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Selectability/Rejectability Measures Approach for Nominal Classification

Ayeley P. Tchangani[1,2,*]
[1] *Université de Toulouse; Toulouse III - UPS; IUT de Tarbes*
1, rue Lautréamont, 65016 Tarbes Cedex, France
[2] *Université de Toulouse; Laboratoire Génie de Production - LGP*
47 Avenue d'Azereix, BP 1629, 65016 Tarbes Cedex, France

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Abstract

In this paper, we consider the problem of assigning objects (peoples, projects, decisions, units, options, etc.) characterized by multiple attributes or criteria to predefined classes characterized by multiple features, conditions or constraints that are functions of object attributes: this is nominal or non ordered classification as opposed to ordinal classification in which case classes are ordered according to some desires of decision maker(s). These problems have retained the attention of a broad community of researchers that have developed methods and algorithms to deal with them because of their applicability in many domains such as social, economics, medical, engineering, ... In this paper we will consider a new approach that is based, given an object to be classified and a class, on the derivation of two measures: the *selectability* that measures to what extent this object can be considered for inclusion in that class and the *rejectability*, a degree that measures the extent to which one must avoid including the considered object to the considered class, in the framework of satisficing game theory. The application of this approach to a real world problem in the domain of banking has shown a real potentiality.

Keywords: nominal classification, multi-attribute, multi-feature, multi-actor, selectability/rejectability measures

1 Introduction

Many decision problems rising in different domains such as social, economics or engineering, among others, concern the assignment or classification of objects to classes according to their scores for a certain number of criteria or attributes that characterize them. These problems constitute then a subset of the so-called multicriteria decision making or multicriteria decision analysis (MCDM or MCDA) problems, see for instance [2, 3, 6, 12, 13, 14, 15, 16, 18, 26]. The majority of contributions to these problems encountered in the literature concern mainly the ordered classification case, classes must be ordered, let say, from most/least desired class to least/most desired one, see for instance [4]. The purpose of classification methods or algorithms is then to establish a procedure that linearly rank classes and assign objects to them; one may notice that this is a relative decision making process as objects are finally compared with each other. But it is being shown that the case of non ordered classification where classes are just defined by some features, conditions or constraints over the attributes or criteria is of great importance in many domains. In finance and banking for instance, decision maker(s) face the problem of classifying customers for a credit or service into classes defined by entrance thresholds with regard to their performance in some attributes for instance, see [13]; in international finance or commerce, countries are often ranked or classified in different categories in terms of risk to which potential investors will be exposed in these countries (country risk ranking or classification) by using a certain number of attributes such as GDP per unit of energy use, telephone mainlines per 1000 people, human development index, percentage of military expenditure of the central government expenditure and others, see [27]; in medical domain, a physician classifies a patient as suffering a fever if its temperature is beyond a threshold and/or if it presents some other symptoms; in engineering a design must satisfy some objectives and constraints; in academic, a student will get his/her diploma or degree if his/her marks in some different disciplines are beyond some thresholds, etc..

*Corresponding author. Email: ayeley.tchangani@iut-tarbes.fr, Ayeley.Tchangani@enit.fr (A.P. Tchangani).

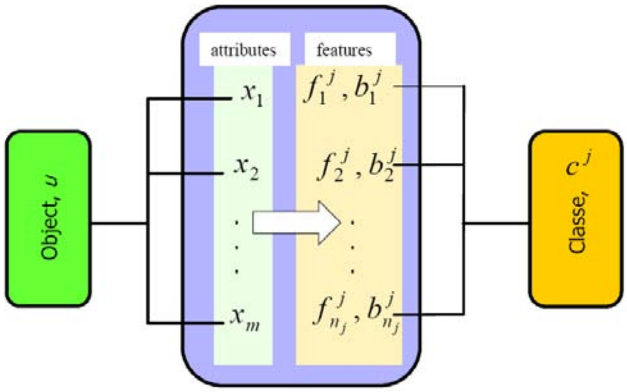

Figure 1: Object (attributes) - class (features) characterisation

Formally the nominal classification problems we will be considering in this paper are defined by the following materials.

- An object u to be classified is characterized by a set of m attributes or criteria and the value (numeric or rendered numeric by a certain procedure) of attribute l is given by $\mathbf{x}_l$ so that this object can be designated by its attributes vector $\mathbf{x} \in \mathbb{R}^m$ (where $\mathbb{R}^m$ represents the set of vectors of dimension m with real entries). For instance in the case of financial portfolio management, an object u is a portfolio constituted by m assets and $\mathbf{x}_l$ is the amount of budget allocated to asset l.

- The former defined object must be assigned to one of the n classes of the set $\mathcal{C} = \left\{c^1,\ c^2,\ ...,\ c^n\right\}$; each class or category c^j is defined by n_j features, conditions or constraints of the form given by equation (1)

$$c^j = \left\{u : \mathbf{f}^j(\mathbf{x}) \geq \mathbf{b}^j,\ \mathbf{b}^j \in \mathbb{R}^{n_j}\right\}; \tag{1}$$

  where inequalities $\mathbf{f}^j(\mathbf{x}) \geq \mathbf{b}^j$ are taken componentwize. Scalar functions $\mathbf{f}_l^j$, $l = 1, 2, ..., n_j$, (components of function $\mathbf{f}^j$), are supposed to be bounded on the attributes domain so that they admit a minimum value $\mathbf{f}_{l,\min}^j$ and a maximum value $\mathbf{f}_{l,\max}^j$; these values could be simply the range of the dedicated feature or constraint; here constraints, objectives or conditions are characterized without restriction by a lower bound $\mathbf{b}_l^j$, (higher is better), the case of upper bound characterization can be easily handled by taking the opposite. This description is summarized on Figure 1.

- A number D of decision makers express their opinions with regard to the importance of each constraint $\mathbf{f}_l^j(\mathbf{x}) \geq \mathbf{b}_l^j$, $l = 1, 2, ..., n_j$, by supplying some weights that are aggregated to obtain a vector of weights $\omega^j \in \mathbb{R}_+^{n_j}$ for each class c^j where $\mathbb{R}_+^n$ represents the set of vectors of dimension n with *positive* real entries. How these weights can be determined in practice and how the aggregation procedure is done are suggested in the subsequent sections. With regard to our previous example of portfolios classification, decision makers that have risk aversion will penalize more the risk going beyond its threshold than the return going below its threshold.

To make it clear what could be represented by parameters $\mathbf{f}^j(\mathbf{x})$ and $\mathbf{b}^j$, let us develop in more details our previous suggested example of portfolio management. Two main measures are generally used in portfolio management (see for instance [10]), the expected return given by equation (2)

$$r(\mathbf{x}) = \sum_{l=1}^{m} r_l \mathbf{x}_l = \mathbf{r}^T \mathbf{x} \tag{2}$$

where r_l is the expected return of asset l ($\mathbf{r}$ is the column vector which entries are constituted by these expected returns) and the risk $R(\mathbf{x})$ measured by the deviation around the mean return given by equation (3)

$$R(\mathbf{x}) = \mathbf{x}^T \mathbf{\Sigma} \mathbf{x} \tag{3}$$

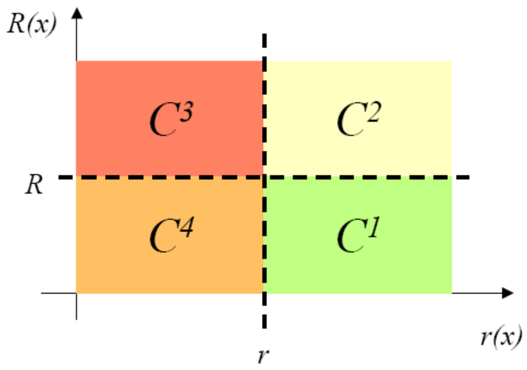

Figure 2: Example of portfolio classification

where $\mathbf{\Sigma}$ is the covariance matrix of the corresponding portfolio. Let us consider a portfolio manager who wants to classify his/her portfolios according to a threshold r with regard to the mean return $r(\mathbf{x})$ and a threshold R with regard to the risk measure $R(\mathbf{x})$ as shown by Figure 2, in which the four classes are defined by parameters of equations (4) to (7), respectively

$$\mathbf{f}^1(\mathbf{x}) = \begin{bmatrix} r^T\mathbf{x} \\ -\mathbf{x}^T\mathbf{\Sigma}\mathbf{x} \end{bmatrix} \text{ and } \mathbf{b}^1 = \begin{bmatrix} r \\ -R \end{bmatrix}, \tag{4}$$

$$\mathbf{f}^2(\mathbf{x}) = \begin{bmatrix} r^T\mathbf{x} \\ \mathbf{x}^T\mathbf{\Sigma}\mathbf{x} \end{bmatrix} \text{ and } \mathbf{b}^2 = \begin{bmatrix} r \\ R \end{bmatrix}, \tag{5}$$

$$\mathbf{f}^3(\mathbf{x}) = \begin{bmatrix} -r^T\mathbf{x} \\ \mathbf{x}^T\mathbf{\Sigma}\mathbf{x} \end{bmatrix} \text{ and } \mathbf{b}^3 = \begin{bmatrix} -r \\ R \end{bmatrix}, \tag{6}$$

$$\mathbf{f}^4(\mathbf{x}) = \begin{bmatrix} -r^T\mathbf{x} \\ -\mathbf{x}^T\mathbf{\Sigma}\mathbf{x} \end{bmatrix} \text{ and } \mathbf{b}^4 = \begin{bmatrix} -r \\ -R \end{bmatrix}. \tag{7}$$

Notice that the characterization of classes considered here include many cases encountered in the literature such as classes defined by entrance threshold on attributes, see [13]; classes defined by a set of representative objects, or by the goal satisfaction constraints as in goal programming approaches, see [7, 8, 18]. For instance classes characterization for the case where they are defined by thresholds on the attributes values is given by the following equation (8)

$$c^j = \left\{u : \mathbf{x} \geq \mathbf{b}^j,\ \mathbf{b}^j \in \mathbb{R}^m\right\} \tag{8}$$

meaning that $\mathbf{f}^j(\mathbf{x}) = \mathbf{x}$. On the contrary if classes are defined by a set of representative objects, let say the class c^j is defined by the subset $\mathcal{U}^j$ given by equation (9)

$$\mathcal{U}^j = \left\{u_1^j,\ u_2^j,\ ...,\ u_{p_j}^j\right\}, \tag{9}$$

where p_j denotes the number of representative objects of class c^j, one can then define the least representative object of class c^j by defining thresholds values $\mathbf{b}_l^j$ as given by the following equation (10)

$$\mathbf{b}_l^j = \min_{u \in \mathcal{U}^j} \left\{\mathbf{f}_l^j(\mathbf{x})\right\} \tag{10}$$

to obtain a characterization similar to (1).

The aim of this paper is to derive a classification algorithm using the materials defined above. A number of multicriteria decision aid (MCDA) methods have been developed for nominal classification. They include multicriteria filtering [11], a method based on concordance and non-discordance principles; PROAFTN, see [1], a multicriteria fuzzy classification method; a method based on fuzzy integrals, see for instance [5]; TRINOMFC method [9] that computes local concordance; or the stochastic multicriteria acceptability analysis (SMAA) method that supports incomplete or inaccurate preference, see [28]. If these methods have been successfully applied in practice, many of them do have usability limitation (with regard to final users) such as complexity of how parameters must be specified by the users. The intention of this paper is not to propose a method that is better than existing ones in terms of final classification performances but to add a method to the panorama of multicriteria decision aid methods that we hope will be easier to use by the final users who in general are non specialists. In this paper we consider a method of nominal classification that is based, for a given object and a given class, on two measures corresponding respectively to what extent the object can be included in the class and to what extent it should be excluded, in the framework of statisficing games theory [19]. The main contribution of this paper consists in a procedure that uses all the information from the problem specification, namely the scores of attributes for each object, the constraints, conditions or features characterizing each class and the relative importance of each constraint (weights supplied by decision maker(s)) to compute the *selectability* and the *rejectability* measures for a pair constituted by an object u and a class c^j, that is formulating the multi-attribute, multi-feature and multi-actor nominal classification problem as a satisficing game. Then for each class c^j, the set $\Sigma_q^{c^j}$ of objects that can be included in it (at the caution or boldness index q) is defined as the objects for which the selectability measure exceeds the rejectability measure multiplied by q and for each object u the set $\mathcal{C}_q(u)$ of classes where it can be included is defined so that the final class in which it will be included can be chosen to optimize a certain ultimate criterion (minimization of the difference between the selectability and the rejetability for instance). It is interesting to notice that an object u can be included in any class of the set $\mathcal{C}_q(u)$ with more or less confidence giving then some flexibility to this approach that is closed to how human beings proceed in practice: contenting oneself with "good enough options" instead of optimal ones respecting by the way the spirit of satisficing game theory. In terms of usability, one can notice that this approach can be totally transparent to the final user because class characterization (in terms of functions $\mathbf{f}_l^j$ and thresholds $\mathbf{b}_l^j$) as well as importance to assign to each feature (in terms of weights ω_l^j) within a class definition are matter of experts; to classify an object u, the user needs just to have its attributes vector $\mathbf{x}$.

The remainder of this paper is organized as follows: in the second section, satisficing game theory is briefly presented (only the necessary materials relevant to our approach is presented, the reader interested in this theory can consult [19]); the third section is the core of this paper and is devoted to formulating the assignment problem defined previously as a satisficing game; in the fourth section we consider a real world application to show potential applicability of the approach developed so far and finally a conclusion is presented in the fifth section.

2 Satisficing Game Theory

The underlying philosophy behind many classification methods encountered in MCDA literature is the substantive rationality that is looking for the best. But the substantive rationality paradigm is not necessarily the way humans evaluate and classify options. Most of the time humans content themselves with options that are just "good enough"; the concept of being good enough allows a certain flexibility because one can always adjust one's aspiration level. On the other hand, decision makers more probably tend to classify units as good enough or not good enough in terms of their positive attributes (degree to which a class must be considered as the assignment class for an object) and their negative attributes (degree to which a class must be excluded from inclusion classes for an object) with regard to classification goal instead of ranking objects with regard to each other. For instance, to evaluate and classify cars for a purchase purpose, we often make a list of positive attributes (driving comfort, speed, robustness, etc.) and a list of negative attributes (price, petrol consumption, maintainability, etc.) of each car and then make a list of cars for which positive attributes "exceed" negative attributes in some sense. This way of evaluation and classification falls into the framework of praxeology or the study of the theory of practical activity (the science of efficient action) derived from epistemic logic (the branch of philosophy that classifies propositions on the basis of knowledge and belief regarding their content; for a proposition to be admissible it must be both believable and informative) and developed by [19]. Here decision maker(s), instead of looking for the best options or classes, look for the satisficing ones. This approach for decision making is rather close to Simon's theory of bounded rationality (see for instance [17]) who suggests to change the optimization paradigm by the satisfaction one as one will never get all the necessary information regarding relationships of different components of a decision problem nor have enough computing power to search for the optimal solution so that one can content himself with options that satisfy its aspiration's level.

Satisficing is a term that refers to a decision making strategy where options, units or alternatives are selected which are "good enough" instead of being the best [19]. Let us consider a universe $\mathcal{U}$ of options, alternatives or units; then for each unit $u \in \mathcal{U}$, a *selectability function or measure* $p_s(u)$ and a *rejectability function or measure* $p_r(u)$ are defined so that $p_s(u)$ measures the degree to which u works towards success in achieving the decision maker's goal and $p_r(u)$ is the cost associated with this unit. This pair of measures called *satisfiability functions* are in general normalized on $\mathcal{U}$. The following definition then gives the set of options which can be considered to be "good enough" because, for these options, the "benefit" expressed by the function p_s exceeds the "cost" expressed by the function p_r with regard to an index of caution or boldness q.

Definition 2.1 *The satisficing set* $\Sigma_q \subseteq \mathcal{U}$ *is the set of units defined by equation (11)*

$$\Sigma_q = \{u \in \mathcal{U} : p_s(u) \geq q p_r(u)\}. \tag{11}$$

Small values of the index of caution q will lead to a lot of units being declared as satisficing whereas large values of q will reduce the number of satisficing units. A sensitivity analysis can be carried up to determine the value $q_{\min}$ below which all the units of $\mathcal{U}$ will be declared satisficing and a value $q_{\max}$ above which non unit will be satisficing. For all units of $\mathcal{U}$ to be declared satisficing the following inequalities (12)

$$p_s(u) \geq q p_r(u) \ \forall \ u \in \mathcal{U} \ \Leftrightarrow \ q \leq q_{\min} = \min_{u \in \mathcal{U}} \left(\frac{p_s(u)}{p_r(u)} \right) \tag{12}$$

must be verified so that for such an indices of caution q we have

$$\Sigma_q = \mathcal{U}. \tag{13}$$

On the contrary, there is no satisficing unit, that is the following equation (14)

$$\Sigma_q = \varnothing \tag{14}$$

is verified if and only if the following inequalities (15)

$$p_s(u) < q p_r(u) \ \forall \ u \in \mathcal{U} \ \Leftrightarrow \ q > q_{\max} = \max_{u \in \mathcal{U}} \left(\frac{p_s(u)}{p_r(u)} \right) \tag{15}$$

are verified. Finally if the index of caution verifies $q \in [q_{\min}, q_{\max}]$ then we have relation of equation (16)

$$\Sigma_q \subseteq \mathcal{U}. \tag{16}$$

In the next section we will present a procedure that formulates and solves nominal classification problems as defined in introduction section using satisficing games theory approach. Basically the procedure will establish a way to compute satisfiability measures of a given object for each class using all the problem specification materials.

3 Proposed Approach

In this section we will formulate a non ordered classification problem as defined in the introductory section using satisficing game theory. This theory (see [19]), originally developed in artificial intelligence and computer science as an alternative to the classical game theory to deal with possible situational altruism that may exist among agents engaged in a decision process, is showing promising application in operational research and decision science domains as demonstrated by the author in [20, 21] for performance evaluation, in [22] for priority setting or load/resources dispatching, in [23] for relevant objects retrieval from a database and in [24, 25] for multiattribute/multiobjective decision making. Formulating the nominal classification problem considered in this paper in the framework of satisficing games return to establishing a procedure to compute selectability measure $p_S^{c^j}(u)$ and rejectability measure $p_R^{c^j}(u)$ given an object u and a class c^j; in the following paragraphs, we will give how to obtain these parameters from problem specifications.

3.1 Features or Constraints Weights Derivation Procedure

Obtaining constraints, features or objectives weights ω_l^j (relative weight of constraint l in the characterization of class j) can be a difficult task for different reasons: it could be non easy to compare constraint or there can be antagonist opinion regarding the importance of each constraint from decision makers (or experts) when there are multiple decision makers, a situation that is common in practice. We consider here that the classification or decision making process involves D decision makers whose principal role is to give their opinions about each constraint within each class in terms of weights. So doing, each decision maker d will be asked to assign a relative weight $\alpha_{l,d}^j$ for the constraint l in class j so that the weight ω_l^j will be obtained as the mean value given by equation (17)

$$\omega_l^j = \frac{1}{D}\sum_{d=1}^{D}\alpha_{l,d}^j. \tag{17}$$

Practically these weights derivation procedure can be carried up remotely and in a distributed manner using web technology for instance; a connected decision maker will be presented only with classes, their description and constraints/features that characterize them. Then for each class, this decision maker will be asked to choose a pivot constraint and compare other constraints to it using AHP (analytic hierarchy process) scales, see [15, 16] for instance; then these comparison notes are used by AHP procedure to compute $\alpha_{l,d}^j$ and finally ω_l^j similar to the approach proposed in [21] for efficiency evaluation of production units.

The weights ω_l^j will be used in the following paragraph along with constraints/objectives values to derive satisfiability (selectability and rejectability) measures $p_S^{c^j}(u)$ and $p_R^{c^j}(u)$ for any couple (u, c^j).

3.2 Satisfiability Measures Derivation Procedure

The stepping stones of satisficing games approach is the satisfiability functions or measures $p_S^{c^j}(u)$ and $p_R^{c^j}(u)$ given a class c^j and an object u. These measures must be established considering two things: the performance of the considered object with regard to the considered class and the opinions of decision makers that are expressed through weights ω_l^j assigned to each constraint or objective l of the class j that can be obtained using a procedure as described in the previous paragraph. Once these materials are obtained, it seems natural to consider that the selectability of an object with regard to a class is proportional to how far this object satisfies each constraint or feature of that class and their respective importance assigned by decision makers and that its rejectability measure is proportional to how far it lacks to satisfy each constraint; the selectability and rejectability range of a constraint or feature k of a class j is given by the following Figure 3.

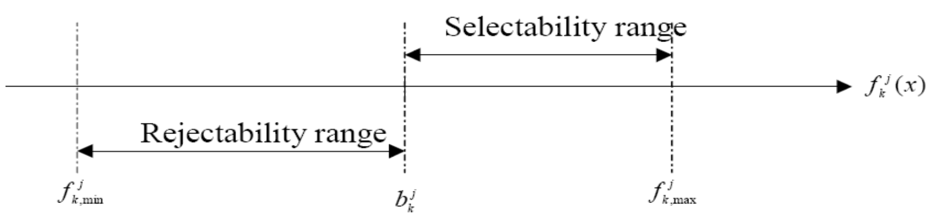

Figure 3: Selectability and rejectability range of the constraint k for the class j

So, given an object u represented by its attributes values vector $\mathbf{x}$ and a class c^j, we define the following parameters that characterize the extent to which this object can be considered for the inclusion or should be excluded from inclusion in the considered class respectively.

- A function $\Psi_S^{c^j}(u)$ that measures how "close" is the object u to the class c^j is given by

$$\Psi_S^{c^j}(u) = \sum_{l=1}^{n_j}\omega_l^j \max\left(0,\ \frac{f_l^j(\mathbf{x}) - b_l^j}{f_{l,\max}^j - b_l^j}\right). \tag{18}$$

The expression $\max\left(0, \frac{f_l^j(\mathbf{x})-b_l^j}{f_{l,\max}^j-b_l^j}\right)$ measures how well the object u satisfies the constraint l in the class c^j that is the relative satisfaction degree for constraint l by the object u; it ranges from 0 for objects that do not satisfy this constraint to 1 for objects having the maximum satisfaction. The function $\Psi_S^{c^j}(u)$ measures then the aggregated degree to which u can be considered to be closed to the class taking into account the importance assigned to each constraint by decision maker(s) through weights ω_l^j.

- A function $\Psi_R^{c^j}(u)$ that measures how "far" is the object u to the class c^j is defined by

$$\Psi_R^{c^j}(u) = \sum_{l=1}^{n_j} \omega_l^j \max\left(0, \frac{b_l^j - f_l^j(\mathbf{x})}{b_l^j - f_{l,\min}^j}\right). \tag{19}$$

Contrary to previous point, here the expression $\max\left(0, \frac{b_l^j-f_l^j(\mathbf{x})}{b_l^j-f_{l,\min}^j}\right)$ measures how far the object u is from satisfying the constraint l in the class c^j, the relative degree of non satisfaction of constraint l by the object u. Similar to $\Psi_S^{c^j}(u)$, $\Psi_R^{c^j}(u)$ is the aggregated degree taking into account decision maker(s) opinion of how far is the object u from the class c^j.

Remark 3.1 *One can imagine other type of characterizations for functions $\Psi_S^{c^j}(u)$ and $\Psi_R^{c^j}(u)$. For instance if there are hard constraints, one can arrange to have $\Psi_R^{c^j}(u)$ being infinite if that constraints are not satisfied. In the case where the constraints are not weighted, another possibility is to consider worst case to determine $\Psi_S^{c^j}(u)$ and $\Psi_R^{c^j}(u)$, as given by the following equations (20) and (21)*

$$\Psi_S^{c^j}(u) = \min_k \left(\max\left(0, \frac{f_k^j(\mathbf{x}) - b_k^j}{f_{k,\max}^j - b_k^j}\right)\right), \tag{20}$$

$$\Psi_R^{c^j}(u) = \max_k \left(\max\left(0, \frac{b_k^j - f_k^j(\mathbf{x})}{b_k^j - f_{k,\min}^j}\right)\right). \tag{21}$$

Once these parameters are obtained, the necessary materials (selectability and rejectability measures $p_S^{c^j}(u)$ and $p_R^{c^j}(u)$ for any pair (u, c^j) and others) for the approach considered in this paper are given by the following definition.

Definition 3.1 *Given an object u and a class c^j, the selectability measure $p_S^{c^j}(u)$ and the rejectability measure $p_R^{c^j}(u)$ are given by equation (22)*

$$p_S^{c^j}(u) = \frac{\Psi_S^{c^j}(u)}{\sum_{l=1}^{n} \Psi_S^{c^l}(u)} \text{ and } p_R^{c^j}(u) = \frac{\Psi_R^{c^j}(u)}{\sum_{l=1}^{n} \Psi_R^{c^l}(u)}; \tag{22}$$

for each class c^j, the subset $\Sigma_q^{c^j}$ of objects that can be included in that class with the index of caution or boldness q is given by equation (23)

$$\Sigma^{c^j} = \left\{u : p_S^{c^j}(u) \geq q p_R^{c^j}(u)\right\}; \tag{23}$$

and finally the subset $\mathcal{C}_q(u)$ of classes to which an object u can be assigned with the index of caution or boldness q is defined by equation (24)

$$\mathcal{C}_q(u) = \left\{c^j : u \in \Sigma_q^{c^j}\right\}. \tag{24}$$

As it has been stated in the introductory section on satisficing game theory, by managing the index of caution or boldness q, one can arrange to have a non empty subset $\mathcal{C}_q(u)$ for any object u. Furthermore, this definition brings the following comments and remarks that show some coherency for the approach.

Remark 3.2 *The selectability (respectively rejetability) measure of a given object with regard to a given class is proportional to:*

- *the number of the features that respect (respectively does not respect) the class conditions;*
- *the amount by which the features satisfy (respectively does not satisfy) the conditions thresholds;*
- *the importance of the features represented by their weight.*

In the next paragraph, a procedure for final assignment will be presented.

3.3 Classification Procedure

At the boldness or caution index q, an object u can be included in any class of the subset $\mathcal{C}_q(u)$ selected using more or less flexible procedure. Thus, the approach presented in this paper falls in the framework of soft computing as opposed to hard computing because it allows certain flexibility and liberty in the interpretation and parameters selection. Soft computing is a paradigm that comprise computing techniques that are tolerant of imprecision, uncertainty, partial truth, and approximation and which guiding principle can be stated as "*exploit the tolerance for imprecision, uncertainty, partial truth, and approximation to achieve tractability, robustness and low cost solution*"; the role model of soft computing is then a human mind. Nevertheless, some optimization techniques can be included for final assignment purpose; with the previous model, the classification of a given object can be carried up in order to optimize some performance index. For instance if decision makers are able to specify the caution or boldness index q, then the final optimal class $c^*(u)$ for a given object u can be considered to be the **maximum discriminant** one given by equation (25)

$$c^*(u) = \arg\left\{\max_{c^j \in \mathcal{C}_q(u)} \left\{p_S^{c^j}(u) - q p_R^{c^j}(u)\right\}\right\}. \tag{25}$$

On the contrary, when there is no information about the desired caution or boldness index, the assigned class $c^*(u)$ can naturally be chosen to maximize this index (**maximum index of caution**) and be given by the following equation (26)

$$c^*(u) = \arg\left\{\max_{c^j \in \mathcal{C}_q(u)} \left\{\frac{p_S^{c^j}(u)}{p_R^{c^j}(u)}\right\}\right\}. \tag{26}$$

Sometimes, decision makers may not worry about the rejectability or the selectability; this case appear mainly when one of this measure is known to be uniformly distributed over classes; in this case the optimal assigned class $c^*(u)$ for an object u can be considered so that the selectability measure is maximized (**maximum selectability**) that is given by equation (27)

$$c^*(u) = \arg\left\{\max_{c^j \in \mathcal{C}_q(u)} \left\{p_S^{c^j}(u)\right\}\right\} \tag{27}$$

or to minimize the rejectability measure (**minimum rejectability**) as given by equation (28)

$$c^*(u) = \arg\left\{\min_{c^j \in \mathcal{C}_q(u)} \left\{p_R^{c^j}(u)\right\}\right\}. \tag{28}$$

In the next section we will consider an application in the domain of banking to show potential applicability of the approach developed in this paper.

4 Application

This application is taken from [13] and concerns the problem of assigning retailers that use EFTPoS (Electronic Fund Transfer at Point of Sale) service of a bank to some classes in order for the bank manager to consider their appropriate strategic treatment. Any retailer is characterized by 13 attributes or criteria ($\mathbf{G}_{01} - \mathbf{G}_{13}$) which signification, scale of evaluation and the relative weight are shown on Figure 4; there are four classes which definition and strategy that will be applied to them by the decision maker are shown on Figure 5.

Classes are characterized by an entrance threshold for each attribute denoted by $\mathbf{b}^j$ ($j = \overline{1,\ 4}$, $\mathbf{b}^j \in \mathbb{R}^{13}$); and finally 20 retailers ($\mathbf{R}_{01} - \mathbf{R}_{20}$) are to be classified; data for this application are shown in Table 4.

Criterion	*Definition*	*Scale*	*Weight*
G1	Retailer Size	1-100(asc)	10
G2	Intensity of EFT/POS usage	1-100(asc)	12
G3	Average value per EFT/POS transaction	1-100(asc)	4
G4	Average cost per EFT/POS Terminal	1-100(asc)	13
G5	EFT/POS Terminal profitability	1-100(asc)	13
G6	Average growth rate	1-100(asc)	8
G7	Merchant category	1-100(asc)	10
G8	Collaboration efficiency	1-100(asc)	4
G9	Exclusivity	1-100(asc)	4
G10	Location	1-100(asc)	8
G11	Opening hours	1-100(asc)	4
G12	Training of employees	1-100(asc)	8
G13	Alternative channels	1-100(asc)	2

Figure 4: Example of portfolio classification

	Category			
	C1	C2	C3	C4
Definition	Retailers with relative low potential and medium to high profitability.	Retailers with relative high potential and medium to high profitability.	Retailers with medium to high potential and medium to low profitability.	Retailers with medium to low potential and low profitability.
Strategy	Bank will allocate substantial resources to strengthen retailer's potential.	Bank will allocate maximum resources to provide high added value innovative services.	Bank will minimize resource allocation and focus to top retailers of the category.	Bank will screen retailers for potential development, allocating a minimum level of resources.

Figure 5: Selectability and rejectability range of the constraint k for the class j

Table 1: Data of the considered application

	$\mathbf{G_{01}}$	$\mathbf{G_{02}}$	$\mathbf{G_{03}}$	$\mathbf{G_{04}}$	$\mathbf{G_{05}}$	$\mathbf{G_{06}}$	$\mathbf{G_{07}}$	$\mathbf{G_{08}}$	$\mathbf{G_{09}}$	$\mathbf{G_{10}}$	$\mathbf{G_{11}}$	$\mathbf{G_{12}}$	$\mathbf{G_{13}}$
$\mathbf{R_{01}}$	29	22	28	25	69	25	61	52	25	39	58	61	68
$\mathbf{R_{02}}$	80	78	88	69	59	30	50	45	48	42	22	15	27
$\mathbf{R_{03}}$	77	90	88	61	63	28	35	33	51	33	22	28	33
$\mathbf{R_{04}}$	16	39	26	25	55	25	50	51	43	65	37	38	73
$\mathbf{R_{05}}$	28	56	51	21	34	8	37	61	30	37	55	66	98
$\mathbf{R_{06}}$	79	75	80	65	60	25	30	34	22	19	22	18	21
$\mathbf{R_{07}}$	50	6	54	25	38	21	47	41	40	57	65	65	88
$\mathbf{R_{08}}$	44	19	31	55	49	29	80	70	73	55	48	29	45
$\mathbf{R_{09}}$	49	43	28	29	61	22	67	42	25	39	51	62	55
$\mathbf{R_{10}}$	30	25	30	51	55	44	82	84	90	74	32	15	32
$\mathbf{R_{11}}$	30	29	32	87	86	80	77	46	28	49	25	29	33
$\mathbf{R_{12}}$	49	17	54	25	37	21	47	39	42	54	65	55	98
$\mathbf{R_{13}}$	42	14	27	51	43	22	74	67	69	53	40	25	92
$\mathbf{R_{14}}$	25	19	26	90	81	79	70	44	32	45	28	24	30
$\mathbf{R_{15}}$	42	14	27	51	56	46	81	78	82	53	40	25	33
$\mathbf{R_{16}}$	80	77	79	69	65	22	31	37	28	22	19	21	29
$\mathbf{R_{17}}$	21	15	22	86	79	83	68	40	30	41	20	19	25
$\mathbf{R_{18}}$	18	12	25	82	81	79	64	38	29	39	19	15	27
$\mathbf{R_{19}}$	22	18	26	49	51	41	80	80	86	69	24	11	26
$\mathbf{R_{20}}$	41	35	44	29	34	21	47	61	50	57	62	61	98
ω	10	12	4	13	13	8	10	4	4	8	4	8	2
$\mathbf{b}^1$	75	70	75	60	55	20	25	35	20	15	15	10	20
$\mathbf{b}^2$	15	10	20	75	70	75	60	30	25	35	15	10	20
$\mathbf{b}^3$	15	10	20	45	45	40	75	70	75	60	15	10	20
$\mathbf{b}^4$	55	10	20	15	10	20	35	30	40	70	75	60	55

The application of the approach developed in this paper leads to selectability and rejectability measures for each pair $(\mathbf{R}_k, c^j)$, $k = \overline{1,\ 20}$; $j = \overline{1,4}$ shown in Table 2 as well as the minimum and maximum caution indices for each retailer. From data of Table 2, we deduce results of Table 3; for illustration we consider the natural value of $q = 1$ for the index of caution or boldness q. So in this table one will find the subset $\Sigma_1^{c^j}$ of retailers that can be included in the class c^j at the index of caution 1 and for each retailer $\mathbf{R}_k$ the subset $\mathcal{C}_1(\mathbf{R}_k)$ of classes where it can be included. The final assigned class $c^*(\mathbf{R}_k)$ for each retailer using some of assignment performance indices (**Max caution**: maximum caution index, **Max sel.**: maximum selectability and **Min rej.**: minimum rejectability) presented in the classification procedure paragraph are also shown on this Table 3; one may notice that the **maximum discriminant** performance index for $q = 1$ gives the same assignment results as **maximum caution** index. We present also on this Table 3 results obtained in [13] using a procedure named **NexClass** for "*Non Excluding Classification*"; this procedure is based on concordance/discordances indices (see for instance [26]). The results of an existing procedure (**Exist. proc.**) that were used by the bank manager are also shown on the same Table 3. We can see that using the natural performance index (when there is no additional information regarding the index of caution) recommended by the procedure established in this paper, that is the maximum caution index, we obtain the same classification results as the existing procedure except for the retailer 1 that is assigned to class 4 instead of class 3; this gives a percentage error of 5% whereas this percentage is 15% for the NexClass procedure of [13]. From this Table 3, we can remark also that the minimum rejectability index gives the same results whereas the maximum selectability index criterion is completely inefficient; an explanation of this observation is the relatively high number of retailers having 0 for the rejectability measure for which maximum caution index and minimum rejectability index are equivalent. So we do think that unless one is sure that one of the measures (selectability or rejectability) is uniformly distributed, one should avoid using a single measure as the final assignment performance index.

Table 2: Obtained selectability and rejectability measures for the considered application

	p_S				p_R					
	c_1	c_2	c_3	c_4	c_1	c_2	c_3	c_4	$q_{\min}$	$q_{\max}$
$\mathbf{R_{01}}$	0.2963	0.1922	0.2449	0.2666	0.3675	0.2129	0.2587	0.1609	0.8062	1.6567
$\mathbf{R_{02}}$	0.1871	0.2119	0.2652	0.3359	0.0000	0.2873	0.3197	0.3930	0.7375	∞
$\mathbf{R_{03}}$	0.1866	0.2221	0.2793	0.3119	0.0057	0.3303	0.3644	0.2996	0.6726	32.5821
$\mathbf{R_{04}}$	0.2578	0.2489	0.2076	0.2857	0.3367	0.2655	0.2156	0.1822	0.7658	1.5683
$\mathbf{R_{05}}$	0.2078	0.2894	0.2569	0.2459	0.2870	0.2733	0.2803	0.1594	0.7240	1.5431
$\mathbf{R_{06}}$	0.1109	0.2232	0.3108	0.3551	0.0021	0.3214	0.3437	0.3328	0.6944	53.9355
$\mathbf{R_{07}}$	0.2734	0.2987	0.2333	0.1947	0.3087	0.3170	0.2809	0.0934	0.8306	2.0835
$\mathbf{R_{08}}$	0.2516	0.2597	0.1745	0.3142	0.4155	0.2875	0.0703	0.2267	0.6055	2.4826
$\mathbf{R_{09}}$	0.2298	0.2461	0.2572	0.2669	0.3068	0.2706	0.2910	0.1316	0.7490	2.0280
$\mathbf{R_{10}}$	0.2412	0.2315	0.2042	0.3230	0.4299	0.2438	0.0000	0.3263	0.5612	∞
$\mathbf{R_{11}}$	0.2707	0.2112	0.2396	0.2784	0.4202	0.0000	0.1467	0.4332	0.6428	∞
$\mathbf{R_{12}}$	0.2614	0.2989	0.2388	0.2009	0.3526	0.3136	0.2769	0.0569	0.7414	3.5282
$\mathbf{R_{13}}$	0.2704	0.2634	0.1390	0.3273	0.4044	0.2806	0.1095	0.2056	0.6686	1.5919
$\mathbf{R_{14}}$	0.2860	0.1910	0.2364	0.2866	0.4306	0.0000	0.1541	0.4152	0.6642	∞
$\mathbf{R_{15}}$	0.2582	0.2310	0.1869	0.3240	0.4519	0.2404	0.0228	0.2848	0.5712	8.1809
$\mathbf{R_{16}}$	0.1455	0.2109	0.3075	0.3361	0.0000	0.3008	0.3687	0.3305	0.7010	∞
$\mathbf{R_{17}}$	0.2898	0.1674	0.2375	0.3053	0.4131	0.0000	0.1611	0.4257	0.7013	∞
$\mathbf{R_{18}}$	0.2977	0.1367	0.2439	0.3217	0.4025	0.0000	0.1704	0.4270	0.7396	∞
$\mathbf{R_{19}}$	0.2687	0.2295	0.1548	0.3470	0.4381	0.2329	0.0000	0.3290	0.6133	∞
$\mathbf{R_{20}}$	0.2545	0.2994	0.2207	0.2254	0.3469	0.3277	0.2567	0.0687	0.7336	3.2803

Table 3: Classification results for the considered application

		$\Sigma_1^{c^j}$				$c^*(\mathbf{R}_k)$			Results from[13]	
		c_1	c_2	c_3	c_4	**Max caution**	**Max sel.**	**Min rej.**	**NeXClass**	**Exist. proc.**
	$\mathbf{R_{01}}$				×	c_4	c_4	c_4	c_4	c_3
	$\mathbf{R_{02}}$	×				c_1	c_1	c_1	c_1	c_1
	$\mathbf{R_{03}}$	×			×	c_1	c_4	c_1	c_1	c_1
	$\mathbf{R_{04}}$				×	c_4	c_4	c_4	c_3	c_4
	$\mathbf{R_{05}}$		×		×	c_4	c_2	c_4	c_4	c_4
	$\mathbf{R_{06}}$	×			×	c_1	c_4	c_1	c_1	c_1
	$\mathbf{R_{07}}$				×	c_4	c_4	c_4	c_4	c_4
	$\mathbf{R_{08}}$			×	×	c_3	c_4	c_3	c_3	c_3
$\mathcal{C}_1(\mathbf{R}_k)$	$\mathbf{R_{09}}$				×	c_4	c_4	c_4	c_4	c_4
	$\mathbf{R_{10}}$			×		c_3	c_3	c_3	c_3	c_3
	$\mathbf{R_{11}}$		×	×		c_2	c_3	c_2	c_2	c_2
	$\mathbf{R_{12}}$				×	c_4	c_4	c_4	c_4	c_4
	$\mathbf{R_{13}}$			×	×	c_3	c_4	c_3	c_4	c_3
	$\mathbf{R_{14}}$		×	×		c_2	c_3	c_2	c_2	c_2
	$\mathbf{R_{15}}$			×	×	c_3	c_4	c_3	c_3	c_3
	$\mathbf{R_{16}}$	×			×	c_1	c_4	c_1	c_1	c_1
	$\mathbf{R_{17}}$		×	×		c_2	c_3	c_2	c_2	c_2
	$\mathbf{R_{18}}$		×	×		c_2	c_3	c_2	c_2	c_2
	$\mathbf{R_{19}}$			×	×	c_3	c_4	c_3	c_3	c_3
	$\mathbf{R_{20}}$				×	c_4	c_4	c_4	c_4	c_4

5 Conclusion

The problem of nominal classification that consists in an assignment of objects characterized by many attributes to predefined classes characterized by different features or conditions over attributes has been considered in this paper. The approach considered to formulate the classification model takes into account the opinions (possibly antagonist) of many actors (decision makers, stakeholders or experts) with regard to the importance of each feature or condition defining a class because we do think that such problems are typically multi-actor ones and the final models is a satisficing game. As humans proceed in practice by balancing "benefit" and "cost" during a decision process, this approach proposes, for a pair constituted by an object to be classified and a class, derivation of two measures by using the attributes values of the object and actors opinions in the framework of satisficing game: the *selectability* that is a degree measuring to what extent this object can be included in that class and the *rejectability* that measures the degree to which one should avoid including the considered object into the specified class. This class is then considered as candidate assignment class with possibly an index of caution or boldness if the selectability measure exceeds the rejectability one in some sense offering a certain flexibility in the assignment process. The application of this approach to a real world problem has shown real potentiality. The next works will be devoted to improving the process of elicitating actors opinions by taking into account possible influence that may exist between them and looking for a new industrial problems to which to apply this approach in order to definitively validate it.

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