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A Default Probability Estimation Model: An Application to Japanese Companies

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Abstract

On the assumption that asset value of a company is the sum of total market value of stock and debt value, we estimate a mean value and variance of the sum with the first moment and second moment. We also assume a new variable for which fluctuation during an evaluation period conforms to these moments and follows geometric Brownian motion. Then we construct a default probability estimation model on condition that the variable is regarded as the asset value of the company. For constructing expected default probability (EDP) model, we partially follow Levy's way [8], in which a new variable used for average option is assumed. Thus its evaluation formula is derived. In addition, concerning estimated values of the default probability, our model is examined by comparing with the conventional structural approach with respect to the company in Japan, where default was actually caused and the company is free from the default.

Keywords: expected default probability (EDP), moment, Black-Scholes-Merton model

1 Introduction

Since 1980, international monetary market has been expanded with great strides. On the other hand, severity of cumulative debts has been deepened and risk control of increasing derivative transactions has become a problematic point. For this reason, it becomes very important not only for financial institutions but also for general companies in possession of stocks or bonds to evaluate management abilities, credit power of companies in an fair and simple manner. Especially with the financial institutions including banks, soundness in management is required in performing international business endeavors and in making transactions competitive enough to cope with foreign financial institutions in a field of international competitions. Thus, quantitative measures of credit risks have become indispensable in completing satisfaction of BIS restriction (Basel Accords), an international unification criterion determined by Basel Committee on Banking Supervision. In controlling such a kind of credit risks, an estimation method of the default probability for the clients with whom credit should be provided becomes important. Proper estimation of the default probability becomes very important information when observation is made with proper pricing of the bond values to which credit risks are applied or with adequate profit brought about by market transactions, resulting in foundation to evaluate these factors.

Modeling of the credit risks are roughly classified into three types: (1) a traditional method where expected loss values are estimated and evaluated on the basis of the estimated values obtained from the past data, (2) structural approach where an event with which a company loses solvency to be defaulted on the given balance sheet is modeled to obtain a default probability of the company, and (3) inductive approach where the default probability is exogenously given in a probability model.

As a representative method of the structural approach taken up in this study, Merton [11], who stipulates as default a situation where asset values become below liabilities as default, modeled default risks of the bonds by using an option pricing theory introduced by Black and Scholes [2]. The approach concerning the evaluation of the company debt of Black and Scholes [2] and Merton [11] has become an indispensable portion of the corporate finance theory. After that, Black and Cox [3] developed a model that default is caused at the time when the value of the profit of the company reaches a certain low threshold value by relaxing the assumption based on a framework of the Merton

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model as a foundation. On the other hand, Longstaff and Schwarz [9] considered that the company is fallen into the default state when the company value following a stochastic process narrowly fails to reach a threshold value. Thus, they expanded the black and cox model by incorporating a probability fluctuation model of an interest rate taking account of uncertainty of the interest. However, with the Black-Scholes-Merton model as the conventional structural approach, the EDP estimation value is occasionally underestimated for heightening of volatility usually caused by critical affairs of the company (e.g., in the vicinity of the default time point) and increase of a ratio of the leverage (e.g., in lowering of total market value of current stock).

In this study, the EDP estimation model is constructed by assuming a new variable which follows geometric Brownian motion allowing the fluctuation during the EDP evaluation period conforms to the first moment and second moment of the sum of the total market value of current stock and debt value with the variable being regarded as asset value of the company. By so doing, we propose a model that gives more influence to EDP estimates than the traditional model for volatility of the total market value of current stock susceptible to influence of trend of the market and a ratio of the leverage showing financial state of the company. Then, the proposed model is examined by comparing with the conventional one, showing difference of the results of default and non-default companies in Japan.

2 Black-Scholes-Merton Model and Parameters Required for EDP Model

Default risks are modeled by using option pricing theory and the Black-Scholes-Merton model in consideration of a simple company comprised of a single type of the debt that is free from profit payment and a single type of capital that is liberated from dividend. This model is based on the assumption that a situation where asset value becomes below the debt is default. Then the assumption below is made with the EDP estimation by means of the Black-Scholes-Merton model.

The asset value $A(t)$ of the company follows a stochastic process below

$$dA(t) = \mu_A A(t)dt + \sigma_A A(t)dW(t), \tag{1}$$

where μ_A is the expected profit ratio of the asset value, σ_A is volatility of the asset value, and $dW(t)$ is standard Brownian motion. From (1), the asset value $A(\tau)$ of $\tau(T_0 \le \tau \le T)$ time point can be expressed for $A(T_0)$,

$$A(\tau) = A(T_0)\exp\left(\left(\mu_A - \sigma_A^2/2\right)\tau + \sigma_A W(\tau)\right), \tag{2}$$

where $W(\tau) \sim N(0,\tau)$. Denoting the debt value at the time point T by $B(T)$, the default event can be expressed as $A(T) \le B(T)$. The default probability EDP on the Black-Scholes-Merton model is given below

$$EDP = P(\ln A(T) \le \ln B(T)) = 1 - N\left(\frac{\ln(A(T_0)/B(T)) + \left(\mu_A - \sigma_A^2/2\right)(T - T_0)}{\sigma_A\sqrt{T - T_0}}\right), \tag{3}$$

where $N(\cdot)$ is an accumulated probability density function of the standard normal distribution.

In estimating the default probability based on (3), it is necessary to set values of 4 parameters $A(T_0)$, $B(T)$, μ_A, and σ_A except for $T - T_0$ given exogenously. In the empirical study, EDP estimation was made by Mercus and Shaked [10], Ronn and Verma [14], Saito and Moridaira [15] and Ando and Marushige [1] with $A(T_0)$, $B(T)$, and risk-free interest rate. On the other hand, the estimation was made by Miyoshi [12] with $A(T_0)$, σ_A, and risk-free interest rate given exogenously. Meanwhile, although risk-free interest rate is used taking up risk neutrality as premise for obtaining these types of EDP estimate, Moridaira [13] proposed that EDP should be estimated by utilizing the Boness [4] model as an option evaluation model with which none of risk neutral evaluation is made.

This study makes the following assumptions for the 4 parameters as shown below.

1) Let the asset value $A(T_0)$ be the sum of the total market value of current stock $E(T_0)$ and debt value $B(T_0)$. That is to say, the asset value can be expressed below on the condition that the stock value $S(T_0)$ and issued stock number n are known.

$$A(T_0) = E(T_0) + B(T_0) = nS(T_0) + B(T_0). \tag{4}$$

2) Debt value $B(T)$ cannot be observed in the market, and therefore let it be understood that $B(T) = B(T_0) =$ "book value of debt."

3) Let the expected profit ratio of the asset value be $\mu_A = r$ with the risk-free interest rate r.

4) Denote volatility of the asset value and the total market value of current stock by σ_A and σ_E, respectively. On the assumption of 1), the expression below is obtained from Itô's lemma because the volatility σ_E is observable on the market,

$$\sigma_E E(T_0) = \frac{\partial E}{\partial A}\sigma_A A(T_0), \tag{5}$$

where

$$\sigma_E E(T_0) = N\left(\frac{\ln(A(T_0)/B(T_0)) + (\mu_A + \sigma_A^2/2)(T - T_0)}{\sigma_A\sqrt{T - T_0}}\right)\sigma_A A(T_0). \tag{6}$$

Then σ_A satisfying (6) is found. With volatility σ_E, historical volatility calculated from stock data in the past is used.
Remark 1: It is easy to see that as $T \to T_0$, $\sigma_A \to E(T_0)/(B(T_0) + E(T_0))\sigma_E$.

3 Derivation of Evaluation Formula based on the First and Second Moments

As stated in the previous section, the asset value of a company is assumed as the sum of the total market value of stock price and debt value. For constructing EDP model, we partially follow Levy's way [8] in which a new variable used for average option is assumed. In general, the fluctuation of the asset value of a company on the *T*th period which comprises of total value of stock price and debt value may not be known. Then, the solution in pricing of the asset value needs to resort to Monte Carlo simulation or approximation formula. However, the first and second moments of the asset value are easily found so that by defining a new variable we may take the following procedure to obtain an evaluation formula.

a) The first and second moments such as a mean value and variance of the sum of the total value of stock price and debt value together are derived.
b) A new variable X following geometric Brownian motion where fluctuation in the EDP evaluation period coincides with the moments of the sum of the total value of stock price and debt value is assumed.
c) By regarding the variable as asset value of the company, EDP estimation is made.

On the assumption that asset value $A(T)$ of the T th period is the sum of the total market value of stock price and debt value, the asset value can be expressed as

$$A(T) = E(T) + B(T). \tag{7}$$

Let the evaluation time point be *t* and the debt value $B(T)$ be a value of $B(T) \equiv B(T_0)$. The first and second moments $E[A(T)]$ and $E[A(T)^2]$ of $A(T)$ can be written as

$$E[A(T)] = E[E(T) + B(T_0)] \ , \ \ E[A(T)^2] = E[(E(T) + B(T_0))^2]. \tag{8}$$

On the assumption that total market value of current stock price $E(T)$ follows geometric Brownian motion, the total value of current price of stock $E(\tau)$ on the $\tau(T_0 \le \tau \le T)$ time point can be expressed as below with the total value of stock price $E(T_0)$ of the T_0 period.

$$E(\tau) = E(T_0)\exp((\mu_E - \sigma_E^2/2)\tau + \sigma_E W(\tau)). \tag{9}$$

From the above description, the first and second moments of the sum of the total value of stock price and debt value are expressed as

$$\begin{aligned} E[A(T)] &= E(T_0)e^{\mu_E(T-T_0)} + B(T_0), \\ E[A(T)^2] &= E(T_0)^2 e^{(2\mu_E + \sigma_E^2)(T-T_0)} + 2E(T_0)B(T_0)e^{\mu_E(T-T_0)} + B(T_0)^2. \end{aligned} \tag{10}$$

At the next stage, define a new variable X which follows the geometric Brownian motion. We also assume that the variable $X(t)$ follows the stochastic process

$$dX(t) = \mu_x X(t)dt + \sigma_x X(t)dW(t) \ , \tag{11}$$

where $dW(t)$ is standard Brownian motion. From (11), it is explained that the variable $X(\tau)$ on the time point $\tau(T_0 \le \tau \le T)$ is expressed with the variable $X(T_0)$ on the T_0 time point.

$$X(\tau) = X(T_0)\exp((\mu_x - \sigma_x^2/2)\tau + \sigma_x W(\tau)). \tag{12}$$

Therefore, the first moment $E[X(T)]$ and second moment $E[X(T)^2]$ of the variable X are expressed as

$$E[X(T)] = X(T_0)e^{\mu_x(T-T_0)} \ , E[X(T)^2] = X(T_0)^2 e^{(2\mu_x + \sigma_x^2)(T-T_0)}. \tag{13}$$

In this study, since we assume that the fluctuation during the evaluation period of the variable coincides with the first and second moments of the sum of the total value of stock price and debt value, drift ratio μ_x and volatility σ_x can be, from (10) and (13), obtained as follows

$$X(T_0)e^{\mu_x(T-T_0)} = E(T_0)e^{\mu_E(T-T_0)} + B(T_0). \tag{14}$$

Hence,

$$\mu_x = \frac{1}{T-T_0}\ln\left(\frac{E(T_0)}{X(T_0)}e^{\mu_E(T-T_0)} + \frac{B(T_0)}{X(T_0)}\right), \tag{15}$$

and for σ_x ,

$$X(T_0)^2 e^{(2\mu_x+\sigma_x^2)(T-T_0)} = E(T_0)^2 e^{(2\mu_E+\sigma_E^2)(T-T_0)} + 2E(T_0)B(T_0)e^{\mu_E(T-T_0)} + B(T_0)^2 \tag{16}$$

gives

$$\sigma_x = \sqrt{\frac{1}{T-T_0}\ln\psi - 2\mu_x}, \tag{17}$$

where

$$\psi = \frac{E(T_0)^2}{X(T_0)^2}e^{(2\mu_E+\sigma_E^2)(T-T_0)} + \frac{2E(T_0)B(T_0)}{X(T_0)^2}e^{\mu_E(T-T_0)} + \frac{B(T_0)^2}{X(T_0)^2}. \tag{18}$$

Remark 2: As $T \to T_0$, $\sigma_x \to E(T_0)/(B(T_0)+E(T_0))\sigma_E$.

On the assumption that the company concerned has not been liquidated till time T the total value of stock price $\tilde{E}(T_0)$ at time T_0 is expressed as the difference between the asset value and debt value. In other words,

$$\tilde{E}(T_0) = X(T_0)e^{(\mu_x-r)(T-T_0)}N(d) - B(T_0)e^{-r(T-T_0)}N\left(d-\sigma_x\sqrt{T-T_0}\right), \tag{19}$$

with

$$d = \frac{\ln(X(T_0)/B(T_0)) + (\mu_x+\sigma_x^2/2)(T-T_0)}{\sigma_x\sqrt{T-T_0}},\ \mu_x = \frac{1}{T-T_0}\ln\left(\frac{E(T_0)}{X(T_0)}e^{\mu_E(T-T_0)} + \frac{B(T_0)}{X(T_0)}\right),$$

$$\sigma_x = \sqrt{\frac{1}{T-T_0}\ln\psi - 2\mu_x},\ \psi = \frac{E(T_0)^2}{X(T_0)^2}e^{(2\mu_E+\sigma_E^2)(T-T_0)} + \frac{2E(T_0)B(T_0)}{X(T_0)^2}e^{\mu_E(T-T_0)} + \frac{B(T_0)^2}{X(T_0)^2}.$$

This result is equivalent to a payoff of European call option by letting the underlying asset be the asset value of the company concerned and the exercise price be the debt value.

From the above description, let the default be defined as an event where the variable X becomes below debt value on the time point T. With the default event, the default probability EDP on the time point T viewed from the evaluation time point T_0 of the variable is given as shown below when $X(T) \le B(T_0)$,

$$EDP = P(\ln X(T) \le \ln B(T_0)) = 1 - N\left(\frac{\ln(X(T_0)/B(T_0)) + (\mu_x-\sigma_x^2/2)(T-T_0)}{\sigma_x\sqrt{T-T_0}}\right). \tag{20}$$

Note that EDP values don't change even by any value of $X(T_0)$ ($0 < X(T_0) < \infty$) at evaluation time. Note that all the parameters in (20) necessary to estimate default probability are those as explained before. Also, it is understood that $\mu_E = r$ and the risk-free interest rate is equivalent to $\mu_A = r$.

4 Empirical Analysis of the Default Company

In this section, a relation between the model proposed in this paper and the estimation method of the default probability by the Black-Scholes-Merton model is examined.

Table 1: Default and non-default companies

A type of business	Default enterprises	Default time point	Non-default enterprises
Electric appliance manufacturer	Akai Denki	2000/11/2	TEAC
Retail business	Mical	2001/9/14	Aeon
Steel industry	Nihon Jyukagaku kogyo	2002/2/22	Taiheiyo Kinzoku

Table 2: Available data

Financial statements	Separate closing accounts
Risk-free interest rate	One year Japanese bond interest rate
Volatility of the total value of current price of stock	Annual conversion of the logarithm earning rate for 60 days of a stock price
Evaluation period	One year

Default probabilities are calculated with a company in Japan in which default was actually caused and another company of the same trade in which no default was caused. After that, comparison is made with both the default probabilities. Details of the companies in which the default probability is compared and data used in the company are listed in Tables 1 and 2.

Financial indices of the individual companies, which are extracted from NEEDS-Financial QUEST, are utilized. Meanwhile with the risk-free interest rate, the Japanese bond interest rate on an annual basis calculated by Bloomberg Corporation was used.

4.1 Transition of the EDP Estimation Value of both the Models

4.1.1 A Default Company and Non-default Company in the Electric Appliance Manufacturer

EDP transition of the proposed model and conventional model in the electric appliance manufacturer is illustrated in Table 3 and Figure 1. With Akai Denki as a default company and TEAC as a non-default company, comparison is made between the model proposed and the Black-Scholes-Merton model. From this it is observed that transition of the EDP shows the same tendency with the individual models. However, the EDP estimation value of the model of this time is always kept in a high state. The tendency, which is strong enough when the EDP is high, is so strong especially in the vicinity of the default time point.

Table 3: Akai denki

		2000/08/1	2000/09/1	2000/10/2	2000/11/2	2000/12/1
	Volatility of stock (%)	200.079	169.756	143.191	133.654	610.963
	B/A	0.864	0.883	0.916	0.930	0.992
B-S-M model	Volatility of total value (%)	37.943	27.492	16.336	12.541	8.990
	EDP(%)	42.086	37.417	32.242	30.089	47.584
Proposed model	Volatility of total value	83.288	45.703	21.555	15.428	526.611
	EDP	59.495	48.213	38.273	34.801	99.575

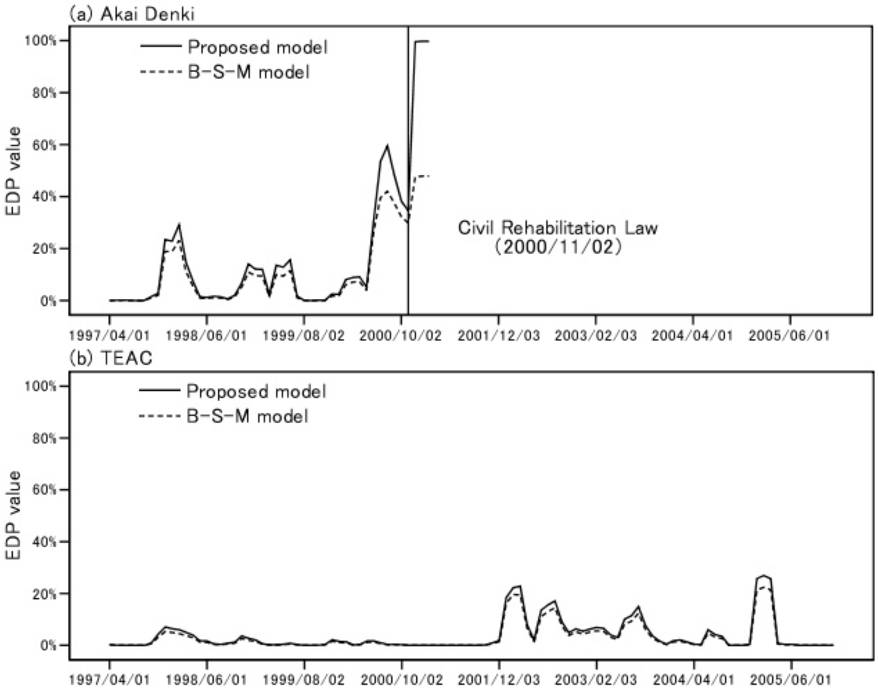

Figure 1: EDP values for Akai Denki and TEAC

With the conventional model, the transition of Akai Denki as a default company, which becomes 42.1% on 1 August, 2000 before default, is once decreased to recover default. The EDP estimation value on the default time point

(November 2, 2000), which is at that time 30.1%, remains 47.6% since that time on as well. As seen above, the transition, which is very low also with respect to the EDP since the default time point does not show adequately the state in the vicinity of the default time point. Contrary to the above, the proposed model, which showed 59.5% on August 1, 2000, is once lowered to reach the time point where default was made, anew rises to 99.6% since that time on. As seen above for example, the state of default is adequately exhibited. In such a manner, a considerable amount of dissociation is seen with the estimation value of the EDP in the vicinity of the default time point.

When transition of the EDP of both the companies is compared, a tendency of time-series-dependently similar at the primary period is observed, and the degree of the transition is very small. Especially in the vicinity of the default time point of Akai Denki, the EDP of TEAC is exceedingly small. After that, the EDP of TEAC repeats fluctuation of heightening and lowering. These kinds of transition can be recognized with both the models.

4.1.2 A Default Company and Non-default Company in Retail Business

In connection with Mical and Aeon as retail dealers, the same tendency is exhibited with the EDP transition of the proposed model and conventional model as illustrated in Table 4 and Figure 2. The EDP estimation value of the model of this time is kept in a very high state.

Table 4: Mical

		2001/08/01	2001/09/14	2001/10/01	2001/11/14	2001/12/03
	Volatility of stock (%)	73.022	112.765	484.904	518.723	578.972
	B/A	0.976	0.987	0.999	0.999	0.9997
B-S-M model	Volatility of total value (%)	1.912	1.922	0.805	0.873	0.187
	EDP(%)	9.451	22.694	39.241	40.084	21.662
Proposed model	Volatility of total value	1.999	2.129	311.907	362.252	411.888
	EDP	11.585	26.835	94.053	96.493	98.027

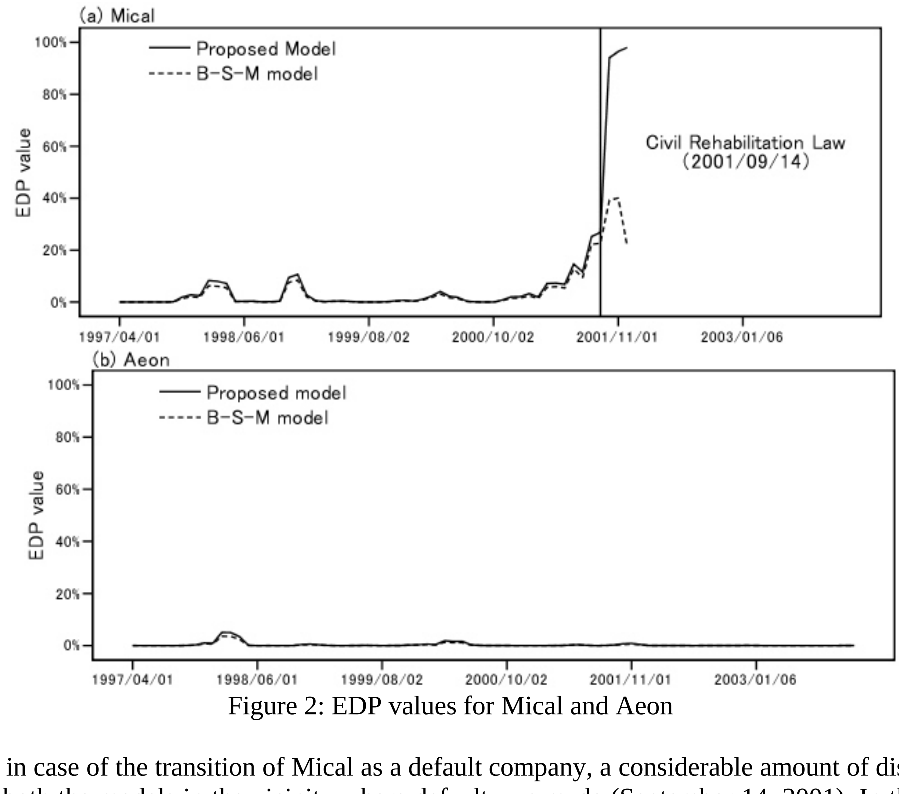

Figure 2: EDP values for Mical and Aeon

Meanwhile, in case of the transition of Mical as a default company, a considerable amount of dissociation is seen with the EDP of both the models in the vicinity where default was made (September 14, 2001). In the meantime, the default point comes up to 26.8% with the proposed model, and this becomes 94.1% in October 2001 to be increased to 98.0% in December 2001. Contrary to the above, the EDP of the default point reaches 22.7% in case of the conventional model. On October 2001, the EDP becomes 39.2% to be decreased to 21.7% in December 2001. Thus contradictory behavior is noticed as seen for example in decrease of the EDP after the default.

When the transition of the EDP of both the companies to the extent of the default time point of Mical is compared, the degree of the transition is, although a time-series-dependently similar tendency is observed, as small as in the case of electric appliance manufacturers. Especially in the vicinity of the default time point of Mical, the EDP of Aeon is extremely small and always shows stabilized low value after that. With these, the same tendency is noticed for both the models.

4.1.3 A Default Company and Non-default Company in the Steel Industry

EDP transition for Nihon Jyukagaku Kogyo as a default company and Taiheiyo Kinzoku as a non-default company are illustrated in Table 5 and Figure 3. As in the previous business, the transition of the EDP shows the same tendency with the individual models. However, it is interesting to observe that the EDP of Taiheiyo Kinzoku repeats fluctuation of heightening and lowering and the values are as high as in the case of Nihon Jyukagaku Kogyo. Then, it seems to become stabilized low value after January 1, 2003.

In this connection, the EDP of the default point for the proposed model becomes very high to reach 50.81%, while the conventional model shows 39.18%.

Taiheiyo Kinzoku seems to be in a critical state for a while but it appears to fortunately recover later.

Table 5: Nihon Jyukagaku Kogyo

		2001/12/03	2002/01/04	2002/02/01	2002/02/22	2002/03/01
	Volatility of stock (%)	93.153	121.747	134.569	212.052	632.361
	B/A	0.943	0.947	0.948	0.972	0.998
B-S-M model	Volatility of total value(%)	6.385	8.554	9.523	9.240	1.688
	EDP(%)	18.137	27.090	30.023	39.180	43.901
Proposed model	Volatility of total value	6.727	9.795	11.690	25.983	519.675
	EDP	19.948	30.523	34.565	50.813	99.531

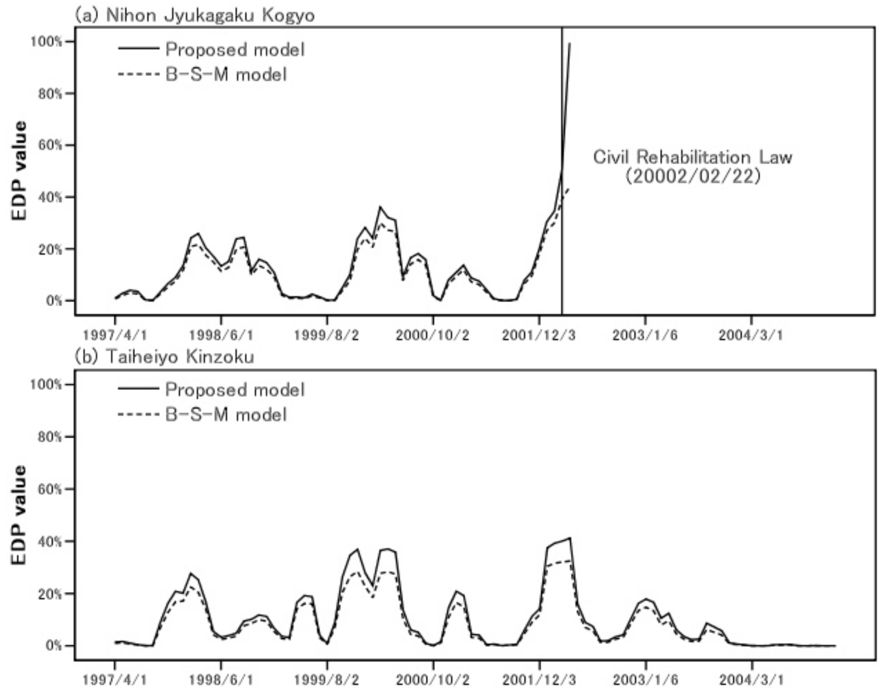

Figure 3: EDP values for Nihon Jyukagaku Kogyo and Taiheiyo Kinzoku

4.2 Relation among the EDP, Leverage, and Volatility of the Total Value of Stock Price

4.2.1 Relation between the EDP and Leverage of both the Models

A relation between the EDP and the leverage (B/A) with both the models with respect to the electric appliance manufacturer, retail sale and steel industry is illustrated, respectively, in Figures 4 to 6. In the electric appliance manufacturer, defaulted Akai Denki makes great change with the value of B/A in a range from 0.32 to 0.99 in comparison with TEAC. There is a tendency that the EDP is increased accompanied with increase of B/A, but variance is large enough even for the same B/A. Thus it cannot be said that their correlation is distinct.

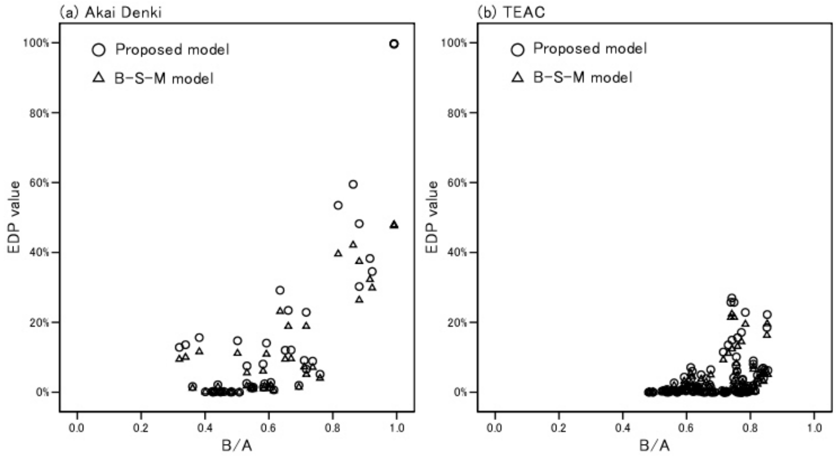

Figure 4: Relation between the EDP values and the leverage for electric appliance manufacturers

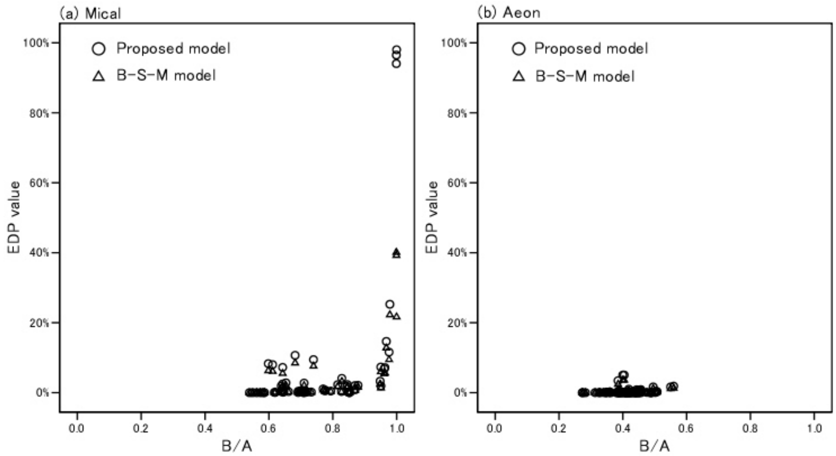

Figure 5: Relation between the EDP values and the leverage for retail business

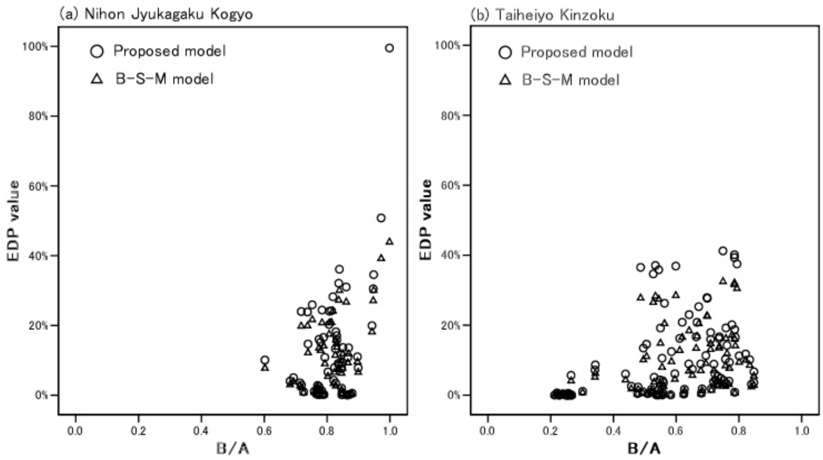

Figure 6: Relation between the EDP values and the leverage for steel industry

In connection with the EDP of Akai Denki in Figure 4(a), the proposed model shows 99.8% when B/A = 0.99. Contrary to the above, the model shows 47.9% in case of the conventional model even when B/A = 0.99. Thus it can certainty be said that great difference exists.

With TEAC as a non-default company, the value of B/A is distributed in a range from 0.48 to 0.85. Meanwhile in comparison with Akai Denki, variance of the EDP for B/A is small. From the spot where B/A comes to exceed 0.7, there appears to be a tendency that the EDP will partially be increased. However, none of exceeding dissociation between the proposed model and the conventional model can be seen.

With Mical as a retail dealer shown in Figure 5(a), the EDP is rapidly increased on the time spot where it approaches B/A = 1.0, so that difference between both the models is extremely great. Aeon as a non-default company exhibits a value low enough to be B/A <0.56, whereas 5.04% is the highest value with the proposed model even in relation to the EDP. Thus, none of great difference can be noticed with both the models.

For Nihon Jyukagaku Kogyo as a default company and Taiheiyo Kinzoku as a non-default company the situation is quite different from other two cases mentioned. The range of B/A of Taiheiyo Kinzoku are more wide, showing 0.2 to 0.8, than Nihon Jyukagaku Kogyo. However, the EDP of Nihon Jyukagaku Kogyo shows 100% with the proposed model when B/A=1.0, while the convention model shows 42% even when B/A=1.0.

4.2.2 Relation between the EDP and the Volatility of the Total Value of Stock Price

In Figures 7, 8 and 9, a relation between the EDP and the σ_E is illustrated with respect to the electric appliance manufacturer, retail sale and steel industry. The EDP and σ_E of Akai Denki as a default company of the electric appliance manufacturer apparently show a positive correlation, and there is a tendency likewise with both the models that the EDP will be increased with the increase of the σ_E. From this, it is understood that the EDP is greatly subjected to influence of the σ_E.

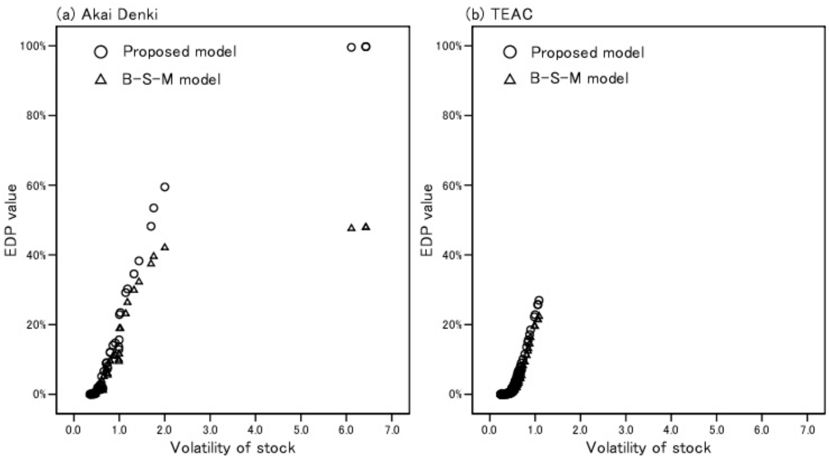

Figure 7: Relation between the EDP values and σ_E for electric appliance manufacturers

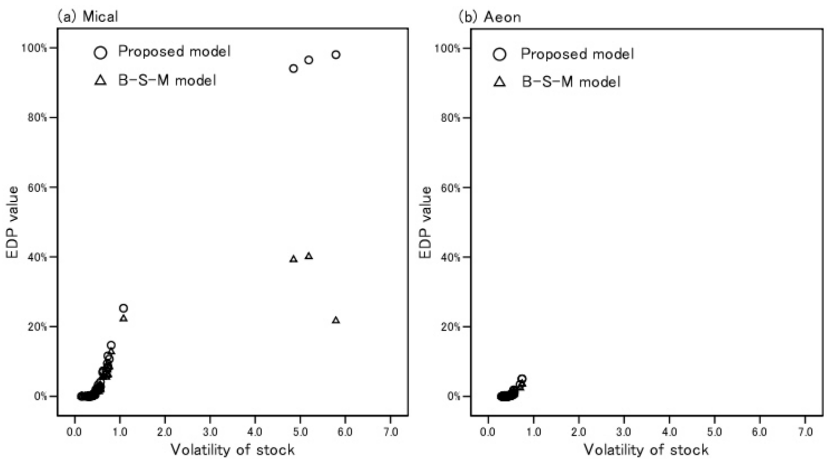

Figure 8: Relation between the EDP values and σ_E for retail business

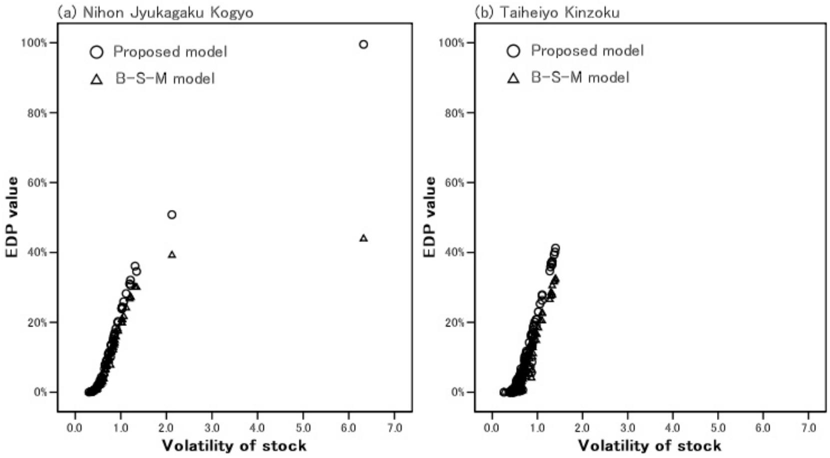

Figure 9: Relation between the EDP values and σ_E for steel industry

In comparison with both the models, it is revealed that an increasing ratio of the EDP to the σ_E of the proposed model is large. Furthermore, it is also revealed that as the σ_E becomes large, difference of the EDP between both the models increased. It is understood that especially with σ_E >6.43 in the vicinity of the default time point of Akai Denki, difference increased exceedingly. In a region where the σ_E is small, dependence of the EDP on the σ_E shows the same tendency with those of Akai Denki and TEAC.

With Mical as a default company of retail sale, a tendency the same as that of Akai Denki is noticed as illustrated in Figure 8(a). With the model of this time, the EDP exceeds 94.1% at σ_E >4.85, approaching 100% with increase of the σ_E. Contrary to the above, the EDP is reduced to 21.7% for increase of the σ_E after the maximum EDP = 40.1% was shown at σ_E = 5.19 with the conventional model.

With Aeon as a non-default company, σ_E is as small as 0.74 at the greatest. However, there is a tendency that the EDP will increase with increment of σ_E with respect to both the models.

For Nihon Jyukagaku Kogyo as a default company the tendency is roughly as similar as that of Akai Denki and Mical. The EDP reaches 100% at σ_E =6.5, while it shows 42% with the conventional model. On the other hand, the tendency of Taiheiyo Kinzoku is more similar to that of TEAC than Aeon, showing that the EDP be 40% at σ_E =1.25.

Remark 3: Comparing the EDP values for Black-Scholes model and the proposed model, it is observed that the proposed model gives always higher probability than those of Black-Scholes model. The reason originates in that companies approaching to default situation shows higher values of B/A, and the volatility of both the models plays a crucial role. In other words, the respective volatility σ_x and σ_A in EDP formulas (3) and (20) dominates each of the whole expression of normal distribution and σ_x^2 in (20) is larger than σ_A^2 in (3) so that the above-mentioned may be understood.

5 Conclusions

We estimated first and second moments such as a mean value or variance of the sum on the assumption that the asset value of a company is the sum of the total value of stock price and debt value. We also defined that a new variable for which fluctuation during the EDP evaluation period conforms to the moments and follow geometric Brownian motion. After that, an EDP estimation model was proposed by regarding the variable as asset value of the company. Thus an evaluation formula of the model was introduced.

By the model of this time, transition of the EDP of 3 types of business of default company and non-default company was obtained. Then, comparison was made with the Black-Sholes-Merton model. The same tendency such as time-series-dependent change of the EDP of both the models, difference between companies, etc. is shown. However on all the time points, the EDP of the proposed model showed a higher value than that of the conventional model. The tendency is so strong when the EDP is high enough, and the movement different from that of the conventional model is observed especially in the vicinity of the default time point.

For heightening of volatility of the total value of current price of stock in the vicinity of the default time point of the company and increase of the ratio of the leverage, the EDP estimation value is occasionally underestimated with the conventional model. However, it is possible for the proposed model, where the EDP estimation value in the default time point of the company shows almost 100 %, to show an actual default situation more distinctly than the conventional model. This stems from the reason that since the volatility of the total value of current price of stock influences on the difference of the EDP of both the models, the parameter of the σ_E liable to be more strongly influenced by the trend of the market has greater influence on the EDP estimation value than the conventional model does.

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