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Designing Cellular Manufacturing Systems under Uncertainty

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Abstract

This paper addresses a stochastic model in order to design the cellular manufacturing systems (CMSs) under uncertain environment. In real-world cases, any parameters such as demand, processing time, inter arrival time and etc may change over the planning horizon. In this research, it's assumed that processing time for parts on machines and arrival time for parts to cells are stochastic and described by exponential distribution which yield more flexibility for the model and the results. In uncertain environments an approach which can analyze this problem is queuing theory. In this paper, we assume that each machine is a server and each part is a customer where servers should service to customers. Therefore, we have a queue system which can be optimized by queue theory where this approach is not researched well in the literature. The aim of this model is to minimize summation of three cost types: (1) the idleness costs for machines which introduced as servers, (2) total cost of sub-contracting for exceptional elements and (3) the cost of resource underutilization. Finally, some numerical examples are illustrated to show effectiveness of the proposed approach. Also, sensitivity analysis will be performed to learn more about behavior of the model.

Keywords: cellular manufacturing system, queuing theory, uncertainty modeling, stochastic processing time, stochastic arrival time, sensitivity analysis

1 Introduction and Literature Review

Cellular manufacturing system (CMS) is a manufacturing concept where aims to group products to part families according to their similarities is manufacturing processing and also, machines are grouped to machine cells based on parts manufactured by them. CMS framework is a major application of group technology (GT) philosophy. Group technology is a management theory that aims to group products with similar process or manufacturing characteristics, or both (Mitrofanov, [1]). Some real-world limitations in cell formation (CF) are: available capacity of machines must not be exceeded, safety and technological necessities must be met, the number of machines in a cell and the number of cells have not be exceeded an upper bound, intercellular and intracellular costs of handling material between machines must be minimized, machines must be utilized in effect (Heragu, [2]).

There exists many considerations in designing and planning of CMS in different areas such as cell formation problem (Uddin *et al.* [3]; Logendran *et al.* [4]; Cabrera-Rios *et al.*, Cabrera-Rios *et al.* [5]), considering layout problem in CMS problem (Bazargan-Lari, [6]), production planning concurrently in CMS (Riezebos *et al.*, [7]), in addition, simultaneously scheduling in CMS (Wemmerlov and Vakharia, Wemmerlov and Vakharia [8], Solimanpur *et al.* [9], Aneja and Kamoun [10]), etc. Of these issues, the cell formation problem is an area that has been more researched in literature (Soleymanpour *et al.* [11], Onwobolu and Mutingi [12]). Exceptional elements are defined as parts which must be processed in different cells and therefore they have intercellular movements. Shafer *et al.* [13] developed a model which introduces different states for exceptional elements considering inter-cell and intra-cell movement, machines duplication and subcontracting costs. Saad [14] proposed an integrated approach to redesign CMS considering emphasis on redesign aspects. His approach used simulation based on scheduling module.

In practice, costs, demands, processing times, set-up times and other inputs to classical CMS problems may be highly uncertain so that it can have impact on results sensitively. Thus, development models for cell formation problem under uncertainty can be suitable area for researchers and belongs to a relatively new class of CMS problems that not researched well in the literature. In addition, parameter estimates may be mistaken due to inaccurate measurement in the modeling process such as aggregated demands. Knowing this, researchers must develop models for CMS under uncertainty. In this way, random parameters can be either continues or described by discrete scenarios.

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If probability information is known, uncertainty is described using a (discrete or continuous) probability distribution on the parameters, otherwise, continuous parameters are normally limited to lie in some pre-determined intervals (Snyder, [15] and Ghezavati *et al.* [16]). There are some approaches such as stochastic programming, queuing theory and robust optimization which can be applied for uncertainty modeling. In this study, it's assumed that random parameters have continues probability distribution. Also, queuing theory will be applied to reach desired results.
Queuing theory can be applied to any manufacturing or service systems (also, in cellular manufacturing systems). For example, in a machine shop, jobs wait to be machined (Heragu; [17]). In a queuing system, customers arrive by some arrival process and wait in a queue for the next available server. In the manufacturing framework, customers can be assumed as parts and servers may be machines. The input process shows how parts arrive at a queue in a cell. An arrival process is commonly identified by the probability distribution of the number of arrivals in any time interval. The service process is usually described by a probability distribution. The service rate is the number of parts (customers) served per unit time. The arrival rate of a queuing system is usually given as the number of parts (customers) arriving per unit time. Thus, measurements of a queue system such as maximization the probability that each server is busy (utilization factor), minimization waiting time in queues (that leads to minimization work in process in cells) and etc can be optimized and cells will be formed optimality. In addition, we consider sub-contracting or outsourcing as a penalty cost for exceptional elements. In this way, if a part needs to be operated on a machine isn't located together in a same cell, due to use subcontracting, the total capacity of machine is not used completely and the machine will be idle. Therefore, by minimizing costs related machines' idleness rate (the probability that a machine is idle), cells with the most similarities in processing and also, optimized part families will be formed, concurrently. Sample of a manufacturing cell modeled as a queuing system is shown in Figure 1.

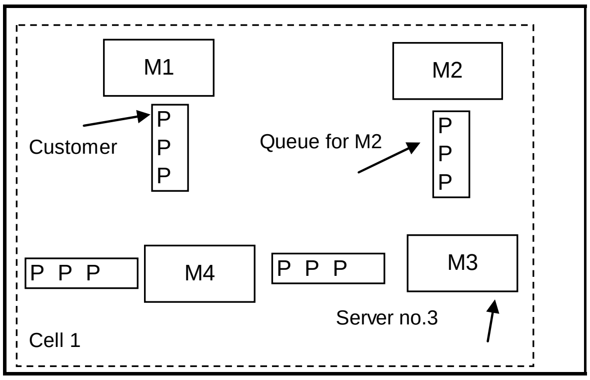

Figure 1: Sample of a cell in a CMS problem formed as a queuing system

In this paper, we'll formulate a CMS problem as a queue system and therefore, it can be optimized by queue theory and finally effectiveness of this approach will be illustrated. The goal of this model is to minimize summation of three cost types: (1) the idleness costs for machines which introduced as servers, (2) total cost of sub-contracting for exceptional elements and (3) the cost of resource underutilization.

The structure of this paper is as follows. In Section 2, we present the stochastic cell formation problem (SCFP) and formulation of the problem is presented. We present computational results and sensitivity analysis in Section 3. In Section 4, we summarize our conclusions and discuss avenues for future research.

2 Model Development

In this section, we describe a new version of mathematical model for stochastic cell formation problem (SCFP) which we are interested. We assume that processing time of parts on machines and arrival time for parts to cells are uncertain that are described by continues distributions. So, in this formulation, we minimize total expected cost included expected idleness rate costs in the cell for machines, total cost of sub-contracting costs for outsourcing exceptional elements and the cost of resource underutilization that is occurred when the parts which have no need to be operated on a machine placed together in a same cell. In the modeling process it's assumed that the inter-arrival time between two sequenced parts is described by exponential distribution with the rate λ_i for each part. Also, processing time for parts follows exponential distribution with the rate μ_j for each machine.

2.1 Application Queuing Theory to CMS

In this research, we assume each machine as a server and each part as a customer where servers should serve customers. Also, assume a birth-death process with constant arrival (birth) and service completion (death) rates. Specifically, let λ and μ be the arrival and service rate of parts, respectively, per unit time. If arrival rate is greater than the service rate, the queue will grow infinitely. The ratio of λ to μ is named utilization factor or the probability that a machine is busy and is defined as $\rho = \lambda / \mu$. Therefore, for a system in steady state, this ratio must be less than one. In this research, we assume M/M/1 queue system for CMS where each part arrived to cells with rate λ_i where parts served by machines. In these conditions, due to operate different parts on each machine and in addition, each part has different arrival rate so, for each machine (server) ρ is computed using the following property.

Property 1: It is known that the minimum of independent exponential random variables is also, exponential with rate

$$\lambda = \lambda_1 + \lambda_2 + ... + \lambda_n .$$

An interesting implication of this property to inter-arrival times is discussed in Hillier and Lieberman [18]. Suppose *n* types of customers, with the *i*th type of customer having an exponential inter-arrival time distribution with parameter λ_i , arrive at a queue system. Let us assume that an arrival has just taken place. Then from a no-memory property of exponential distribution, it follows that the time remaining until the next arrival is also exponential. Using mentioned property, we can see that the inter-arrival time for entire queue system (which is the minimum among all inter-arrival times) has an exponential distribution with parameter $\sum_{i=1}^{N} \lambda_i$.

Hence, utilization factor or the probability that each machine is busy is $\rho_j = \sum_{i=1}^{N} \lambda_i / \mu_j$.

2.2 Notation

Indexes

- i Part index.
- j Machine index.
- k Cell index.

Parameters

- P Number of parts.
- M Number of machines.
- C Number of cells.

$$a_{ij} = \begin{cases} 1, & \text{if part } i \text{ require to be processed on machine } j \\ 0, & \text{otherwise.} \end{cases}$$

- c_i Penalty cost of sub-contracting for part *i*.
- u_j Cost machine *j* not utilizes its capacity.
- $M_{\max}$ Maximum number of machines permitted in a cell.
- C_u Maximum number of cells permitted.
- λ_i Mean arrival rate for part *i*.
- μ_j Number of customers served per unit time by machine *j* (Mean Service Rate).
- $\overline{U}_{ij}$ Cost part i not utilizing machine *j*.

Decision variables

$$x_{ik} = \begin{cases} 1, & \text{if part i processed in cell k} \\ 0, & \text{otherwise.} \end{cases}$$

$$y_{jk} = \begin{cases} 1, & \text{if machine j assigned to cell k} \\ 0, & \text{otherwise.} \end{cases}$$

ρ_j : Utilization factor for machine *j* (or the probability that the machine *j* is busy).
(Definition: idleness rate is the probability that a machine is idle and is defined as 1-ρ**).**

2.3 Model Formulation

$$Min\ Z = \sum_j \left[1-\rho_j\right]\times u_j + \sum_k\sum_j\sum_i c_i a_{ij} X_{ik}(1-Y_{jk}) + \sum_k\sum_j\sum_i \overline{U_{ij}} X_{ik} Y_{jk}(1-a_{ij}) \tag{1}$$

Subject to: $\sum_k x_{ik} = 1 \qquad i = 1,2,\ldots,p$ (2)

$$\sum_k y_{jk} = 1 \qquad j = 1,2,\ldots,m \tag{3}$$

$$\rho_j - \sum_k \frac{\sum_i \lambda_i a_{ij} X_{ik} Y_{jk}}{\mu_j} = 0 \qquad j = 1,2,\ldots,m \tag{4}$$

$$\sum_j y_{jk} \le M_{\max} \qquad k = 1, 2, \ldots, c \tag{5}$$

$$\rho_j \le 1 \qquad j = 1,2,\ldots,m \tag{6}$$

$$x_{ik}, y_{jk} \in \{0,1\},\ \rho_j \ge 0 \tag{7}$$

The objective function (1) minimizes total cost made of expected idleness costs for machines in cells, subcontracting cost as well as the cost of resource underutilization. Set constraint (2) says that each part must be assigned to a single cell. Set constraint (3) states that each machine can be assigned only to one cell. Set constraint (4) computes utilization factor or the probability that each machine is busy. The major point to compute ρ is that arrival rate for a part is considered in summation for total arrival rate of each machine if the part needs to be operated on the machine and also, the part and the machine are located together in a same cell. Otherwise, sub-contract method is selected to do operation for the part. Set constraint (5) specifies maximum number of machines allowed in any cell. Set constraints (6) guarantees that the probability that each machine is busy must not be exceeded one. Set constraint (7) specifies type of decision variables.

2.4 Linearization of the Proposed Model

Unfortunately, the proposed model is nonlinear, and nonlinear models are usually much harder to solve for optimality than linear models. We reformulate the model as a mixed-integer linear programming model by introducing new set of variables XY_{ijk} to replace the x_{ik} and y_{jk}. Also, three constraints are added to the previous model to guarantee correctness of this replacement. By doing this procedure, all constraint of new model will be linear and therefore, solutions obtained from exact and optimal solvers will show global solutions.

2.4.1 Model Linearization

$$Min\ Z = \sum_j [1-\rho_j]\times u_j + \sum_k\sum_j\sum_i c_i a_{ij} x_{ij} - \sum_k\sum_j\sum_i c_i a_{ij} XY_{ijk} + \sum_k\sum_j\sum_i u_{ij}(1-a_{ij}) XY_{ijk} \tag{8}$$

Subject to: Constraints (2), (3), (5), (6) and (7)

$$XY_{ijk} \le x_{ik} \qquad \forall i, k, j \tag{9}$$

$$XY_{iksr} \le y_{jk} \qquad \forall i, k, j \tag{10}$$

$$x_{ik} + y_{jk} - XY_{ijk} \le 1 \qquad \forall i, k, j \tag{11}$$

Constraint (4) is changed as follows:

$$\rho_j - \sum_k \frac{\sum_i \lambda_i a_{ij} XY_{ijk}}{\mu_j} = 0 \qquad \forall j. \tag{12}$$

3 Computational Results

To measure the effectiveness of the proposed approach, we generate some random examples and solve them by branch-and-bound algorithm using Lingo 8 software package. All algorithms considered in this paper are run on a Pentium IV PC with 3 GHz CPU and 512 MB RAM.

Suppose a manufacturer wants to design new manufacturing cells, in which there are 40 parts (customers) and 25 machines (servers). The decision maker needs to assign parts and machines to cells to serve these customers. In Table 1, associated solutions when proposed example is solved for 8 times where all parameters applied in the model are fix expect idleness rate cost are obtained. The aim of this example is to show a vital role of utilization factor (or the probability that a machine is busy) of machines in order to determine cell formation decisions. It's known that for a cellular manufacturing system one of the important factors in order to identify ideal cells is to have minimum intercellular movements. This major point will be base for our computational experiments. Therefore, in this example which is run for 8 times, idleness rate cost is varied and number of intercellular movement is measured. Table 1 illustrates characteristics, the other problem information and the results.

Table1: Effectiveness of queuing approach in a CMS problem

problem info.							
Prob. No.	No. of parts	No. of machines	No. of cells	Max machine allowed in each cell	Idleness rate Cost	Average Utilization factor	No. of Intercelluar movement
P1	$40 \times 25 \times 6$			6	50.00	28.33%	18
P2	$40 \times 25 \times 6$			6	55.00	33.58%	17
P3	$40 \times 25 \times 6$			6	60.00	35.11%	15
P4	$40 \times 25 \times 6$			6	65.00	37.95%	14
P5	$40 \times 25 \times 6$			6	70.00	40.02%	12
P6	$40 \times 25 \times 6$			6	75.00	41.58%	11
P7	$40 \times 25 \times 6$			6	80.00	43.23%	11
P8	$40 \times 25 \times 6$			6	85.00	45.14%	10

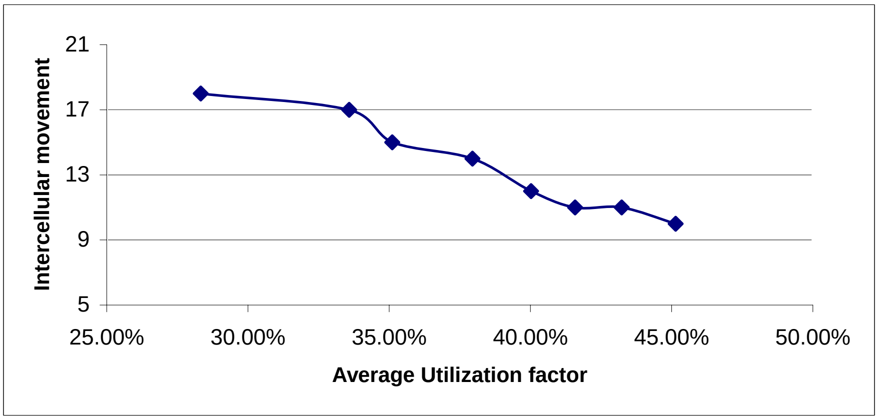

Figure 2: Relationship curve between average utilization factor and no. of intercellular movements

As it can be found from Table 1, if idleness rate cost increases, average utilization factor for machines increases, too. In other words, by increasing idleness rate cost, in order to minimize total cost for CMS problem, in order to decrease value of term $\sum_j [1-\rho_j]\times u_j$, the model must increase the probability that each machine is busy. On the other hand, by increasing the probability that each machine is busy, total number of intercellular movements will be decreased since more operation will be handled by each machine. This means that a queue system will be busier and can work efficiently and in addition, CMS decisions will be made optimality in minimal cost. Figure 2 illustrates relation between average utilization factor and intercellular movement.

Figure 2 shows relation between average utilization factor of the queue system and number of intercellular movement. As it can show, this queuing measurement (average utilization factor) has an important role in determining machine cells and part families where leads to minimum intercellular movements and this leads to maximum similarities in work cells. If a queue system works more efficiently (or the servers will be more busy), part families will be formed more efficiently, too. So, it can be found that queuing measurements are suitable tools to analysis and optimize a CMS problem.

4 Conclusion and Future Directions

In this paper, we defined a notation of stochastic cell formation problem considering stochastic inter-arrival and processing times which have been described by exponential distribution. A conceptual framework and a mathematical model were proposed as a queue system and therefore by optimizing queue system measurements, work cells and part families can be formed optimality. Our contributions research field consists of: considering stochastic parameters which yield to more flexibility and practical aspects in real world cases and formulating the stochastic problem as a queue system.

For future research, we suggest three directions, which remain critical issues for future study:

(a) Development of the model under more and the other stochastic parameters such as costs, processing routes and machine availability.

(b) Optimizing the other queue measurements such as average waiting time and average queue length for parts (customers) in cells (servers) to form cells and part families with high quality.

(c) Aggregating proposed model with the other assumptions like layout problem considerations.

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