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Anti-synchronization Between Two Different Hyperchaotic Systems

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Abstract

The purpose of this paper is to study hyperchaos anti-synchronization of two identical and different hyperchaotic systems using active control. The sufficient conditions for achieving anti-synchronization of two high dimensional chaotic systems are derived based on Lyapunov stability theory, where the controllers are designed by using the sum of the relevant variables in hyperchaotic systems such that the hyperchaotic Lü system is controlled to be hyperchaotic Lorenz system. Theoretical analysis and numerical simulations are shown to verify the results.

Keywords: hyperchaotic, anti-synchronization, active control, Lyapunov direct method

1 Introduction

Since the pioneering work of Pecora and Carroll [1], chaos synchronization, as a very active topic in nonlinear science, has been intensively studied in the last few years and widely explored in a variety of fields including physical, chemical and ecological system [2]. Hence various synchronization schemes such as adaptive control [3, 4, 5], linear and nonlinear feedback synchronization methods [6, 7], active control [8], and backstepping design technique [9] have been successfully applied to chaos synchronization. The concept of synchronization has been extended to the scope such as generalized synchronization [10, 11], phase synchronization [10], lag synchronization [12], and even anti phase synchronization (APS) [13, 14, 15]. APS can also be interpreted as anti-synchronization (AS), which is a phenomenon that the state vectors of the synchronized systems have the same amplitude but opposite signs as those of the driving system. Therefore, the sum of two signals are expected to converge to zero when either AS or APS appears. Recently, active control has been applied to anti-synchronize two identical chaotic systems [16, 17, 18]. Moreover, it is examined in different types of chaotic systems [19]. In fact, in engineering, it is hardly the case that every component can be assumed to be identical. Thus, it is much more attractive and challengeable to realize anti-synchronization of two different chaotic systems. The aim of this work is to further develop the state observer method for constructing anti-synchronized of the high dimensional system, since the aforementioned method is mainly concern with the synchronization of chaotic systems with low dimensional attractor which is characterized by one positive Lyapunov exponent. This feature limits the complexity of the chaotic dynamics. It is believed that the chaotic systems with higher dimensional attractor have much wider application. In fact, the adoption of higher dimensional chaotic systems has been proposed for secure communication and the presence of more than one Lyapunov exponent clearly improves security of the communication scheme by generating more complex dynamics. Recently, hyperchaotic systems were also considered with quickly increasing interest. Hyperchaotic system is usually classified as a chaotic system with more than one positive Lyapunov exponent.

The rest of the paper is organized as follows. Section 2 presents a brief description of the active control method. Section 3 gives a brief description of the two systems. In Sections 4 and 5, we present chaos anti-synchronization of two identical hyperchaotic systems, and in Section 6 we present chaos anti-synchronization of two different hyperchaotic systems. Finally, concluding remark is given in Section 7.

2 Anti-synchronization

Consider a chaotic continuous system described by

$$\dot{x} = f(x(t), t), \tag{1}$$

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where $x \in R^n$ is a n-dimensional state vector of the system, and $f : R^n \rightarrow R^n$ defines a vector field in n-dimensional space. We decompose the function $f(x(t), t)$ as

$$f(x(t), t) = g(x(t), t) + h(x(t), t), \tag{2}$$

where $g(x(t), t)$ is the linear part of $f(x(t), t)$, and described by

$$g(x(t), t) = Ax(t), \tag{3}$$

where A is a full rank constant matrix and all eigenvalues of A have negative real parts, $h(x(t), t) = f(x(t), t) - g(x(t), t)$ is the nonlinear part of $f(x(t), t)$. Then the system (1) can be written as

$$\dot{x} = g(x(t), t) + h(x(t), t). \tag{4}$$

The chaotic anti-synchronization discussed in this paper is defined as the complete Anti-synchronization, which means that the state vectors of synchronized systems have the same absolute values but opposite signs. We say that anti-synchronization of two system $M_1(t)$ and $M_2(t)$ is achieved if the following equation holds:

$$\lim_{t\rightarrow\infty} \|x_1(t) + x_2(t)\| = 0, \tag{5}$$

where $x_i(t)$, $(i = 1, 2)$ are the state vectors of the system $M_i(t), (i = 1, 2)$. We construct a new system

$$\dot{w}(t) = g(w(t), t) - h(x(t), t). \tag{6}$$

The synchronization error between system (1) and system (6) is defined as $e(t) = x(t) + w(t)$, their evolution is determined by the following equation:

$$\dot{e}(t) = \dot{x}(t) + \dot{w}(t) = A(x(t) + w(t)) = Ae(t). \tag{7}$$

Since all eigenvalues of the matrix A have negative real parts, the zero point of synchronization error is asymptotically stable and $e(t)$ tends to zero when $t \rightarrow \infty$. Then the state vectors $x(t)$ and $w(t)$ of different systems can be anti-synchronized. The simplest configuration of matrix A=diga$(\lambda_1, \lambda_2, ..., \lambda_n)$ through separation of systems can be used for the adjustment of parameters to satisfy the stability criterion of linear system that all eigenvalues have negative real.

3 Systems Description

The hyperchaotic Lorenz system [20, 21] is given by

$$\begin{cases} \dot{x} = a(y - x) + w \\ \dot{y} = -xz + rx - y \\ \dot{z} = xy - bz \\ \dot{w} = -xz + dw, \end{cases} \tag{8}$$

where x, y, z and w are state variables, and a, b, c and d are real constants. When parameters $a = 10, r = 28, b = 8/3$ and $d = 1.3$, system (8) shows hyperchaotic behavior, the projections of the hyperchaotic attractor is shown in Figure 1.

Chen *et al.* [22] constructed hyperchaotic system by introducing an additional state into the third-order Lü chaotic system. The four-dimensional autonomous hyperchaotic system is described by

$$\begin{cases} \dot{x} = a(y - x) \\ \dot{y} = -xz + cy \\ \dot{z} = xy - bz \\ \dot{w} = xz + dw, \end{cases} \tag{9}$$

where x, y, z, and w are state variables and a, b, c, and d, are constant parameters. When parameters $a = 36, b = 3, c = 20$ and $d = 1.3$, system (9) is hyperchaotic, the projections of the hyperchaotic attractor are shown in Figure 2.

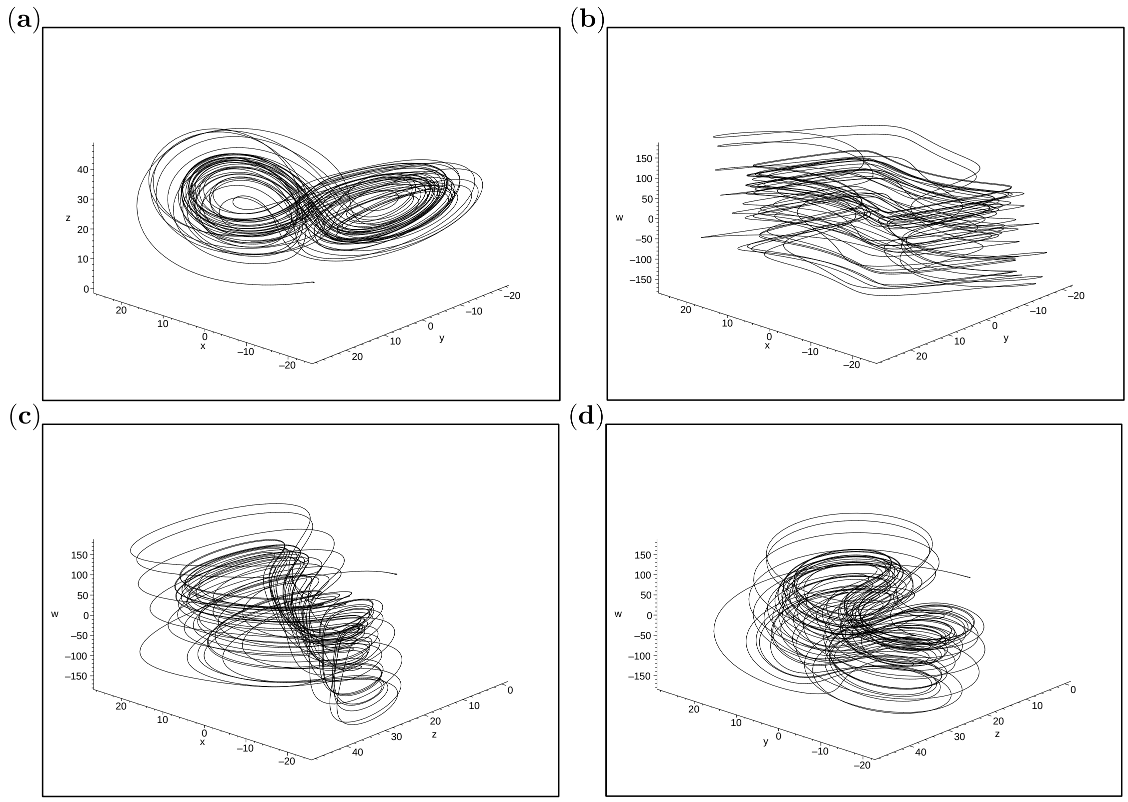


Figure 1: Typical dynamical behaviors of system (8): **(a)** Projection in (x, y, z) space; **(b)** Projection in (x, y, w) space;**(c)** Projection in (x, z, w) space; **(d)** Projection in (y, z, w) space.

4 Anti-synchronization of Hyperchotic Lorenz System

In order to observe the anti-synchronization behavior in the Lorenz hyperchaotic system (8), we have two Lorenz hyperchaotic systems where the drive system with four state variables denoted by the subscript 1 and the response system having identical equations denoted by the subscript 2. However, the initial condition on the drive system is different from that of the response system. The two Lorenz systems are described, respectively, by the following equations:

$$\begin{cases} \dot{x}_1 = a(y_1 - x_1) + w_1 \\ \dot{y}_1 = -x_1 z_1 + r x_1 - y_1 \\ \dot{z}_1 = x_1 y_1 - b z_1 \\ \dot{w}_1 = -x_1 z_1 + d w_1 \end{cases} \tag{10}$$

and

$$\begin{cases} \dot{x}_2 = a(y_2 - x_2) + w_2 + u_1(t) \\ \dot{y}_2 = -x_2 z_2 + r x_2 - y_2 + u_2(t) \\ \dot{z}_2 = x_2 y_2 - b z_2 + u_3(t) \\ \dot{w}_2 = -x_2 z_2 - d w_2 + u_4(t) \end{cases} \tag{11}$$

be the response system, where we have introduced four control functions $u_1(t), u_2(t), u_3(t)$ and $u_4(t)$ in Eq. (11). These functions are to be determined for the purpose of anti-synchronizing the two Lorenz hyperchaotic dynamical system with the same unknown parameters and different initial conditions. Let us define the state errors between the response system that is to be controlled and the controlling derive system as

$$e_1 = x_2 + x_1, \quad e_2 = y_2 + y_1, \quad e_3 = z_2 + z_1, \quad e_4 = w_2 + w_1. \tag{12}$$

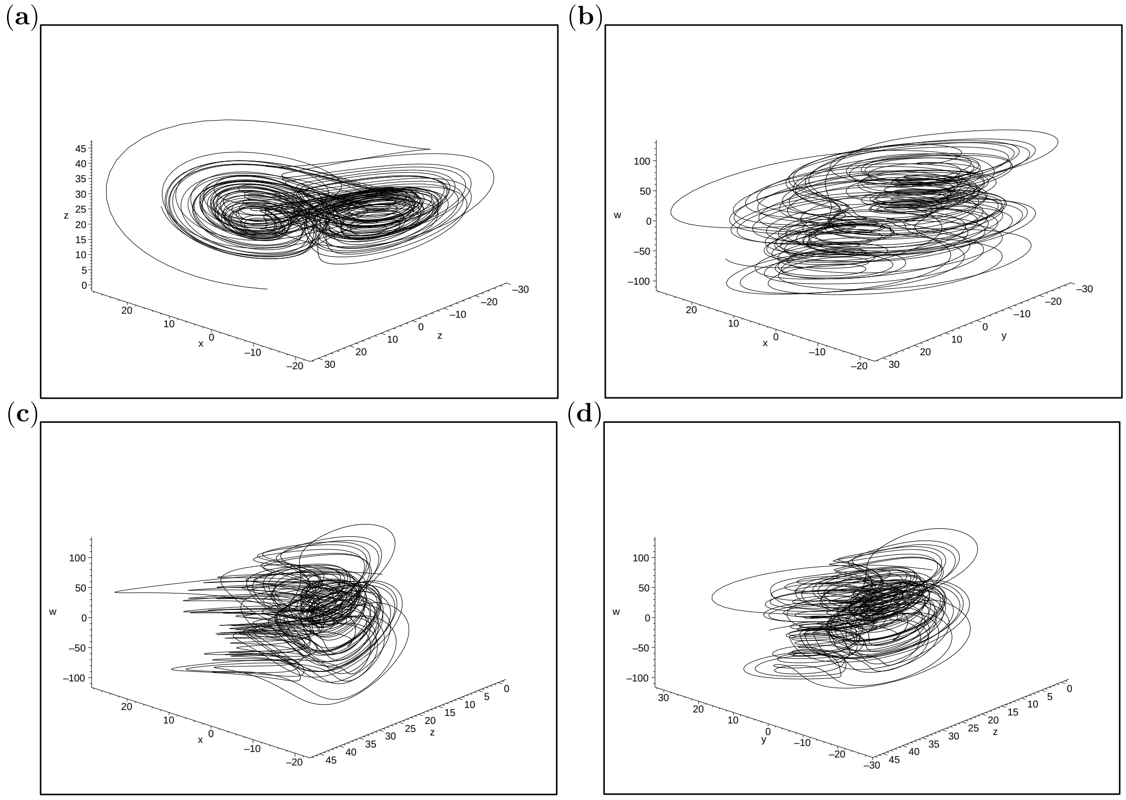


Figure 2: Typical dynamical behaviors of system (9): **(a)** Projection in (x, y, z) space; **(b)** Projection in (x, y, w) space;**(c)** Projection in (x, z, w) space; **(d)** Projection in (y, z, w) space.

By adding Eq.(10) to Eq.(11) yields the error dynamical system between Eqs.(10) and (11)

$$\begin{cases} \dot{e}_1 = a(e_2 - e_1) + e_4 + u_1(t) \\ \dot{e}_2 = -x_2 z_2 - x_1 z_1 + r e_1 - e_2 + u_2(t) \\ \dot{e}_3 = x_2 y_2 + x_1 y_1 - b e_3 + u_3(t) \\ \dot{e}_4 = -x_1 z_1 - x_2 z_2 + d e_4 + u_4(t). \end{cases} \tag{13}$$

The anti-synchronization problem for Lorenz hyperchaotic dynamical system is to achieve the asymptotic stability of the zero solution of the error system (13). To this end we take the active control functions $u_1(t), u_2(t), u_3(t)$ and $u_4(t)$ as follows:

$$\begin{aligned} &u_1(t) = V_1(t),\ u_2(t) = V_2(t) + x_2 z_2 + x_1 z_1, \\ &u_3(t) = V_3(t) - x_2 y_2 - x_1 y_1,\ u_4(t) = V_4(t) + x_1 z_1 + x_2 z_2. \end{aligned} \tag{14}$$

Hence the error system (13) becomes

$$\dot{e}_1 = a(e_2 - e_1) + e_4 + V_1,\ \dot{e}_2 = r e_1 - e_2 + V_2,\ \dot{e}_3 = -b e_3 + V_3,\ \dot{e}_4 = d e_4 + V_4. \tag{15}$$

Eq.(15) describe the error dynamics and can be considered in terms of a control problem where the system to be controlled is a linear system with a control input $V_1(t), V_2(t), V_3(t)$ and $V_4(t)$ as function of e_1, e_2, e_3 and e_4. As long as these feedbacks stabilize the system, e_1, e_2, e_3 and e_4 converge to zero as time t goes to infinity. This implies that two Lorenz hyperchaotic systems are anti-synchronized with feedback control. There are many possible choices for the control $V_1(t), V_2(t), V_3(t)$ and $V_4(t)$. If we choose

$$V_1(t) = -a e_2 - e_4,\ V_2(t) = -r e_1,\ V_3(t) = 0,\ V_4(t) = -(1 + d) e_4, \tag{16}$$

then the error dynamical system is

$$\dot{e}_1 = -a e_1,\ \dot{e}_2 = -e_2,\ \dot{e}_3 = -b e_3,\ \dot{e}_4 = -e_4. \tag{17}$$

Eq.(17) describes the error dynamics. Now we define the Laypunov function for the system (17) as follows:

$$V(e_1,\ e_2,\ e_3,\ e_4) = \tfrac{1}{2}e_1^2 + \tfrac{1}{2}e_2^2 + \tfrac{1}{2}e_3^2 + \tfrac{1}{2}e_4^2. \tag{18}$$

This function is positive definite and equal zero at the equilibrium of the system (17). Moreover, the derivative of the Laypunov function (18) has the form

$$\dot{V} = -\left[ae_1^2 + e_2^2 + be_3^2 + e_4^2\right], \tag{19}$$

which is negative definite. From Lyapunov direct method, we have that the zero solution of the system (17) is asymptotically stable. This implies that the two Lorenz hyperchaotic systems are anti-synchronized.

4.1 Numerical Results

Fourth order Runge-Kutta integration method is used to solve the systems of differential equations (10) and (11). In addition, a time step size 0.001 is employed. We select the parameters of Lorenz hyperchaotic system (8) as $a = 10, b = 8/3, r = 28,\ d = 1.3$, so that system (8) exhibits a hyperchaotic behavior. The initial values of the drive and response systems are $x_1(0) = -0.1, y_1(0) = 0.2, z_1(0) = -0.6, w_1(0) = 0.4$ and $x_2(0) = -1, y_2(0) = 0.4, z_2(0) = -0.2, w_2(0) = 1$, respectively, and the initial states of the error system are $e_1(0) = -1.1, e_2(0) = 0.6, e_3(0) = -0.8, e_4(0) = 1.4$. Figure 3(a)–(d) display the time response of states x_1, y_1, z_1, w_1 for the drive system (10) and the states x_2, y_2, z_2, w_2 for the response system (11).

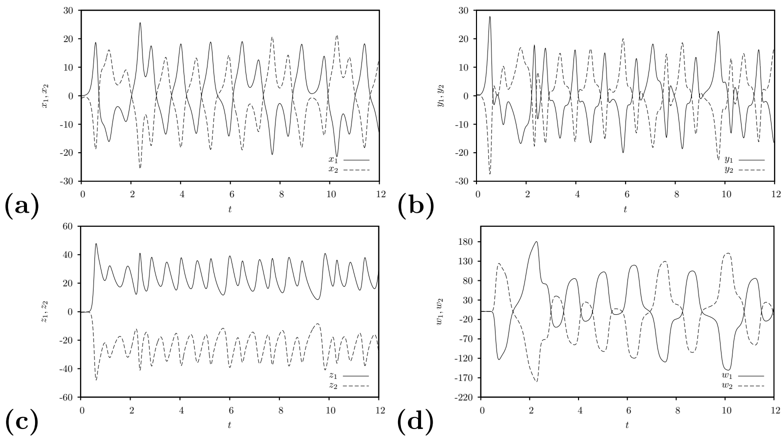


Figure 3: The time response of states for drive system (10) and the response system (11) via active control (**a**) signals x_1 and x_2; (**b**) signals y_1 and y_2; (**c**) signals z_1 and z_2; (**d**) signals w_1 and w_2

5 Anti-synchronization of Hyperchotic Lü System

In order to observe the anti-synchronization behavior in the hyperchotic Lü system, we have two identical hyperchotic Lü system where the the drive system with four state variables denoted by the subscript 1 drives the response system having identical equations denoted by the subscript 2. However, the initial condition on the drive system is different from that of the response system. The two hyperchotic Lü system are described, respectively, by the following equations:

$$\begin{cases} \dot{x}_1 = a(y_1 - x_1) + w_1 \\ \dot{y}_1 = -x_1 z_1 + r x_1 - y_1 \\ \dot{z}_1 = x_1 y_1 - b z_1 \\ \dot{w}_1 = -x_1 z_1 + d w_1 \end{cases} \tag{20}$$

and

$$\begin{cases} \dot{x_2} = a(y_2 - x_2) + w_2 + \beta_1(t) \\ \dot{y_2} = -x_2 z_2 + c y_2 + \beta_2(t) \\ \dot{z_2} = x_2 y_2 - b z_2 + \beta_3(t) \\ \dot{w_2} = x_2 z_2 + d w_2 + \beta_4(t). \end{cases} \tag{21}$$

We have introduced four control functions $\beta_1(t), \beta_2(t), \beta_3(t)$ and $\beta_4(t)$ in (21). These functions are to be determined. Let us define the error states between the response system (21) that is to be controlled and the controlling drive system (20) as

$$e_1 = x_2 + x_1, \quad e_2 = y_2 + y_1, \quad e_3 = z_2 + z_1, \quad e_4 = w_2 + w_1. \tag{22}$$

By adding Eq.(20) to Eq.(21) yields the error dynamical system between Eqs.(20) and (21)

$$\begin{cases} \dot{e}_1 = a(e_2 - e_1) + e_4 + \beta_1(t) \\ \dot{e}_2 = -x_2 z_2 - x_1 z_1 + c e_2 + \beta_2(t) \\ \dot{e}_3 = x_2 y_2 + x_1 y_1 - b e_3 + \beta_3(t) \\ \dot{e}_4 = x_2 z_2 + x_1 z_1 + d e_4 + \beta_4(t). \end{cases} \tag{23}$$

We define the active control functions $\beta_1(t), \beta_2(t), \beta_3(t)$ and $\beta_4(t)$ as follows:

$$\begin{aligned} &\beta_1(t) = V_1(t),\ \beta_2(t) = V_2(t) - x_2 z_2 - x_1 z_1, \\ &\beta_3(t) = V_3(t) + x_2 y_2 + x_1 y_1,\ \beta_4(t) = V_4(t) + x_2 z_2 + x_1 z_1. \end{aligned} \tag{24}$$

Hence the error system (23) becomes

$$\dot{e}_1 = a(e_2 - e_1) + e_4 + V_1,\ \dot{e}_2 = c e_1 + V_2,\ \dot{e}_3 = -b e_3 + V_3,\ \dot{e}_4 = d e_4 + V_4. \tag{25}$$

Eq.(25) describe the error dynamics and can be considered in terms of a control problem where the system to be controlled is a linear system with a control input $V_1(t), V_2(t), V_3(t)$ and $V_4(t)$ as function of e_1, e_2, e_3 and e_4. As long as these feedbacks stabilize the system, e_1, e_2, e_3 and e_4 converge to zero as time t goes to infinity. This implies that two Lü hyperchaotic systems are anti-synchronized with feedback control. There are many possible choices for the control $V_1(t), V_2(t), V_3(t)$ and $V_4(t)$. If we choose

$$V_1(t) = -a e_2 - e_4,\ V_2(t) = -(1 + c) e_2,\ V_3(t) = 0,\ V_4(t) = -(1 + d) e_4, \tag{26}$$

then the error dynamical system is

$$\dot{e}_1 = -a e_1,\ \dot{e}_2 = -e_2,\ \dot{e}_3 = -b e_3,\ \dot{e}_4 = -e_4. \tag{27}$$

Eq.(27) describes the error dynamics. Now we define the Laypunov function for the system (27) as follows:

$$V(e_1,\ e_2,\ e_3,\ e_4) = \tfrac{1}{2} e_1^2 + \tfrac{1}{2} e_2^2 + \tfrac{1}{2} e_3^2 + \tfrac{1}{2} e_4^2 \tag{28}$$

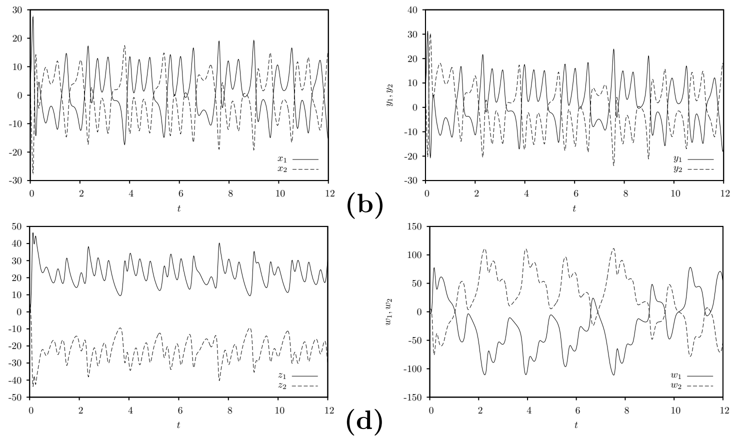
This function is positive definite and equal zero at the equilibrium of the system (27). Moreover, the derivative of the Laypunov function (28) has the form

$$\dot{V} = -\left[a e_1^2 + e_2^2 + b e_3^2 + e_4^2,\right] \tag{29}$$

which is negative definite. From Lyapunov direct method, we have that the zero solution of the system (27) is asymptotically stable. This implies that the two Lü hyperchaotic systems are anti-synchronized.

5.1 Numerical Results

Fourth order Runge-Kutta integration method is used to solve the systems of differential equations (20) and (21). In addition, a time step size 0.001 is employed. We select the parameters of lorenz hyperchaotic system (9) as $a = 36, b = 3, c = 20,\ d = 1.3$, so that system (8) exhibits a hyperchaotic behavior. The initial values of the drive and response systems are $x_1(0) = 5, y_1(0) = 8, z_1(0) = -1, w_1(0) = -3$ and $x_2(0) = 3, y_2(0) = 4, z_2(0) = 5, w_2(0) = 5$, respectively, and the initial states of the error system are $e_1(0) = 8, e_2(0) = 12, e_3(0) = 4, e_4(0) = 2$. Figure 4(a)–(d) display the time response of states x_1, y_1, z_1, w_1 for the drive system (20) and the states x_2, y_2, z_2, w_2 for the response system (21).



(**a**) (**b**) (**c**) (**d**)

Figure 4: The time response of states for drive system (20) and the response system (20) via active control (**a**) signals x_1 and x_2; (**b**) signals y_1 and y_2; (**c**) signals z_1 and z_2; (**d**) signals w_1 and w_2

6 Anti-synchronization between Two Hyperchaotic Systems

In this section, the hyperchaotic Lü system (9) is controlled to be Lorenz hyperchaotic system (8). Therefore, we consider Lorenz hyperchaotic system as the drive system and Lü hyperchaotic system as the response system. The drive system is

$$\begin{cases} \dot{x_1} = a_1(y_1 - x_1) + w_1 \\ \dot{y_1} = -x_1 z_1 + r x_1 - y_1 \\ \dot{z_1} = x_1 y_1 - b_1 z_1 \\ \dot{w_1} = -x_1 z_1 + d_1 w_1, \end{cases} \tag{30}$$

and the response system is

$$\begin{cases} \dot{x_2} = a_2(y_2 - x_2) + w_2 + \eta_1(t) \\ \dot{y_2} = -x_2 z_2 + c y_2 + \eta_2(t) \\ \dot{z_2} = x_2 y_2 - b_2 z_2 + \eta_3(t) \\ \dot{w_2} = x_2 z_2 + d_2 w_2 + \eta_4(t), \end{cases} \tag{31}$$

where $\eta_1(t), \eta_2(t), \eta_3(t)$ and $\eta_4(t)$ are the active control functions introduced in system (31). These functions are to be determined. Let the error states between the response system (30) and the drive system (31) are

$$e_1 = x_2 + x_1, \quad e_2 = y_2 + y_1, \quad e_3 = z_2 + z_1, \quad e_4 = w_2 + w_1. \tag{32}$$

By adding Eq.(30) to Eq.(31) yields the error dynamical system between Eqs.(30) and (31)

$$\begin{cases} \dot{e}_1 = a_2(e_2 - e_1) + e_4 + (a_1 - a_2)(y_1 - x_1) + \eta_1(t) \\ \dot{e}_2 = c e_2 - (1 + c) y_1 + r x_1 - x_2 z_2 - x_1 z_1 + \eta_2(t) \\ \dot{e}_3 = -b_2 e_3 + (b_2 - b_1) z_1 + x_2 y_2 + x_1 y_1 + \eta_2(t) \\ \dot{e}_4 = d_2 e_4 + (d_1 - d_2) w_1 - x_2 z_2 + x_1 z_1 + \eta_4(t). \end{cases} \tag{33}$$

We define the active control functions $\eta_1(t), \eta_2(t), \eta_3(t)$ and $\eta_4(t)$ as follows:

$$\begin{cases} \eta_1(t) = V_1(t) - (a_1 - a_2)(y_1 - x_1) \\ \eta_2(t) = V_2(t) + (1 + c) y_1 - r x_2 + x_2 z_2 + x_1 z_1 \\ \eta_3(t) = V_3(t) - (b_2 - b_1) z_1 - x_2 y_2 - x_1 y_1 \\ \eta_4(t) = V_4(t) - (d_1 - d_2) w_1 + x_2 z_2 - x_1 z_1. \end{cases} \tag{34}$$

Hence the error system (33) becomes

$$\dot{e}_1 = a_2(e_2 - e_1) + e_4 + V_1, \ \dot{e}_2 = c e_2 + V_2, \ \dot{e}_3 = -b_2 e_3 + V_3, \ \dot{e}_4 = d_2 e_4 + V_4. \tag{35}$$

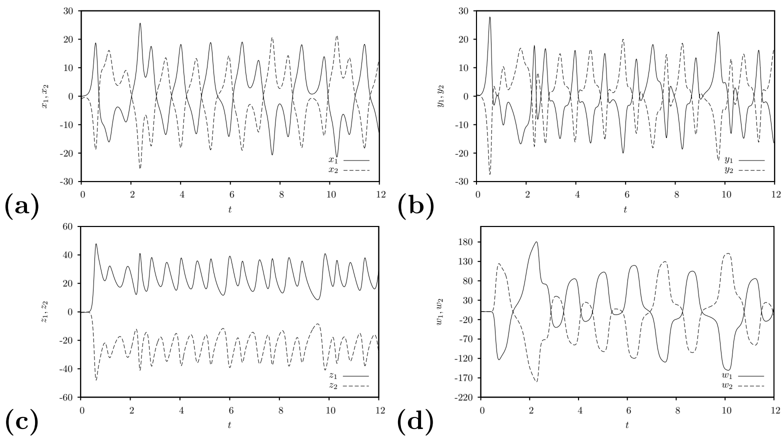


Figure 5: The time response of states for drive system (31) and the response system (30) via active control (**a**) signals x_1 and x_2; (**b**) signals y_1 and y_2; (**c**) signals z_1 and z_2; (**d**) signals w_1 and w_2

Eq.(35) describes the error dynamics and can be considered in terms of a control problem where the system to be controlled is a linear system with a control input $V_1(t), V_2(t), V_3(t)$ and $V_4(t)$ as function of e_1, e_2, e_3 and e_4. As long as these feedbacks stabilize the system, e_1, e_2, e_3 and e_4 converge to zero as time t goes to infinity. This implies that two Lü hyperchaotic systems are anti-synchronized with feedback control. There are many possible choices for the control $V_1(t), V_2(t), V_3(t)$ and $V_4(t)$. If we choose

$$V_1(t) = -a_2 e_2 - e_4,\ V_2(t) = -(1+c)e_3,\ V_3(t) = 0,\ V_4(t) = -(1+d_2)e_4, \tag{36}$$

then the error dynamical system is

$$\dot{e}_1 = -a_2 e_1,\ \dot{e}_2 = -e_2,\ \dot{e}_3 = -b_2 e_3,\ \dot{e}_4 = -e_4. \tag{37}$$

Eq.(37) describes the error dynamics. Now we define the Laypunov function for the system (27) as follows:

$$V(e_1,\ e_2,\ e_3,\ e_4) = \tfrac{1}{2}e_1^2 + \tfrac{1}{2}e_2^2 + \tfrac{1}{2}e_3^2 + \tfrac{1}{2}e_4^2. \tag{38}$$

This function is positive definite and equal to zero at the equilibrium of the system (37). Moreover, the derivative of the Laypunov function (38) has the form

$$\dot{V} = -\left[ae_1^2 + e_2^2 + be_3^2 + e_4^2\right], \tag{39}$$

which is negative definite. From Lyapunov direct method, we have that the zero solution of the system (37) is asymptotically stable. This implies that the two Lü hyperchaotic systems are anti-synchronized.

6.1 Numerical Results

Fourth order Runge-Kutta integration method is used to solve the systems of differential equations (30) and (31). In addition, a time step size 0.001 is employed. We select the parameters of lorenz hyperchaotic system (8) as $a = 10, b = 8/3, r = 28,\ d = 1.3$, and the parameters of Lü hyperchaotic system (9) as $a = 36, b = 3, c = 20$ and $d = 1.3$, so that each of them exhibits a hyperchaotic behavior. The initial values of the drive and response systems are $x_1(0) = -0.1, y_1(0) = 0.2, z_1(0) = -0.6, w_1(0) = 0.4$ and $x_2(0) = -1, y_2(0) = 0.4, z_2(0) = -0.2, w_2(0) = 1$, respectively, and the initial states of the error system are $e_1(0) = -1.1, e_2(0) = 0.6, e_3(0) = -0.8, e_4(0) = 1.4$. The diagram of the hyperchaotic Lü system is controlled to be hyperchaotic Lorenz is shown in Figure 5(a)–(d).

7 Conclusions

This study demonstrated that anti-synchronization can coexist in two different hyperchaotic systems ratchets moving in different asymmetric potentials by active control method. Several numerical simulations were provided to illustrate the anti-synchronization approach.

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