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Fuzzy Based Genetic Algorithm for Multicriteria Entropy Matrix Goal Game

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Abstract

This paper analyzes a multicriteria matrix goal game under entropy environment. Here a new game model known as multicriteria entropy matrix goal Game is formulated. Multiobjective non-linear programming model for each player is established. The concept of Pareto-optimal security strategy assures the property of security in the individual criteria against an opponent's deviation in strategy; however the idea is based on expected values, so that this security might be violated by mixed strategy when replications are not allowed. To avoid this inconvenience, we propose the G-goal security strategy, which includes as a part of solution with the probability of obtaining presanctified values of the payoff functions when the players are want to maximize the information about their strategies. A numerical example is included to illustrate the results in the paper.

Keywords: multicriteria matrix game, G-goal security strategy, entropy, genetic algorithm, fuzzy programming

1 Introduction

Nowadays more attention has been paid to multicriteria matrix game because it gives better realistic application of game theory. In fact, each competitive situation that can be modeled as a scaler zero-sum game has its counterpart as a multicriteria zero-sum game when more than one scenario has to be compared simultaneously. In this situation, once the same strategy has to be used in different scenarios, conflicting interest appear between different decision markers as well as within each individual. For instance, the production policies of two firms which are competing for a market can be seen as a scalar game. However, when they compete simultaneously in several markets, the multicriteria approach is to be adopted.

Again each player is interested in making moves which will be as surprising and as uncertain to the other player as possible. For this reason, the two players are involved in maximizing their entropies. Consequently, in the mathematical models of multicriteria game we introduce entropy function as one of their objectives.

In this paper, some references are presented including their work. Fernandez, Puerto and Monroy [9] considered to solve the two-person multicriteria zero-sum games. As they have considered a multicriteria game, the solution concept was based on Pareto optimality and finally they obtained the Pareto efficient solution for their proposed games. Fernandez and Puerto [8] developed a methodology to get the whole set of Pareto-optimal security strategies which are based on solving a multiple criteria linear program. This approach shows the parallelism between these strategies in multicriteria games and minimax strategies in scalar zero-sum matrix games. This notion of security is based on expected payoffs. For this reason, only when the game has been played many times, can these strategies provide us a real sense of security. On the contrary, if the game has been played only once; as in one shot games, a better analysis should consider not only the payoffs but also the probability to get them. Ghose and Prasad [11] have proposed a solution concept based on Pareto-optimal security strategies for these games. They also introduced the concept based on the similarity with security levels determined by the saddle points in scalar matrix games. This concept is independent of the notion of equilibrium so that the opponent is only taken into account to establish the security levels for one's own payoffs. When it is used to select strategies, the concept of security levels has

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important property that the payoff obtained by these strategies cannot be diminished by the opponent's deviation in strategy. Roy [17] presented the study of two different solution procedures for the two-person bimatrix game. The first solution procedure was applied to the game on getting the probability to achieve some specified goals along the player's strategy. The second specified goals along with the player's strategy by defining the fuzzy membership function defined on the pay-off matrix of the bimatrix game. Das and Roy [2] proposed a new solution concept by considering the entropy function to the objectives of the player. These models are known as entropy optimization models on two-person zero-sum game. Solution concept is based on the Kuhn-Tucker conditions, maximum entropy principle, and minimum cross-entropy principle. Without considering the pay-offs of the players, we have shown that the optimal strategy and the value of the game for each player are equivalent to the results of classical game theory. Das and Roy [3] proposed a new solution concept by considering the entropy function into the objectives of the players to the matrix goal game and then formulated some models known as entropy matrix goal game model. Basically, the paper develops on G-goal security strategy (GGSS) and obtains the probability of getting at least goal value of the players.

Several methodologies have been proposed to solve multicriteria game. Most of these methods are based on the concept of Pareto-optimal security strategies and equilibrium solution for linear models. However, no studies have been made on multicriteria entropy matrix goal game and the corresponding G-goal security strategy (GGSS). Genetic algorithm and fuzzy programming technique are applied for determining the solution of the proposed model.

2 Mathematical Model

In a matrix game, a payoff matrix of the player PI and player PII are defined as follows:

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}.$$

Each element a_{ij} of matrix A is a k-dimensional vector $[a_{ij}(1), \dots, a_{ij}(k)]$. We define individual matrix of m rows and n columns as $A(l) = [a_{ij}(l)]$ for $l = 1, \dots, k$.

The players are represented by PI (the maximizer, who chooses rows) and PII (the minimizer, who chooses columns). As usual, the mixed strategy for players PI and PII are $Y = \{y \in R^m; \sum_{i=1}^m y_i = 1; y_i \geq 0, i = 1, 2, \dots, m\}$ and $Z = \{z \in R^n; \sum_{j=1}^n z_j = 1; z_j \geq 0, j = 1, 2, \dots, n\}$. We remark that the pure strategies for both players are the extreme points of Y and Z. We analyze the problem under PI point of view.

Let $G = (G_1, \dots, G_k)$ be a goal specified by PI. In order to determine the strategies which are based on the probability to achieve the goal G, we formulate a model nameed multicriteria matrix goal game model.

Definition 2.1 *The expected payoff of the multicriteria matrix goal game with goal* $G = (G_1, \dots, G_k)$ *and the matrix* $A(l) = [a_{ij}(l)], l = 1, \dots, k$ *for each strategy pair* $y \in Y$ *and* $z \in Z$*, is* $v(y, z) = y^t A_G z = [v_1(y, z), \dots, v_k(y, z)]$*, where* $v_l(y, z) = y^t A_G(l) z, A_G(l) = [\delta_{ij}(l)]$*, and*

$$\delta_{ij}(l) = \begin{cases} 1 & \text{if } a_{ij}(l) \geq G_l \\ 0 & \text{otherwise} \end{cases}$$

for $l = 1, \dots, k$.

Definition 2.2 *The G-goal security level for PI of a multicriteria matrix game with goal* $G = (G_1, \dots, G_k)$*, and matrices* $A(l) = [a_{ij}(l)], \quad l = 1, \dots, k$*, for each* $y \in Y$ *is* $v(y) = (v_1, \dots, v_k)$*, where*

$$v_l(y) = \min_{z \in Z} v_l(y, z) = \min_{z \in Z} y^t A_G(l) z = \min_{1 \leq j \leq n} \sum_{i=1}^m y_i \delta_{ij}(l), \quad l = 1, \dots, k,$$

v_l*,* $l = 1, \dots, k$ *is the probability to achieve at least goal value* G_l *in each game when PI chooses strategy* y.

Definition 2.3 *A strategy* $y^* \in Y$ *is a G-goal security strategy for player PI if there is no* $y \in Y$ *that* $v(y^*) \leq v(y), v(y^*) \neq v(y)$.

2.1 Determination of G-goal Security Strategies

We consider the following multi-objective linear problem

$$\begin{array}{ll} \max: & v_1, \dots v_k \\ \text{subject to:} & \sum_{i=1}^m y_i \delta_{ij}(l) \geq v_l, \quad j = 1, 2, \dots, n; \quad l = 1, \dots, k \\ & \sum_{i=1}^m y_i = 1; \quad y_i \geq 0, \quad i = 1, 2, \dots, m. \end{array} \tag{1}$$

Definition 2.4 *A solution (v^*, y^*) is an efficient solution of above model (1) if there does not exist any (v, y) such that (v, y) dominates (v^*, y^*).*

Theorem 2.1 *A strategy $y^* \in Y$ is a G-goal security strategies (GGSS) and $v^* = ({v_1}^*, \dots, {v_k}^*)$ is its G-goal security level vector iff (v^*, y^*) is an efficient solution of model (1).*

Proof: Let y^* be a GGSS. Then there is no $y \in Y$ such that $v(y^*) \leq v(y), v(y^*) \neq v(y)$. This is equivalent to

$$(\min\ y^t A_G(1), \dots, \min\ y^t A_G(k)) \geq (\min\ {y^*}^t A_G(1), \dots, \min\ {y^*}^t A_G(k)),$$
$$(\min\ y^t A_G(1), \dots, \min\ y^t A_G(k)) \neq (\min\ {y^*}^t A_G(1), \dots, \min\ {y^*}^t A_G(k)).$$

Hence, (v^*, y^*) is an efficient solution of model (1).

Consequently the above solution is equivalent to the solution of model (1).

Conversely, suppose that an efficient solution (v^*, y^*) of model (1) is not a GGSS. Then there exists $\underline{y} \in Y$ such that

$$(\min\ \underline{y}^t A_G(1), \dots, \min\ \underline{y}^t A_G(k)) \geq (\min\ \underline{y}^{*t} A_G(1), \dots, \min\ \underline{y}^{*t} A_G(k)),$$
$$(\min\ \underline{y}^t A_G(1), \dots, \min\ \underline{y}^t A_G(k)) \neq (\min\ \underline{y}^{*t} A_G(1), \dots, \min\ \underline{y}^{*t} A_G(k)).$$

Taking $\underline{v} = (\underline{v}_1, \dots, \underline{v}_k)$, where $\underline{v_l} = \min\ \underline{y}^t A_G(l),\ l = 1, \dots, k$, then the vector $(\underline{v}, \underline{y})$ is a feasible solution of model (1) which dominates (v^*, y^*). This is a contradiction. Hence the theorem is proved.

Similarly, we introduce the following multiple-objective linear program called the multicriteria linear game problem for player PII as follows

$$\begin{array}{ll} \min: & w_1, \dots w_k \\ \text{subject to:} & \sum_{j=1}^n z_j(l) \delta_{ij}(l) \leq w_l, \quad i = 1, 2, \dots, m; \quad l = 1, \dots, k \\ & \sum_{j=1}^n z_j = 1; z_j \geq 0, \quad j = 1, 2, \dots, n. \end{array} \tag{2}$$

Theorem 2.2 *A strategy $z^* \in Z$ is a GGSS for player PII and $w^* = (w_1^*, \dots, w_k^*)$ is its G-goal security level vector iff (w^*, z^*) is an efficient solution of model (2).*

Proof: Let z^* be a GGSS. Then there is no $z \in Z$ such that $w(z) \leq w(z^*), w(z) \neq w(z^*)$, which is equivalent to

$$(\max\ A_G(1)z, \dots, \max\ A_G(k)z) \leq (\max\ A_G(1)z^*, \dots, \max\ A_G(k)z^*),$$
$$(\max\ A_G(1)z, \dots, \max\ A_G(k)z) \neq (\max\ A_G(1)z^*, \dots, \max\ A_G(k)z^*).$$

Hence (w^*, z^*) is an efficient solution of model (2).

Consequently, the above solution is equivalent to the solution of model (2).

Conversely, suppose that an efficient solution (w^*, z^*) of model (2) is not a GGSS. Then there exists $\overline{z} \in Z$ such that

$$(\max\ A_G(1)\overline{z}, \dots, \max\ A_G(k)\overline{z}) \leq (\max\ A_G(1)\overline{z}^*, \dots, \max\ A_G(k)\overline{z}^*),$$
$$(\max\ A_G(1)\overline{z}, \dots, \max\ A(_Gk)\overline{z}) \neq (\max\ A_G(1)\overline{z}^*, \dots, \max\ A_G(k)\overline{z}^*).$$

Taking $\overline{w} = (\overline{w}_1, \dots, \overline{w}_k)$, where $\overline{w_l} = max\ A_G(l)\overline{z},\ l = 1, \dots, k$, then the vector $(\overline{w}, \overline{z})$ is a feasible solution of model (2) which dominates (w^*, z^*). This is a contradiction. Hence the theorem is proved.

Again each player is interested in making moves which will be as surprising and uncertain to the other player as possible. For this reason, the two players are involved in maximizing their entropies. The mathematical form of entropies are as follows:

$$H_1 = -\sum_{i=1}^m y_i\ ln(y_i), \tag{3}$$

$$H_2 = -\sum_{j=1}^n z_j\ ln(z_j), \tag{4}$$

i.e., they are interested in making their strategies as spread out as possible. However they are primarily interested in maximizing their expected payoffs.

2.2 Multicriteria Entropy Matrix Goal Game Models

Without loss of generality, let us combine the model (1) and the entropy function (3) to formulate a new mathematical model. This model is named as multicriteria entropy matrix goal game model, which is a multi-objective non-linear programming model. This model is defined for player PI as follows:

$$\begin{array}{ll} \max & v_1, \ldots v_k \\ \max & v_{k+1} = H_1 \\ \text{subject to:} & \sum_{i=1}^{m} y_i \delta_{ij}(l) \geq v_l, \quad j = 1, 2, \ldots, n; \quad l = 1, \ldots, k \\ & H_1 = -\sum_{i=1}^{m} y_i \; ln(y_i) \\ & \sum_{i=1}^{m} y_i = 1; \quad y_i \geq 0, \quad i = 1, 2, \ldots, m. \end{array} \tag{5}$$

Definition 2.5 *A solution $y^* \in Y$ of model (5) is an efficient solution iff there does not exist a feasible y such that $v(y) \geq v(y^*)$, $v(y) \neq v(y^*)$, where $v(y) = [\ v_1(y), \ldots, v_{k+1}(y)\ ]$.*

Theorem 2.3 *A strategy $y^* \in Y$ is a GGSS in model (5) for player PI and $v^* = (v_1^*, \ldots, v_{k+1}^*)$ are its security level vector iff (v^*, y^*) is an efficient solution of model (5).*

Proof: Let y^* be a GGSS. Then there is no $y \in Y$ such that $v(y) \geq v(y^*), v(y) \neq v(y^*)$. This is equivalent to

$$(\min\ y^t A_G(1), \ldots, \min\ y^t A_G(k)) \geq (\min\ {y^*}^t A_G(1), \ldots, \min\ {y^*}^t A_G(k)),$$

$$(\min\ y^t A_G(1), \ldots, \min\ y^t A_G(k)) \neq (\min\ {y^*}^t A_G(1), \ldots, \min\ {y^*}^t A_G(k)).$$

Thus, $v_{k+1}(y) \geq v_{k+1}(y^*)$, i.e., $H_1(y) \geq H_1(y^*)$. Hence (v^*, y^*) is an efficient solution of model (5).

Consequently, the above solution is equivalent to the solution of model (5).

Conversely, suppose that an efficient solution (v^*, y^*) of model (5) is not a GGSS. Then there exists $\underline{y} \in Y$ such that

$$(\min\ \underline{y}^t A_G(1), \ldots, \min\ \underline{y}^t A_G(k)) \geq (\min\ \underline{y^*}^t A_G(1), \ldots, \min\ \underline{y^*}^t A_G(k)),$$

$$(\min\ \underline{y}^t A_G(1), \ldots, \min\ \underline{y}^t A_G(k)) \neq (\min\ \underline{y^*}^t A_G(1), \ldots, \min\ \underline{y^*}^t A_G(k)).$$

$v_{k+1}(\underline{y}) \geq v_{k+1}(\underline{y^*})$ i.e., $H_1(\underline{y}) \geq H_1(\underline{y^*})$. Taking $\underline{v} = (\underline{v}_1, \ldots, \underline{v}_k)$, where $\underline{v_l} = \min\ \underline{y}^t A_G(l),\ l = 1, \ldots, k$, $v_{k+1}(\underline{y}) = \underline{H_1} = H_1(\underline{y})$, then the vector $(\underline{v}, \underline{y})$ is a feasible solution of model (5) dominating (v^*, y^*). This is a contradiction. Hence the theorem is proved.

Similarly, the multicriteria entropy matrix goal game model for player PII is as follows:

$$\begin{array}{ll} \min & w_1, \ldots w_k \\ \max & H_2 \\ \text{subject to:} & \sum_{j=1}^{n} z_j(l) \delta_{ij}(l) \leq w_l, \quad i = 1, 2, \ldots, m; \quad l = 1, \ldots, k \\ & H_2 = -\sum_{j=1}^{n} z_j \; ln(z_j) \\ & \sum_{j=1}^{n} z_j = 1; z_j \geq 0, \quad j = 1, 2, \ldots, n. \end{array} \tag{6}$$

Theorem 2.4 *A strategy $z^* \in Z$ is a GGSS for player PII and $w^* = (w_1^*, \ldots, w_k^*)$, H_2^* is its security level vector iff (w^*, H_2^*, z^*) is an efficient solution of model (6).*

Proof: Let z^* be a GGSS. Then there is no $z \in Z$ such that $w(z) \leq w(z^*)$, $w(z) \neq w(z^*)$, $H_2(z) \geq H_2(z^*)$. This is equivalent to

$$(max\ A_G(1)z, \ldots, max\ A_G(k)z) \leq (max\ A_G(1)z^*, \ldots, max\ A_G(k)z^*),$$

$$(max\ A_G(1)z, \ldots, max\ A_G(k)z) \neq (max\ A_G(1)z^*, \ldots, max\ A_G(k)z^*),$$

$H_2(z) \geq H_2(z^*)$. Hence (w^*, H_2^*, z^*) is an efficient solution of model (6).

Consequently the above solution is equivalent to the solution of model (6).

Conversely, suppose that an efficient solution (w^*, H_2^*, z^*) of model (6) is not a GGSS. Then there exists $\overline{z} \in Z$ such that

$$(max\ A_G(1)\overline{z}, \ldots, max\ A_G(k)\overline{z}) \leq (max\ A_G(1)\overline{z}^*, \ldots, max\ A_G(k)\overline{z}^*),$$

$$(max\ A_G(1)\overline{z}, \ldots, max\ A_G(k)\overline{z}) \neq (max\ A_G(1)\overline{z}^*, \ldots, max\ A_G(k)\overline{z}^*).$$

$H_2(\overline{z}) \geq H_2(\overline{z}^*)$. Taking $\overline{w} = (\overline{w}_1, \ldots, \overline{w}_k)$, where $\overline{w_l} = max\ A_G(l)\overline{z},\ l = 1, \ldots, k,\ \overline{H_2} = H_2(\overline{z})$, then the vector $(\overline{w}, \overline{H_2}, \overline{z})$ is a feasible solution of model (6) which dominates (w^*, H_2^*, z^*). This is a contradiction. Hence the theorem is proved.

3 Solution Procedure

3.1 Basic Concepts of Fuzzy Set and Membership Function

Fuzzy sets was first introduced by Zadeh [18] in 1965 as a mathematical way to represent impreciseness or vagueness in everyday life.
Fuzzy set: A fuzzy set A in a discourse X is defined as the following set of pairs $A = (x, \mu_A) : x \in X$, where $\mu_A : X \longrightarrow [0, 1]$ is a mapping, called membership function of the fuzzy set A and $\mu_A(x)$ is called the membership value or degree of membership of $x \in X$ in the fuzzy set A. The larger $\mu_A(x)$ is the stronger grade of membership form in A.
Fuzzy number: A fuzzy number is a fuzzy set in the universe of discourse X that is both convex and normal. A fuzzy number A is a fuzzy set of real line R whose membership function $\mu_A(x)$ has the following characteristic with $a < b$

$$\mu_A(x) = \begin{cases} 0 & \text{if } \ x \leq a \\ \frac{x-a}{b-a} & \text{if } \ a \leq x \leq b \\ 1 & \text{if } \ x \geq b, \end{cases} \tag{7}$$

and any one may form other type of fuzzy number.

3.2 Fuzzy Programming Technique

Fuzzy programming technique to conversion single objective non-linear problem from multi-objective non-linear problem(MONLP):
A MONLP or a vector maximization problem(VMP) may be taken in the following form:

$$\begin{array}{ll} \max & f(x) = [f_1(x), \ldots, f_k(x)]^T \\ \text{subject to:} & x \in X = \{x \in R^n; g_j \leq \text{or} =\geq b_j \ \text{ for } \ j = 1, \ldots, m; x \geq 0\}. \end{array} \tag{8}$$

Zimmermann [19] showed that fuzzy programming technique could be used nicely to solve the multi-objective non-linear programming problem. To convert VMP (8) to a single objective, following steps are used:

Step 1. Solve the VMP (8) as a single objective non-linear programming problem using only one objective at a time and ignoring the others. These solutions are known as ideal solutions.

Step 2. From the results of Step 1., determine the corresponding values for every objectives at each solution derived. Let $x^1, \ldots, x^k$ be the ideal solutions of the objectives $f_1(x), \ldots, f_k(x)$ respectively. Then $U_r = \max\{f_r(x^1), \ldots, f_r(x^k)\}$ and $L_r = \min\{f_r(x^1), \ldots, f_r(x^k)\}$, where L_r and U_r are the lower and upper bounds of the r-th objective function $f_r(x)$ for $r = 1, \ldots, k$, respectively.

Step 3. Using aspiration levels of each objective of VMP to the problem (8), formulate a new problem as follows: Find x to satisfy

$$f_r(x) \geq U_r, r = 1, \ldots, k, x \in X.$$

Here the objective functions in (8) are considered as constraints. This type of constraints can be quantified by eliciting a corresponding membership functions as follows:

$$\mu_r\{f_r(x)\} = \begin{cases} 0 & \text{if } f_r(x) \le L_r \\ \frac{f_r(x)-L_r}{U_r-L_r} & \text{if } L_r \le f_r(x) \le U_r, r = 1,\dots,k \\ 1 & \text{if } f_r(x) \ge U_r. \end{cases} \tag{9}$$

Having elicited the above membership functions $\mu_r\{f_r(x)\}$ for $r = 1,\dots,k$, the VMP (8) can be defined as

$$\begin{array}{ll} \max: & \lambda \\ \text{subject to:} & \lambda \le \mu_r\{f_r(x)\}, \text{for } r = 1,\dots,k \\ & x \in X, 0 \le \lambda \le 1. \end{array} \tag{10}$$

In the previous section, we can see that the models (5) and (6) are multi-objective non-linear programming (MONLP) problem. In fuzzy programming technique, first we construct the membership function for each objective function in model (5). Let $\mu_{1l}(v_l), l = 1,\dots,k$, and $\mu_1(H_1)$ be the membership functions for objective functions respectively and they are defined as

$$\mu_{1l}(v_l) = \begin{cases} 0 & \text{if } v_l \le v_l^- \\ \frac{v_l-v_l^-}{v_l^+-v_l^-} & \text{if } v_l^- \le v_l \le v_l^+, \qquad l = 1,\dots,k, \\ 1 & \text{if } v_l \ge v_l^+ \end{cases} \tag{11}$$

and

$$\mu_1(H_1) = \begin{cases} 0 & \text{if } H_1 \le H_1^- \\ \frac{H_1-H_1^-}{H_1^+-H_1^-} & \text{if } H_1^- \le H_1 \le H_1^+ \\ 1 & \text{if } H_1 \ge H_1^+, \end{cases} \tag{12}$$

where v_l^+, v_l^- represent upper and lower bounds of $v_l, l = 1,\dots,k$, respectively, and H_1^+, H_1^- represent upper and lower bounds of H_1, respectively, for player PI.

Similarly we can construct the membership functions for objective functions in model (6). Let $\mu_{2l}(w_l), l = 1,\dots,k$, $\mu_2(H_2)$ be the membership functions for objective functions respectively and they are defined as

$$\mu_{2l}(w_l) = \begin{cases} 1 & \text{if } w_l \le w_l^- \\ \frac{w_l^+-w_l}{w_l^+-w_l^-} & \text{if } w_l^- \le w_l \le w_l^+ \\ 0 & \text{if } w_l \ge w_l^+ \end{cases} \tag{13}$$

for $l = 1,\dots,k$, and

$$\mu_2(H_2) = \begin{cases} 0 & \text{if } H_2 \le H_2^- \\ \frac{H_2-H_2^-}{H_2^+-H_2^-} & \text{if } H_2^- \le H_2 \le H_2^+ \\ 1 & \text{if } H_2 \ge H_2^+, \end{cases} \tag{14}$$

where w_l^+, w_l^- represent upper and lower bounds of $w_l, l = 1,\dots,k$, respectively, and H_2^+, H_2^- represent upper and lower bounds of H_2, respectively for player PII.

Applying fuzzy programming technique in model (5) and using the membership functions (11) and (12), we formulate the following model

$$\begin{array}{ll} \max & \lambda \\ \text{subject to:} & \lambda \le \frac{v_l-v_l^-}{v_l^+-v_l^-}, \quad l = 1,\dots,k \\ & \lambda \le \frac{H_1-H_1^-}{H_1^+-H_1^-} \\ & \sum_{i=1}^m \delta_{ij}(l) y_i \ge v_l, \quad j = 1,2,\dots,n; \quad l = 1,\dots,k \\ & H_1 = -\sum_{i=1}^m y_i \; ln(y_i) \\ & \sum_{i=1}^m y_i = 1; \quad y_i \ge 0, \quad i = 1,2,\dots,m, \end{array} \tag{15}$$

and for player PII, by the help of (13), (14) and model (6), the similar model can be built as

$$\begin{array}{ll} \max & \delta \\ \text{subject to:} & \delta \le \frac{w_l^+ - w_l}{w_l^+ - w_l^-}, \quad l = 1, \ldots, k \\ & \delta \le \frac{H_2 - H_2^-}{H_2^+ - H_2^-} \\ & \sum_{j=1}^{n} \delta_{ij}(l) z_j \le w_l, \quad i = 1, 2, \ldots, m; \quad l = 1, \ldots, k \\ & H_2 = -\sum_{j=1}^{n} z_j \; ln(z_j) \\ & \sum_{j=1}^{n} z_j = 1; \quad z_j \ge 0, \quad j = 1, 2, \ldots, n. \end{array} \tag{16}$$

To solve models (15) and (16), we apply fuzzy programming technique as stated above but we determine the upper and lower bounds of $v_l, w_l, l = 1, \ldots, k, H_1$ and H_2 by genetic algorithm depicted in the next subsection.

3.3 Genetic Algorithm

The revised genetic algorithm for the matrix game of order 4 (any order may be consider also) is illustrated as follows.

Gene Type: The gene is defined as $(y_1^a, y_2^a, y_3^a, y_4^a)$ in this study: y_1^a, y_2^a, and y_3^a are randomly given values. Please notice a gene must satisfy that $y_1^a + y_2^a + y_3^a + y_4^a = 1$.

Generating genes: This process randomly generates each element in $(y_1^a, y_2^a, y_3^a, y_4^a)$ and $y_1^a + y_2^a + y_3^a + y_4^a = 1$. Moreover, the number of gene is limited to 25 when each new run begins.

Crossover; Since it is not easy to design a crossover between genes such that $y_1^a + y_2^a + y_3^a + y_4^a = 1$, no crossover is applied in this study.

Mutation: Mutation is designed as a order of elements in $(y_1^a, y_2^a, y_3^a, y_4^a)$ by randomly determined cut-point. Consider an example: if the original gene is $(y_1^a, y_2^a, y_3^a, y_4^a)$ and cut-point is randomly determined between the string: y_1^a and y_2^a, y_3^a, y_4^a, then moreover newly mutated gene (y_1', y_2', y_3', y_4') is $(y_2^a, y_3^a, y_4^a, y_1^a)$.

Reproduction: The reproduction is also omitted to prevent the early- matured solution, which will limit the variety of solution.

Evaluation: Once $(y_1^a, y_2^a, y_3^a, y_4^a)$ is determined, the corresponding $v_l^a, l = 1, \ldots, k$ and H_1^a can be computed by the first constraint of (1) and (3).

Optimum 1: For 25 genes we get 25 values of $v_l^a, l = 1, \ldots, k$, and 25 values of H_1^a. Among these values of v_l^a for $l = 1, \ldots, k$, we store maximum in v_l^{a+} and minimum in v_l^{a-}. In each iteration, these maximum and minimum values are globally stored in $VMAX_l$ and $VMIN_l$, respectively for $l = 1, \ldots, k$. Similarly, among 25 values of H_1^a we store maximum in H_1^{a+} and minimum in H_1^{a-} and then they are stored in another locations $HMAX1$ and $HMIN1$, respectively in each iteration.

Iteration: The number of iteration is set to 800 runs, each of which begins with different random seed.

Optimum 2: After completing all the iterations, we determine v_l^+ as the maximum among all $VMAX_l, l = 1, \ldots, k$, and v_l^- as the minimum among all $VMIN_l, l = 1, \ldots, k$. Similarly, H_1^+ is the maximum among all $HMAX1$, and H_1^- is the minimum among all $HMIN1$ are determined.

Similar technique can also be applied for player PII.

4 Numerical Example

Consider the following payoff matrix

$$A = \begin{bmatrix} (1,3) & (2,1) \\ (3,1) & (1,2) \\ (1,1) & (3,3) \end{bmatrix}.$$

Let $G = (3, 2)$ be a vector of goal specified by PI. Then

$$A_G(1) = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, A_G(2) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}.$$

Table 1: Results computed by genetic algorithm

	Maximum value	Minimum value
v_1	$v_1^+ = 0.4774$	$v_1^- = 0.001850$
v_2	$v_2^+ = 0.48594$	$v_2^- = 0.001850$
w_1	$w_1^+ = 0.99795$	$w_1^- = 0.501595$
w_2	$w_2^+ = 0.99795$	$w_2^- = 0.501595$
H_1	$H_1^+ = 1.097173$	$H_1^- = 0.108341$
H_2	$H_2^+ = 0.6931472$	$H_2^- = 0.014737$

The following results are summarized in Table 1, which is computed by genetic algorithm.
With the values in Table 1, the mathematical models for the players PI and PII are redefined as

$$\begin{array}{ll} \max & \lambda \\ \text{subject to:} & \lambda \leq \frac{v_1 - 0.001850}{0.4774 - 0.001850} \\ & \lambda \leq \frac{v_2 - 0.001850}{0.48594 - 0.001850} \\ & \lambda \leq \frac{H_1 - 0.108341}{1.097173 - 0.108341} \\ & y_2 \geq v_1, y_3 \geq v_1, y_1 \geq v_2, y_2 + y_3 \geq v_2 \\ & H_1 = -\sum_{i=1}^{3} y_i ln(y_i) \\ & \sum_{i=1}^{3} y_i = 1; \quad y_i \geq 0, \quad i = 1, 2, 3, \end{array} \tag{17}$$

and

$$\begin{array}{ll} \max & \delta \\ \text{subject to:} & \delta \leq \frac{0.99795 - w_1}{0.99795 - 0.501595} \\ & \delta \leq \frac{0.99795 - w_2}{0.99795 - 0.501595} \\ & \delta \leq \frac{H_2 - 0.014737}{0.6931472 - 0.014737} \\ & z_1 \leq w_1, z_2 \leq w_1, z_1 \leq w_2, z_2 \leq w_2 \\ & H_2 = -\sum_{j=1}^{2} z_j ln(z_j) \\ & z_1 + z_2 = 1, z_1, z_2 \geq 0, \end{array} \tag{18}$$

respectively. The aspiration levels with two objectives for a given solution λ^* and δ^* are obtained from above models (17) and (18) by Lingo package. The efficient solutions for players PI and player PII are represented in the following Table 2. Hence we conclude that when players want to maximize the uncertainty for their moves, then if PI plays strategy $(0.3372, 0.3314, 0.3314)$, he/she gets $G_1 = 3$ with a probability at least .3313609 and $G_2 = 2$ with a probability at least .3372783; and if PII plays strategy $(.5, .5)$ he/she gets $G_1 = 3$ with a probability at least 0.52295 and $G_2 = 2$ with a probability at least 0.52295.

Table 2: Efficient solutions for player PI and player PII

Aspiration level	Efficient solution		
	Probability of goal	Entropies	GGSS
$\lambda^* = 0.692905$	$v^* = (.3313609, .3372783)$	$H_1^* = 1.098577$	$y^* = (0.3372, 0.3314, 0.3314)$
$\delta^* = 1.00$	$w^* = (0.52295, 0.52295)$	$H_2^* = 0.6931472$	$z^* = (0.5, 0.5)$

5 Conclusions

This paper is the study of a multicriteria matrix goal game and analyzes the game under entropy environment. Using goal, we considered the solution not only strategy played by the player, but also the probability of getting at least goal value of the players. Therefore, with this approach, each player has getting the information about the probability of achieving the possible outcomes of the multicriteria entropy matrix goal game.

A methodology to obtain the GGSS was developed through fuzzy based genetic algorithm and we have shown that all these strategies, together with their associated probabilities, can be obtained as a G-goal efficient solution of the formulated models. These models are highly significant to the real world practical problem.

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