A Resource-Constrained Project Scheduling Problem with Fuzzy Random Duration

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Abstract

In this paper, first a fuzzy random resource-constrained project scheduling problem is presented. The object of the problem is to find the optimal scheduling of project activities. In this model, duration of project activities is a fuzzy random variable. Then, the proposed model is formulated by using the expected value of fuzzy random variables as an IP model. An illustrative example is also provided to clarify the concept.

Keywords: resource-constrained project scheduling problem, fuzzy random variable, integer programming

1 Introduction

The basic assumption in a project scheduling is complete information about the scheduling of activities. Furthermore, it is also assumed that there is a static deterministic environment within which the pre-computed baseline schedule will be executed. However, in the real world, project activities are subject to considerable uncertainty, which is gradually resolved during project execution.

The problem addressed in this research is the Resource-Constrained Project Scheduling (RCPS) problem. RCPS has been extensively studied in the literature [1, 5, 13, 15]. Hapke and Slowinski [8] considered fuzzy activity duration and applied fuzzy dispatching rules to generate a set of schedules for solving the RCPS problem. The object of this paper is to develop a fuzzy stochastic methodology to schedule the resource-constraint product development project. We apply the fuzzy stochastic framework [9, 14] to handle uncertain and flexible temporal information contain both possibility and probability aspects.

The main goal of the stochastic version of RCPS is to schedule projects with uncertain duration in order to minimize the expected project duration. The constraints of the model are zero-lag finish-start precedence constraints and renewable resource constraints. Scheduling policies method [12], Branch-and-Bound algorithms [16, 17] and heuristic procedures have been applied for stochastic RCPS problem.

The study of fuzzy model of resource-constrained project scheduling has been initiated by Hapke et al. [7] and Hapke and Slowinski [8]. In 1999, Wang [19] has developed a fuzzy set approach to schedule product development projects having imprecise temporal information. He assumed a fuzzy ready time and fuzzy deadline for the project. The activities have fuzzy durations, all described by trapezoidal fuzzy numbers.

Fuzzy stochastic theory [18, 19, 20] can provide a useful framework for managing the uncertain and flexible temporal information. There are two types of common uncertainties in the real-life words: randomness and fuzziness. Fuzzy Random Variable (F.R.V.) initiated by Kwakernaak [10] is one of the appropriate ways to describe this type of uncertainty. In this research, fuzzy random variable times have been considered for ready-time, duration time, deadline, etc..

This paper is organized as follows: in the first section, some preliminaries on fuzzy stochastic theory are presented. Then, the Fuzzy Random Resource-Constrained Project Ccheduling Problem (FR-RCPS) is introduced. The IP model and an algorithm to solve the FR-RCPS are presented in Sections 4 and 5. The concluding remarks and suggestions for further research are discussed in the last section.

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2 Preliminaries

This section reviews some technical terms presented by Puri and Ralescu [14]. In the following definitions, we assume that $(\Omega, \Im, P)$ is a probability space and $(\Theta, \mathrm{P}(\Theta), Pos)$ is a possibility space where Θ is universe, $\mathrm{P}(\Theta)$ is the power set of Θ and Pos is a possibility measure defined on fuzzy sets. Furthermore, $F_c(\Re)$ is a collection of all normalized fuzzy numbers whose $\alpha -$ level sets are convex subsets of $\Re$ and $\mathrm{K}_c(\Re)$ is the class of all nonempty compact convex subset of $\Re$.

Definition 1 *An LR fuzzy number* $\tilde{A}$ can be described with the following membership function

$$\tilde{A}(x) = \begin{cases} L(\frac{A^0 - x}{A^-}), & if\ A^0 - A^- \le x \le A^0 \\ 1, & if\ A^0 \le x \le A^1 \\ R(\frac{x - A^1}{A^+}), & if\ A^1 \le x \le A^1 + A^+ \\ 0, & otherwise \end{cases}$$

where $[A^0, A^1]$ is the peak of fuzzy number $\tilde{A}$, A^0 and A^1 are the lower and upper modal values, $A^-, A^+ > 0$ represent the left and right spread respectively, $L, R : [0,1] \to [0,1]$ with $L(0) = R(0) = 1$ and $L(1) = R(1) = 0$ are strictly decreasing, continuous functions. We will use the notation $\tilde{A} = (A^0, A^1, A^-, A^+)_{LR}$.

Definition 2 A F.R.V. is a function $\xi : \Omega \to F_c(\Re)$ such that $\{(\omega,t) \mid t \in \xi_\alpha(\omega)\} \in \Im \times \mathrm{B}$ for every $\alpha \in [0,1]$ where $\xi_\alpha : \Omega \to \mathrm{K}_c(\Re)$ is a random set defined by $\xi_\alpha(\omega) = \{t \in \Re \mid \mu_{\xi(\omega)}(t) \ge \alpha\}$ and B denotes the collection of Borel subsets of $\Re$.

Three kinds of common F.R.V. are triangular, trapezoidal and normal. The definition of trapezoidal F.R.V. is given as follows.

Definition 3 Let ξ be a F.R.V.. It is said to be trapezoidal, if for each ω, $\xi(\omega)$ is a trapezoidal fuzzy variable, denoted by $(r_1(\omega), r_2(\omega), r_3(\omega), r_4(\omega))$ where r_i is a random variable defined on the probability space Ω.

Lemma 1 Let X be a F.R.V.. Then $X(\omega) = \bigcup_{\alpha \in (0,1]} \alpha . X_\alpha(\omega)$, $\forall \omega \in \Omega$.

Proof: If A is a fuzzy number, then $A = \bigcup_{\alpha \in (0,1]} \alpha . A_\alpha$. Since we have

$$(\bigcup_\alpha \alpha . A_\alpha)(x) = Sup\ \{ \alpha .(A_\alpha)(x) \mid \alpha \in (0,1]\} = Sup\ \{ \alpha \mid x \in A_\alpha \} = A(x)$$

for any $x \in \Re$, then $A = \bigcup_{\alpha \in (0,1]} \alpha . A_\alpha$. Therefore, $X(\omega) \in F_c(\Re)$ and proof is completed.□

Definition 4 The expected value of a F.R.V. such as X, denoted by $E(X)$, is defined as follows

$$E(X) = \int_\Omega X(\omega) p(d\omega) = \bigcup_{\alpha \in (0,1]} \alpha \int_\Omega X_\alpha(\omega) p(d\omega) = \bigcup_{\alpha \in (0,1]} \alpha \left[\int_\Omega X_\alpha^-(\omega) p(d\omega), \int_\Omega X_\alpha^+(\omega) p(d\omega) \right].$$

Therefore, the expectation of a F.R.V. is defined as a unique $U \in F$ whose $\alpha -$ cut is $U_\alpha = E(X_\alpha) = \left[E(X_\alpha^-), E(X_\alpha^+) \right]$, that is $(E(X))_\alpha = E(X_\alpha)$.

Expected value is a fundamental concept for F.R.V.. In order to define the expected value of a F.R.V., several operators were introduced in literature [11, 19].

Definition 5 If a F.R.V. such as ξ degenerates to a random variable, then the expected value is defined as follows

$$E(\xi) = \int_\Omega \xi(\omega) P(d\omega) \cdot$$

If ξ is a trapezoidal F.R.V., then for each ω, $\xi(\omega)$ is a trapezoidal fuzzy variable $(r_1(\omega), r_2(\omega), r_3(\omega), r_4(\omega))$, whose expected value is $E(\xi) = (E(r_1(\omega)), E(r_2(\omega)), E(r_3(\omega)), E(r_4(\omega)))$.

Colloraly 1 Let X and Y be F.R.V. and $\lambda \in \Re$. Then

i) $E(\lambda) = \lambda$;

ii) $E(X + \lambda Y) = E(X) + \lambda E(Y)$.

Definition 6[4] Let $\overline{X}=(x^0,x^-,x^+)$ and $\overline{X}=(y^0,y^-,y^+)$ be two fuzzy numbers. Then we have

$$\overline{X}\le\overline{Y}\Leftrightarrow x^0\le y^0\ \&\ x^0-x^-\le y^0-y^-\ \&\ x^0+x^+\le y^0+y^+ .$$

Now we discuss a method to evaluate the fuzzy random inequality $X\tilde{\leqq}Y$ or $X\tilde{\geqq}Y$ where X and Y are fuzzy random variables. It is obvious that $E(X)$ and $E(Y)$ in this case are fuzzy numbers according to Definition 5 and can be compared based on Definition 6.

Definition 7 Let X and Y be fuzzy random variables. Then the relations "$\tilde{\leqq}$" and "$\tilde{\geqq}$" are defined respectively as follows

i) $X\tilde{\leqq}Y$ iff $E(X)\le E(Y)$;

ii) $X\tilde{\geqq}Y$ iff $E(X)\ge E(Y)$.

3 Problem Formulation

This paper applies fuzzy stochastic theory to model the uncertain and flexible temporal information in a resource-constrained project scheduling problem. The uncertainty in this model is in the both probability and possibility aspects. The uncertain activity duration $\tilde{d}$ is represented by a fuzzy random variable and can be defined by a trapezoidal F.R.V. $\tilde{d}(\omega)=(t_1(\omega),t_2(\omega),t_3(\omega),t_4(\omega))$, $\omega\in\Omega$ [21].

The flexible ready-time and deadline of a project are also assumed to be trapezoidal F.R.V. and denoted by $\tilde{b}$ and $\tilde{e}$ respectively. Despite an activity duration, both possibility-probability distributions are used to represent the preference of a project manager for $\tilde{b}$ and $\tilde{e}$. For example, a project manager may prefer that a project should be completed before $e_1(\omega)$, but not later than $e_2(\omega)$. In this case, the preferred project deadline $\tilde{e}$ can be represented as $\tilde{e}(\omega)=(e_1(\omega),e_1(\omega),e_1(\omega),e_2(\omega))$, $\omega\in\Omega$. In the same way, the preferred ready-time of project $\tilde{b}$ can also be represented as $\tilde{b}(\omega)=(b_1(\omega),b_2(\omega),b_2(\omega),b_2(\omega))$, $\omega\in\Omega$ [21].

The definition of Fuzzy Random Resource-Constrained Project Scheduling (FR-RCPS) model is described as follows. A project p has a ready-time $\tilde{b}$ and a deadline $\tilde{e}$, both are fuzzy random variables, and all of its activities must be performed during the period $[\tilde{b},\tilde{e}]$. Furthermore, activity i has specific F.R.V. duration $\tilde{d}_i$ and its execution requires the exclusive use of a number of resources defined by a vector $N_i=(n_{i1},n_{i2},...,n_{iq})$ whose elements determine the usage of resources type $1,2,...,q$ by activity i.

The resource availability for the project is also defined by a vector $R=(m_1,m_2,...,m_q)$ where m_k indicates the availability of resource type k, $k=1,...,q$. Now, the decision variable and constraints of our model are defined as follows [21]:

Decision Variable

$\tilde{f}_i$: finish time of activity i which is a trapezoidal F.R.V..

Constraints

(1) Earliest-start-time constraints: For all activities, the real finish time of activity i denoted by f_i should be greater than or equal to its earliest finish time which is F.R.V and denoted by $\tilde{eft_i}$. Therefore, we have $f_i\in[\tilde{eft_i},+\infty)$.

The earliest finish time of activity i is equal to $\tilde{est_i}+\tilde{d}_i$, where

$$\tilde{est_i}=\begin{cases}\tilde{\max_j}\{\tilde{est_i},\tilde{est_j}+\tilde{d}_j\}, & \text{if activity } i \text{ is preceded by activity } j\\ \tilde{b}, & \text{if activity } i \text{ has no predecessors.}\end{cases}$$

(2) Latest-start-time constraints: For all activities, the real finish time of activity i should be less than or equal to its latest finish time denoted by $\tilde{lft_i}$. Hence, we have $f_i\in(-\infty,\tilde{lft_i}]$.

The latest finish time of activity i is defined as

$$\tilde{lft_i} = \begin{cases} \tilde{\min_j} \{ \tilde{lft_i}, \tilde{lft_j} - \tilde{d}_j \}, & \text{if activity } j \text{ is preceded by activity } i \\ \tilde{b}, & \text{if activity } i \text{ has no successors.} \end{cases}$$

(3) *Precedence constraints:* If activity i precedes j in a partial order, then $f_j - f_i \in [\tilde{d}_j, +\infty)$.
This constraint implies that the finish time of activity i should be less than or equal to the start time of activity j. In order words, $f_j - f_i$ should be greater than or equal to the duration of activity j.

(4) *Resource capacity constraints:* For any time t, let $J_t = \{i \mid f_i - \tilde{d}_i \le t < f_i\}$. Then for time t, and all resource types $k = 1,...,q$, we have $\sum_{i \in J_t} n_{ik} \le m_k$.

We want to find a feasible schedule that minimize the project makespan $\tilde{f}_n$.

4 Fuzzy Linear Programming Approach

The mathematical formulation of FR-RCPS problem is presented as follows:

(FR-RCPS)

$$\begin{aligned} \text{Min} \quad & \tilde{f}_n \\ \text{s.t.} \quad & \tilde{f}_i \tilde{\le} \tilde{f}_j - \tilde{d}_j \quad \text{for all } (i,j) \in A \\ & \tilde{f}_1 \cong \tilde{b} + \tilde{d}_1 \\ & \sum_{i \in J_t} n_{ik} \le m_k, \; k = 1,...,q, \; J_t = \{i \mid f_i - \tilde{d}_i \le t < f_i\}, \; \forall t \\ & \tilde{f}_i \text{ is F.R.V. for } i = 1, \dots, n \end{aligned}$$

where the first and the third constraints are related to precedence and resource capacity constraints respectively.

In order to transform the above model to a mathematical programming model, we need to explain how to consider J_t as a constraint.

We use the Alvarz-Valas and Tamarit's approach [2] to linearization of FR-RCPS problem. This approach is based on the definition of a set, IS, of all minimal resource incompatibles sets S. In order to resolve a resource conflict that would originate from parallel processing of activities of a resource incompatible set , one needs to introduce at least one precedence relation between a pair (i, j) of the activities in that set. The binary decision variable x_{ij}, which equals 1 if activity i precedes activity j and 0 otherwise (i.e., if activity j is scheduled before i or in parallel with i). We obtained the following formulation:

(Model 1)

$$\begin{aligned} \text{Min} \quad & \tilde{f}_n \\ \text{s.t.} \quad & x_{ij} = 1; x_{ji} = 0 \quad \text{for all } (i,j) \in A \\ & x_{ij} + x_{ji} \le 1 \text{ for } i, j = 1, \dots\dots, n \text{ and } i \ne j \\ & x_{ij} + x_{jk} - x_{ik} \le 1 \text{ for } i, j, k = 1,..., n \text{ and } i \ne j, j \ne k, i \ne k \\ & \sum_{i, j \in S, i \ne j} x_{ij} \ge 1 \text{ for all } S \in IS \\ & \tilde{f}_i \tilde{\le} \tilde{f}_j - x_{ij}(\tilde{d}_j + M) + M \quad \text{for } i, j = 1,..., n \text{ and } i \ne j \\ & \tilde{f}_1 \cong \tilde{b} + \tilde{d}_1 \\ & x_{ij} \in \{0, 1\} \quad \text{for } i, j = 1,..., n \text{ and } i \ne j \\ & \tilde{f}_i \text{ is F.R.V. for } i = 1, \dots, n. \end{aligned}$$

By using the concept of expected value of fuzzy random variable and Corollary 1, the above model can be converted to the following fuzzy integer programming problems without fuzzy random parameters:

(Model 2)

$$
\begin{aligned}
\text{Min} \quad & E(\tilde{f}_n) \\
\text{s.t.} \quad & x_{ij}=1; x_{ji}=0 \quad \text{for all } (i,j)\in A \\
& x_{ij}+x_{ji}\le 1 \quad \text{for } i,j=1,\dots,n \text{ and } i\ne j \\
& x_{ij}+x_{jk}-x_{ik}\le 1 \quad \text{for } i,j,k=1,\dots,n \text{ and } i\ne j,\ j\ne k,\ i\ne k \\
& \sum_{i,j\in S, i\ne j} x_{ij}\ge 1 \quad \text{for all } S\in IS \\
& E(\tilde{f}_i)\,\tilde{\le}\, E(\tilde{f}_j)-x_{ij}(E(\tilde{d}_j)+M)+M \quad \text{for } i,j=1,\dots,n \text{ and } i\ne j \\
& E(\tilde{f}_1)=E(\tilde{b}+\tilde{d}_1) \\
& x_{ij}\in\{0,1\} \quad \text{for } i,j=1,\dots,n \text{ and } i\ne j \\
& \tilde{f}_i \text{ is F.R.V. for } i=1,\dots,n.
\end{aligned}
$$

By replacing the expected values of variables, we have

(Model 3)

$$
\begin{aligned}
\text{Min} \quad & \bar{f}_n \\
\text{s.t.} \quad & x_{ij}=1; x_{ji}=0 \quad \text{for all } (i,j)\in A \\
& x_{ij}+x_{ji}\le 1 \quad \text{for } i,j=1,\dots,n \text{ and } i\ne j \\
& x_{ij}+x_{jk}-x_{ik}\le 1 \quad \text{for } i,j,k=1,\dots,n \text{ and } i\ne j,\ j\ne k,\ i\ne k \\
& \sum_{i,j\in S, i\ne j} x_{ij}\ge 1 \quad \text{for all } S\in IS \\
& \bar{f}_i\,\tilde{\le}\, \bar{f}_j-x_{ij}(\bar{d}_j+M)+M \quad \text{for } i,j=1,\dots,n \text{ and } i\ne j \\
& \bar{f}_1=\bar{b}+\bar{d}_1 \\
& x_{ij}\in\{0,1\} \quad \text{for } i,j=1,\dots,n \text{ and } i\ne j \\
& \bar{f}_i\ge 0 \quad \text{for } i=1,\dots,n.
\end{aligned}
$$

In the above model, $\bar{f}$, $\bar{d}$ and $\bar{b}$ are fuzzy trapezoidal values of $\tilde{f}$, $\tilde{d}$ and $\tilde{b}$ respectively. Now, by using the concept of trapezoidal fuzzy numbers, (Model 3) can be written as follows

(Model 4)

$$
\begin{aligned}
\text{Min} \quad & (f_{n1}, f_{n2}, f_{n3}, f_{n4}) \\
\text{s.t.} \quad & x_{ij}=1; x_{ji}=0 \quad \text{for all } (i,j)\in A \\
& x_{ij}+x_{ji}\le 1 \quad \text{for } i,j=1,\dots,n \text{ and } i\ne j \\
& x_{ij}+x_{jk}-x_{ik}\le 1 \quad \text{for } i,j,k=1,\dots,n \text{ and } i\ne j,\ j\ne k,\ i\ne k \\
& \sum_{i,j\in S, i\ne j} x_{ij}\ge 1 \quad \text{for all } S\in IS \\
& (f_{i1}, f_{i2}, f_{i3}, f_{i4})\,\tilde{\le}\,(f_{j1}, f_{j2}, f_{j3}, f_{j4})-x_{ij}((d_{j1},d_{j2},d_{j3},d_{j4})+M)+M \\
& \qquad \text{for } i,j=1,\dots,n \text{ and } i\ne j \\
& (f_{11}, f_{12}, f_{13}, f_{14})=(b_1,b_2,b_2,b_2)+(d_{11},d_{12},d_{13},d_{14}) \\
& x_{ij}\in\{0,1\} \quad \text{for } i,j=1,\dots,n \text{ and } i\ne j \\
& \bar{f}_i=(f_{i1}, f_{i2}, f_{i3}, f_{i4})\ge 0 \quad \text{for } i=1,\dots,n.
\end{aligned}
$$

By using the maximizing fuzzy number method and fuzzy inequality approaches [4, 6, 22], the following model is generated

(Model 5)

$$\text{Max} \quad [+f_{n2} - f_{n1}, -f_{n2}, -f_{n3}, -f_{n4} + f_{n3}]$$

$$\begin{aligned}
\text{s.t.} \quad & x_{ij} = 1; x_{ji} = 0 \quad \text{for all } (i,j) \in A \\
& x_{ij} + x_{ji} \le 1 \quad \text{for } i,j = 1,\dots,n \text{ and } i \ne j \\
& x_{ij} + x_{jk} - x_{ik} \le 1 \quad \text{for } i,j,k = 1,\dots,n \text{ and } i \ne j,\ j \ne k,\ i \ne k \\
& \sum_{i,j \in S, i \ne j} x_{ij} \ge 1 \quad \text{for all } S \in IS \\
& f_{i2} \le f_{j2} - x_{ij}(d_{j2} + M) + M \quad \text{for } i,j = 1,\dots,n \text{ and } i \ne j \\
& f_{i3} \le f_{j3} - x_{ij}(d_{j3} + M) + M \quad \text{for } i,j = 1,\dots,n \text{ and } i \ne j \\
& f_{i1} \le f_{j1} - x_{ij}(d_{j1} + M) + M \quad \text{for } i,j = 1,\dots,n \text{ and } i \ne j \\
& f_{i4} \le f_{j4} - x_{ij}(d_{j4} + M) + M \quad \text{for } i,j = 1,\dots,n \text{ and } i \ne j \\
& (f_{11}, f_{12}, f_{13}, f_{14}) = (b_1, b_2, b_2, b_2) + (d_{11}, d_{12}, d_{13}, d_{14}) \\
& x_{ij} \in \{0,1\} \quad \text{for } i,j = 1,\dots,n \text{ and } i \ne j \\
& f_{i1} \le f_{i2} \le f_{i3} \le f_{i4} \quad \text{for } i = 1,\dots,n \\
& f_{i1}, f_{i2}, f_{i3}, f_{i4} \ge 0 \quad \text{for } i = 1,\dots,n.
\end{aligned}$$

Let Z^1, Z^2, Z^3 and Z^4 be upper bounds of $+f_{n2} - f_{n1}$, $-f_{n2}$, $-f_{n3}$ and $-f_{n4} + f_{n3}$ respectively and p^1, p^2, p^3 and p^4 be their initial tolerance values. By consideration the membership function of fuzzy objective function and using Bellman and Zadeh's max-min operator [3], (Model 5) can be converted to the following model [23, 24]:

(Model 6)

$$\text{Max} \quad \eta$$

$$\begin{aligned}
\text{s.t.} \quad & +f_{n2} - f_{n1} \ge Z^1 - (1-\eta)p^1 \\
& -f_{n2} \ge Z^2 - (1-\eta)p^2 \\
& -f_{n3} \ge Z^3 - (1-\eta)p^3 \\
& -f_{n4} + f_{n3} \ge Z^4 - (1-\eta)p^4 \\
& x_{ij} = 1; x_{ji} = 0 \quad \text{for all } (i,j) \in A \\
& x_{ij} + x_{ji} \le 1 \quad \text{for } i,j = 1,\dots,n \text{ and } i \ne j \\
& x_{ij} + x_{jk} - x_{ik} \le 1 \quad \text{for } i,j,k = 1,\dots,n \text{ and } i \ne j,\ j \ne k,\ i \ne k \\
& \sum_{i,j \in S, i \ne j} x_{ij} \ge 1 \quad \text{for all } S \in IS \\
& f_{i2} \le f_{j2} - x_{ij}(d_{j2} + M) + M \quad \text{for } i,j = 1,\dots,n \text{ and } i \ne j \\
& f_{i3} \le f_{j3} - x_{ij}(d_{j3} + M) + M \quad \text{for } i,j = 1,\dots,n \text{ and } i \ne j \\
& f_{i1} \le f_{j1} - x_{ij}(d_{j1} + M) + M \quad \text{for } i,j = 1,\dots,n \text{ and } i \ne j \\
& f_{i4} \le f_{j4} - x_{ij}(d_{j4} + M) + M \quad \text{for } i,j = 1,\dots,n \text{ and } i \ne j \\
& (f_{11}, f_{12}, f_{13}, f_{14}) = (b_1, b_2, b_2, b_2) + (d_{11}, d_{12}, d_{13}, d_{14}) \\
& x_{ij} \in \{0,1\} \quad \text{for } i,j = 1,\dots,n \text{ and } i \ne j \\
& f_{i1} \le f_{i2} \le f_{i3} \le f_{i4} \quad \text{for } i = 1,\dots,n \\
& f_{i1}, f_{i2}, f_{i3}, f_{i4} \ge 0 \quad \text{for } i = 1,\dots,n \\
& 0 \le \eta \le 1.
\end{aligned}$$

The above model is a mixed integer programming model and can be solved by one of the MIP solution approaches such as Branch and bound. Then, expected-optimal solution of the original problem with trapezoidal fuzzy numbers can be obtained by $\bar{f}_i^* = (f_{i1}^*, f_{i2}^*, f_{i3}^*, f_{i4}^*)$ for $i = 1, ..., n$.

In our model, finish times of activities are assumed to be fuzzy random variables which are more suitable to real world problems. However, it is not possible to determine the finish time of project theoretically in this case due to fuzzy randomness of activity duration. Therefore, we used the concept of expected value of fuzzy random variables to overcome this problem. This replacement enabled us to convert the original complex model to a mixed integer programming model. Another advantage of using expected value is its linear property in compare to the other moments like the variance. However, using the expected value is tantamount to focus on the center of the distribution while neglecting other parameters of the distribution.

Unfortunately, obtaining the finish time of activities is not an easy task due to the complexity of the final model if some other parameters of the fuzzy random variables are taken into consideration and the generalization of the model in this case can be an interesting idea for future researches.

5 An Algorithm for Solving FR-RCPS Problem

In the following steps, we describe the general steps of algorithm for solving FR-RCPS problem.

Data Entry

-Define a membership function for each fuzzy random variable in (Model 1) and determine the expected values of the fuzzy random variables.

Model Structure

-Convert (Model 1) to (Model 2) by using the concept of random expected value of fuzzy random variables.

-Convert (Model 2) to (Model 4) by using the concepts of fuzzy numbers.

-Convert (Model 4) to (Model 5) by fuzzy inequality approaches.

-Convert (Model 5) to (Model 6) by Bellman and Zadeh's max-min operator. (Model 6) is a mixed integer programming model.

Solution Procedure

-Solve (Model 6) as a Mixed Integer Programming model by one of the MIP solvers. Let $f_{i1}^*, f_{i2}^*, f_{i3}^*, f_{i4}^*$ be expected solutions. Obtain an optimal solution of the original problem by $\bar{f}_i^* = (f_{i1}^*, f_{i2}^*, f_{i3}^*, f_{i4}^*)$.

Example [21]: Assume that a project consists of seven activities and as it is represented by the precedence graph shown in Figure 1. The corresponding activity information is listed in Table 1 and supposed that the resource usage of each activity is 1. The fuzzy project ready-time and deadline are set to (0, 1, 1, 1) and (57, 57, 57, 63) respectively. Only one type of resource is required for the project and its resource availability is 2. Each activity resource usage is 1.

Table 1: Activity information in Figure 1

Activity	Expected of Duration
a_1	(5, 7, 8, 10)
a_2	(8, 10, 15, 18)
a_3	(14, 17, 20, 24)
a_4	(9, 12, 16, 20)
a_5	(3, 5, 7, 9)
a_6	(5, 9, 12, 15)
a_7	(20, 24, 28, 33)

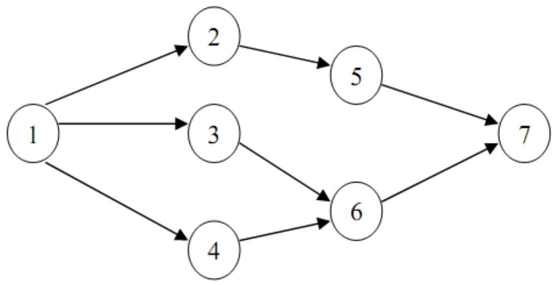

Figure 1: Precedence graph with seven activities

Apply the proposed algorithm and solve the obtained MIP problem by one of the MIP solver. The following MIP model is generated

$$
\begin{aligned}
\text{Min} \quad & [f_{72}, f_{73}, f_{74} - f_{73}, f_{71} - f_{72}] \\
\text{s.t.} \quad & x_{ij} = 1; x_{ji} = 0 \quad \text{for all } (i,j) \in A \\
& x_{ij} + x_{ji} \le 1 \quad \text{for } i,j = 1,\dots,7 \text{ and } i \ne j \\
& x_{ij} + x_{jk} - x_{ik} \le 1 \quad \text{for } i,j,k = 1,\dots,7 \text{ and } i \ne j,\ j \ne k,\ i \ne k \\
& x_{23} + x_{32} + x_{24} + x_{42} + x_{34} + x_{43} \ge 1 \\
& x_{53} + x_{35} + x_{54} + x_{45} + x_{34} + x_{43} \ge 1 \\
& f_{ik} \le f_{jk} - x_{ij}(d_{jk} + M) + M \quad \text{for } i,j = 1,\dots,7 \text{ and } i \ne j,\ k = 1,\dots,4 \\
& (f_{11}, f_{12}, f_{13}, f_{14}) = (0,1,1,1) + (5,7,8,10) = (5,8,9,11) \\
& x_{ij} \in \{0,1\} \quad \text{for } i,j = 1,\dots,7 \text{ and } i \ne j \\
& f_{i1} \le f_{i2} \le f_{i3} \le f_{i4} \quad \text{for } i = 1,\dots,7 \\
& f_{i1}, f_{i2}, f_{i3}, f_{i4} \ge 0 \quad \text{for } i = 1,\dots,7.
\end{aligned}
$$

The above model is solved by LINGO 8.0 and the optimal solution of the model is summarized in the second column of Table 2.

Table 2: Schedule generated for the problem

	FR-RCPS	RCPSP	
Activity	$\overline{f}_i^*$	f_i^*	Precedence Relations
1	(5, 8, 9, 11)	8.5	
2	(22, 30, 40, 49)	35	$4 \to 2 \to 5 \to 7$
3	(19, 25, 29, 35)	27	
4	(14, 20, 25, 31)	22.5	$1 \to 3 \to 6$
5	(25, 35, 47, 58)	41	
6	(24, 35, 41, 50)	37.5	$4 \to 5$
7	$\overline{f}_7^*$=(45, 59, 75, 91)	f_7^*=67	

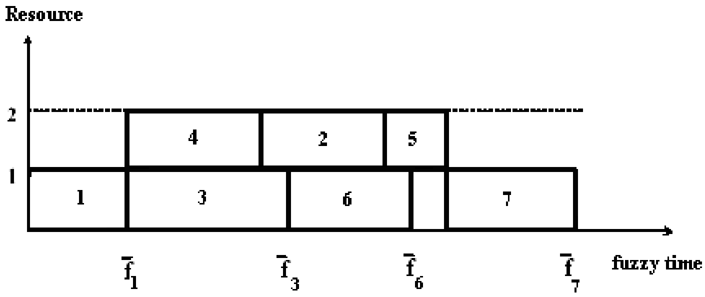

Figure 2: Schedule with seven activities

In order to compare the result of our model with the original RCPS problem in which only duration of project activities are assumed to be real variables, this example was also solved by considering the following assumptions: $d_1 = 7.5$, $d_2 = 12.5$, $d_3 = 18.5$, $d_4 = 14$, $d_5 = 6$, $d_6 = 10.5$, $d_7 = 26$. For the real model the following MIP model is generated in which the variables are real

$$\begin{aligned}
\text{Min } & f_7 \\
\text{s.t. } & x_{ij} = 1; x_{ji} = 0 \quad \text{for all } (i, j) \in A \\
& x_{ij} + x_{ji} \le 1 \quad \text{for } i, j = 1,\dots,7 \text{ and } i \ne j \\
& x_{ij} + x_{jk} - x_{ik} \le 1 \quad \text{for } i, j, k = 1,\dots,7 \text{ and } i \ne j, j \ne k, i \ne k \\
& x_{23} + x_{32} + x_{24} + x_{42} + x_{34} + x_{43} \ge 1 \\
& x_{53} + x_{35} + x_{54} + x_{45} + x_{34} + x_{43} \ge 1 \\
& f_i \le f_j - x_{ij}(d_j + M) + M \quad \text{for } i, j = 1,\dots,7 \text{ and } i \ne j, \\
& f_1 = e + d_1 \\
& x_{ij} \in \{0,1\} \quad \text{for } i, j = 1,\dots,7 \text{ and } i \ne j \\
& f_i \ge 0 \quad \text{for } i = 1,\dots,7.
\end{aligned}$$

The optimal solution in this case when only the mean value of the duration of project activities is important is also shown in the third column of Table 2. Furthermore, the optimal precedence relations of this example in both fuzzy and real state are summarized in the forth column of Table 2. Finally, Figure 2 indicates the optimal precedence relations of activities according to resource constraints. As this result shows, precedence relations have not been changed but finish time of activity of our model are more confident. There is no difference between optimal precedence relations of two models because we have used exact linear programming in both fuzzy and real models. In the fuzzy random model the finish time of activates and the project makespan are trapezoidal fuzzy variables which are critical for a top manager of a project and he/she can apply them in order to schedule more confidently in compare with the real model which has real variables and has more risk for the manager.

6 Concluding Remarks

In this paper, we introduced the FR-RCPS problem. Then, we proposed a method for solving it. This method is based on linear programming formulation [2]. We applied it to the problem in order to transform non-linear constraints of FR-RCPSP to linear constraints. Furthermore, we used the expected value of fuzzy random variables, fuzzy inequality approaches, and Bellman-Zadeh's max-min operator for multi-objective programming. In the fuzzy random model the finish time of activates and the project makespan are both trapezoidal fuzzy variables which are more flexible for a project manager.

For further future research, different suggestions and remarks can be considered. For example, a meta-heuristic approach can be used to obtain a near-optimal schedule. Furthermore, the usage of resource availability can also be considered as fuzzy random numbers.

As it was mentioned before, there is no doubt that by replacing the fuzzy random variables by their expected values some information will be lost. To improve the method discussed in this paper, the reader can generalize the model by adding the effect of other factors like variance of a fuzzy random variable to model.

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