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A Novel Hyperchaotic System and Its Control

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Abstract

In this paper, a new hyperchaotic system is constructed via state feedback control. Some basic dynamical properties are studied, such as continuous spectrum, Lyapunov exponents, fractal dimensions, strange attractor and bifurcation diagram of the new hyperchaotic system. In addition, adaptive controllers are designed for stabilizing hyperchaos to unstable equilibrium with the unknown parameters. Numerical simulations are given to illustrate and verify the results.

Keywords: hyperchaos, dynamical behaviors, Lyapunov exponents, bifurcation, adaptive control, Lyapunov stability theory

1 Introduction

In 1963, Lorenz discovered the first chaotic system when he studied atmospheric convection [1]. Since then, the Lorenz system has been extensively studied in the field of chaos theory and dynamical systems. In 1999, Chen and Ueta found the Chen system, which is the dual system of Lorenz system via chaotification approach [2]. In 2002, Lü and Chen found a new chaotic system [3], bearing the name of the Lü system. In the same year, Lü *et al.* unified above the three chaotic systems into one chaotic system which is called unified chaotic system [4].

In recent years, hyperchaos generation and control have been extensively studied due to its theoretical and practical applications in the fields of communications, laser, neural work, nonlinear circuit, mathematics, and so on, [5-19]. Hyperchaotic system, possessing more than one positive Lyapunov exponent, has more complex behavior and abundant dynamics than chaotic system. Historically, hyperchaos was firstly reported by Rössler. That is, the noted four-dimensional (4D) hyperchaotic Rössler system [5], which exhibits complex and abundant hyperchaotic dynamics behaviors according to the detailed numerical and theoretical analyses. Very recently, hyperchaos was found numerically and experimentally by adding a simple state feedback controller [12-19].

In this paper, a new hyperchaotic system is constructed, and stabilization of the hyperchaotic system is achieved. We have briefly studied and analyzed its some basic dynamical properties and behaviors. Further, the new hyperchaotic system is suppressed via adaptive control theory.

This paper is organized as follows: In Section 2, a new hyperchaotic system is constructed based on a three-dimensional system by introducing a nonlinear state feedback controller. In Section 3, basic properties and behaviors are investigated numerically and analytically. In Section 4, simple but effective controllers are designed for stabilizing the hyperchaotic system to unstable equilibrium. The final section summarizes this work.

2 Construction of the New Hyperchaotic System

Recently, Liu *et al.* constructed a three-dimensional autonomous chaotic system [20], which has a new reversed butterfly-shaped attractor. The system is described by

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$$\begin{cases} \dot{x} = a(y - x) \\ \dot{y} = bx + kxz \\ \dot{z} = -cz - hxy, \end{cases} \tag{1}$$

in which *a*, *b*, *c*, *h* and *k* are positive real constants. When *a*=10, *b*=40, *c*=2.5, *h*=1 and *k*=16, the system (1) has a reversed butterfly-shaped attractor shown in Fig.1.

To generate hyperchaos from system (1), one needs to extend the dimension of system (1). Moreover, the improved system has to satisfy some requirements. Here, two basic requirements are listed as follows:

(i) Hyperchaos exists only in higher-dimensional systems, i. e., not less than four-dimensional (4D) autonomous system for the continuous time cases.

(ii) It was suggested that the number of terms in the coupled equations that give rise to instability should be at least two, in which one should be a nonlinear function.

On the basis of the system (1), a new hyperchaotic system can be generated by adding an additional stat variable *u* to it. Then, we can get the following four-dimensional autonomous system

$$\begin{cases} \dot{x} = a(y - x) + u \\ \dot{y} = bx + kxz \\ \dot{z} = -cz - hxy \\ \dot{u} = xz - dy, \end{cases} \tag{2}$$

where *a*, *b*, *c*, *h*, *k* and *d* are also positive real parameters.

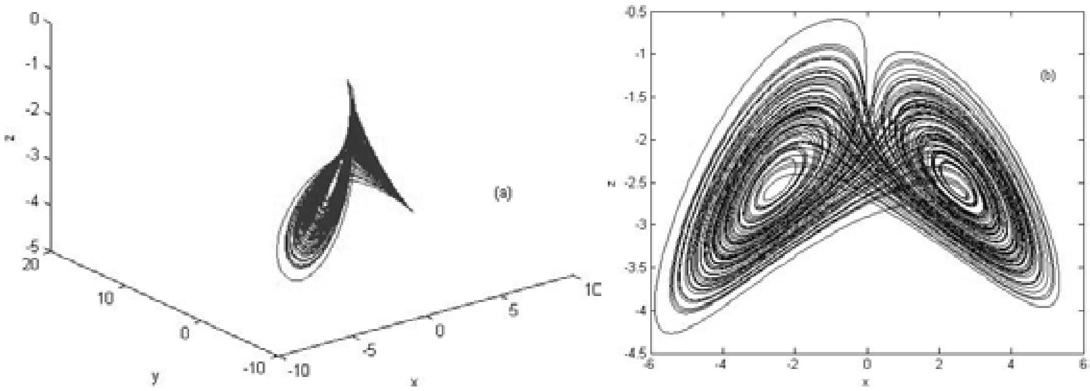
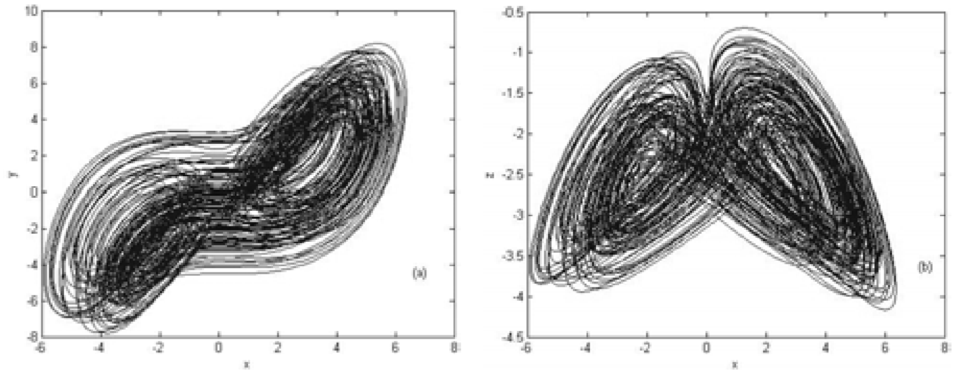


Fig.1: (a) Three-dimensional view *x*-*y*-*z* space (b) *x*-*z* phase plane strange attractor

Let the parameters be *a*=10, *b*=40, *c*=2.5, *h*=1, *k*=16 and *d*=2, thus, the new 4D system (2) is hyperchaotic in this case. Its attractors are shown in Fig. 2. We will reveal the hyperchaotic dynamical properties and behaviors of this four-dimensional system.



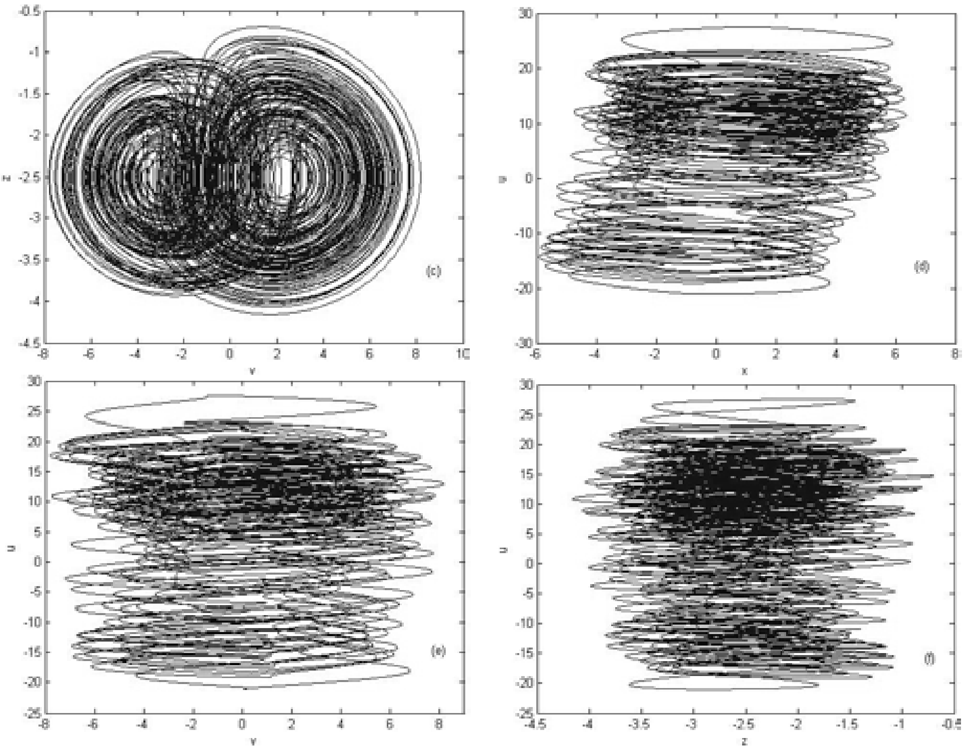


Fig.2: Phase portraits of hyperchaotic system (2), with a=10, b=40, c=2.5, h=1, k=16 and d=2: (a) x-y plan; (b) x-z plan; (c) y-z plan; (d) x-u plan; (e) y-u plan; (f) z-u plan.

3 Properties and Dynamical Behaviors Analysis of Hyperchaotic System (2)

In this section, basic properties and complex dynamics of the new system (2) are invested, such as continuous spectrum, Lyapunov exponents, fractal dimensions, strange attractors and bifurcation diagram. The new hyperchaotic system (2) has the following basic properties.

1) Symmetry and invariance

Note that the invariance of the system (2) under the transformation (x, y, z, u)→(-x, -y, z, u), i.e. under reflection in the z-axis. The symmetry persists for all values of the system parameters.

2) Dissipativity and the existence of attractor

For system (2), one has

$$\nabla V = \frac{\partial \dot{x}}{\partial x} + \frac{\partial \dot{y}}{\partial y} + \frac{\partial \dot{z}}{\partial z} + \frac{\partial \dot{u}}{\partial u} = -a - c = -12.5 \,.$$

So system (2) is dissipative, with an exponential contraction rate: $dv/dt=e^{-12.5}$. That is a volume element V_0 is contracted by the flow into a volume element $V_0e^{-12.5t}$ in time t. This means that each volume containing the system trajectory shrinks to zero as $t\to\infty$ at an exponential rate -12.5. In fact, numerical simulations have shown that system orbits are ultimately confined into a specific limit set of zero volume, and the system asymptotic motion settles onto an attractor.

3) Equilibrium and stability

The equilibrium of system (2) can be easily found by solving the four equations $\dot{x} = \dot{y} = \dot{z} = \dot{u} = 0$, which lead to $a(y-x)+u=0, bx+kxz=0, -cz-hxy=0$, and $xz-dy=0$. It can be easily verified that there is only one equilibrium E (0, 0, 0, 0), because the parameters are all positive constants.

For equilibrium E (0, 0, 0, 0), system (2) is linearized, the Jacobian matrix is defined as

$$J_0 = \begin{pmatrix} -a & a & 0 & 1 \\ b+kz & 0 & kx & 0 \\ -hy & -hx & -c & 0 \\ z & -d & x & 0 \end{pmatrix} = \begin{pmatrix} -10 & 10 & 0 & 1 \\ 40 & 0 & 0 & 0 \\ 0 & 0 & -2.5 & 0 \\ 0 & -2 & 0 & 0 \end{pmatrix}$$

To gain its eigenvalues, let $\left|\lambda I - J_0\right| = 0$.

These eigenvalues corresponding to equilibrium $E\,(0, 0, 0, 0)$ are respectively obtained as follows:

$$\lambda_1 = -25.6909,\ \lambda_2 = 15.4899,\ \lambda_3 = -2.5 \text{ and } \lambda_4 = 0.201.$$

Here λ_2 and λ_4 are two positive real number, λ_1 and λ_3 are two negative real numbers. Therefore, the equilibrium E (0, 0, 0, 0) is a saddle point, which is unstable.

4) Lyapunov exponents and Lyapunov dimension

In order to discover the effective of the parameters on the dynamics of the new 4D system (2), we fix the parameters a=10, b=40, c=2.5, h=1 and k=16, let the parameter d vary in the interval (0,12]. Given the initial condition (0.1, 0.1, 0.1, 0.1), according to the detailed numerical as well as theoretical analysis [21-22], the Lyapunov exponent spectrum are proposed to show how the system (2) changes with in creasing value of parameter d shown in Fig. 3. And the simulation results are obtained by using 4-order Runge-Kutta method with the step length taken as 0.001.

Suppose that LE_i (i=1, 2, 3, 4) are the Lyapunov exponents of the dynamical system, satisfying the condition $LE_4 \le LE_3 \le LE_2 \le LE_1$. From Fig. 3, the abundant dynamics that the system (2) undergoes versus the parameter d are summarized as follows.

When $d \in (0, 0.79]$, the largest Lyapunov exponent is positive, implying the system shows chaotic behavior. Fig.5. displays chaotic attractors for different values of parameter d respectively.

When $d \in [0.80, 12]$, the system (3) has two positive Lyapunov exponents except a very narrow periodic window near 6.81 (shown in Fig.6), representing that the hyperchaos occurs. See Fig.2.

Now, we consider properties of the system (2) when a=10, b=40, c=2.5, h=1, k=16 and d=2. From Fig.4, we know the two largest values of positive Lyapunov exponents of nonlinear system are obtained as λ_{L1}=1.0088 and λ_{L2}=0.1063. It is related to the expanding nature of different direction in phase space. Another one Lyapunov exponent λ_{L3}=0.0038. It is related to the critical nature between the expanding and the contracting nature of different direction in phase space.

While negative Lyapunov exponent λ_{L4}=-13.6191. It is related to the contracting nature of different direction in phase space.

So, we can obtain the Lyapunov dimension of the new hyperchaos attractors of system (2), it is described as

$$D_L = j + \frac{1}{\left|\lambda_{Lj+1}\right|}\sum_{i=1}^{j}\lambda_{Li} = 3 + \frac{\lambda_{L1}+\lambda_{L2}+\lambda_{L3}}{\left|\lambda_{L4}\right|} = 3 + \frac{1.0088+0.1063+0.0038}{\left|-13.6191\right|} = 3.0822.$$

The Lyapunov dimension of the new system (2) is also fractional dimension, therefore, there is really hyperchaos in this system. The hyperchaotic strange attractors are shown in Fig.2.

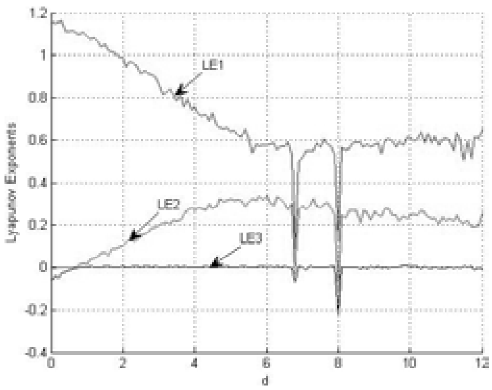


Fig.3: Lyapunov exponents spectrum of the system (2) versus parameter d.

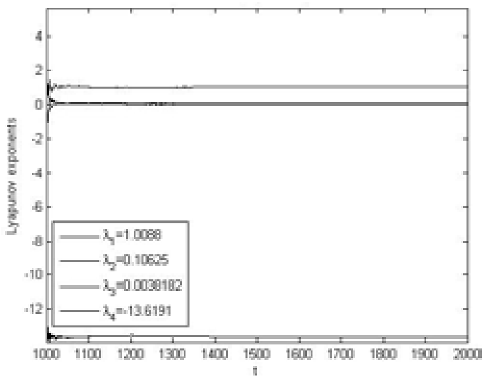


Fig.4: Lyapunov exponents of ystem (2) when d=2

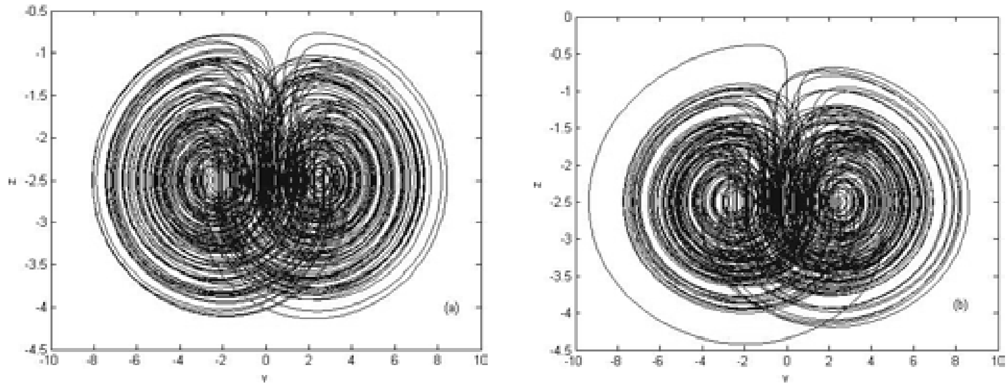


Fig.5: Chaos phase portraits of the system (2) in the *y*-*z* plane for different *d*: (a) *d*=0.05; (*b*) *d*=0.5

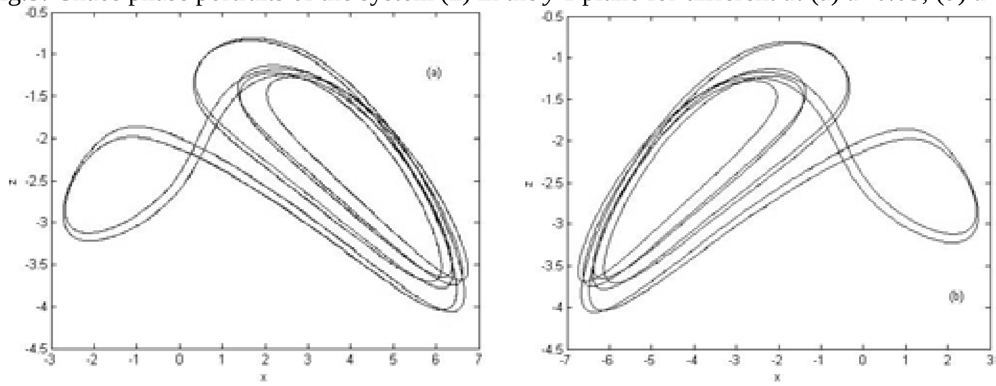


Fig.6: Periodic phase portraits of the system (2) in the *x*-*z* plane for different *d*:(a) *d*=6.81; (b) *d*=6.8105.

5) Waveform, Spectrum, Poincaré mapping and Bifurcation diagram

According to the above analyses, obviously, they are also new reverse butterfly-shape chaotic attractors, while the sensitive dependence on the initial conditions is a prominent characteristic of chaotic behavior: when the initial values are changed, the chaotic dynamical behavior of this system disappears immediately.

The waveforms of x(t) in time domain are shown in Fig.7, apparently, they are non-periodic in system (2), which is one of basic chaotic dynamical properties.

The spectrum of system (2) is also studied, and its spectrum is continuous as shown in Fig.8.

The Poincaré mapping of system (2) is also analyzed. It is clear that the Poincaré mappings are these points in confusion as shown in Fig.9.

The bifurcation diagram of x with increasing a is given in Fig. 10, and it shows abundant and complex dynamical behaviors.

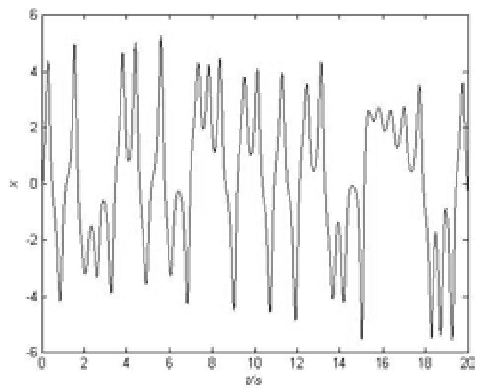


Fig.7: x(t) waveform of system (2)

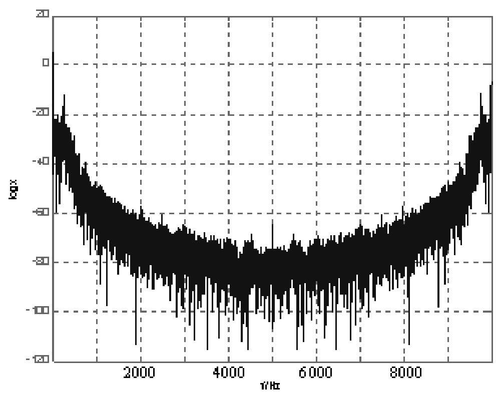


Fig. 8: Spectrum of $|x|$ in system (2)

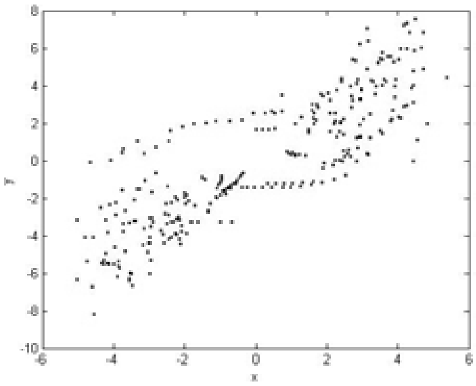


Fig. 9: The Poincaré map of *x*-*y* plane of the system (2)

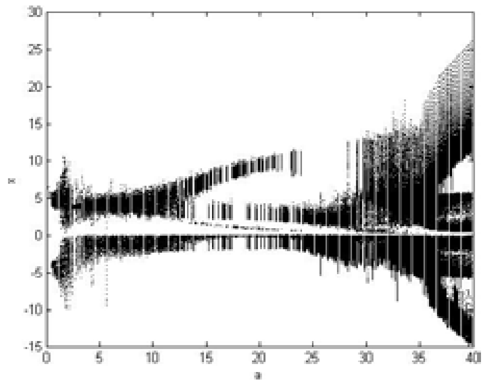


Fig.10: Bifurcation diagram for increasing *a* with *b*=40, *c*=2.5, *d*=2, *h*=1 and *k*=16 of system (2)

The above theoretical analysis and numerical simulation both show that system (2) is really a new hyperchaotic system and has more sophisticated topological structure and abundant hyperchaotic dynamical properties.

4 Stabilizing the Unstable Equilibrium E_0 (0, 0, 0, 0)

In this section, by using adaptive control theory the hyperchaos in dynamical system (2) is controlled to unstable equilibrium in the presence of unknown system parameters. For this purpose, let us assume that the controlled system is as follows:

$$\begin{cases} \dot{x} = a(y-x)+u+\mu_1 \\ \dot{y} = bx+kxz+\mu_2 \\ \dot{z} = -cz-hxy+\mu_3 \\ \dot{u} = xz-dy+\mu_4, \end{cases} \tag{3}$$

in which *a*, *b*, *c*, *k*, *d*, and *h* are unknown parameters, $\mu_1, \mu_2, \mu_3, \mu_4$ are the controllers to be designed.

Choose the controllers $\mu_1, \mu_2, \mu_3, \mu_4$ as follows:

$$\begin{cases} \mu_1 = (\hat{a}-1)x-\hat{a}y-u \\ \mu_2 = -\hat{b}x-\hat{k}xz-y \\ \mu_3 = (\hat{c}-1)z+\hat{h}xy \\ \mu_4 = \hat{d}y-xz-u, \end{cases} \tag{4}$$

and the parameters estimation update law as follows:

$$\begin{cases} \dot{\hat{a}} = xy-x^2 \\ \dot{\hat{b}} = xy \\ \dot{\hat{c}} = -z^2 \\ \dot{\hat{d}} = -yu \\ \dot{\hat{h}} = -xyz \\ \dot{\hat{k}} = xyz. \end{cases} \tag{5}$$

We choose Lyapunov function for (2) as follows:

$$V = \frac{1}{2}(x^2 + y^2 + z^2 + w^2 + \tilde{a}^2 + \tilde{b}^2 + \tilde{c}^2 + \tilde{d}^2 + \tilde{h}^2 + \tilde{k}^2),$$

where $\tilde{a} = \hat{a} - a, \tilde{b} = \hat{b} - b, \tilde{c} = \hat{c} - c, \tilde{d} = \hat{d} - d, \tilde{h} = \hat{h} - h$ and $\tilde{k} = \hat{k} - k$, $\hat{a}, \hat{b}, \hat{c}, \hat{d}, \hat{h}, \hat{k}$ are the estimate values of these unknown parameters, respectively.

The time derivate of *V* along trajectories (3) is

$$\begin{aligned} \dot{V} &= x(a(y-x)+u+\mu_1)+y(bx+kxz+\mu_2)+z(-cz-hxy+\mu_3) \\ &\quad + w(xz-dy+\mu_4)+\tilde{a}\dot{\tilde{a}}+\tilde{b}\dot{\tilde{b}}+\tilde{c}\dot{\tilde{c}}+\tilde{d}\dot{\tilde{d}}+\tilde{h}\dot{\tilde{h}}+\tilde{k}\dot{\tilde{k}} \end{aligned} \tag{6}$$

Substituting Eqs. (4) and (5) into Eq. (6) yields

$$\dot{V} = -x^2 - y^2 - z^2 - w^2 < 0 .$$

It is clear that *V* is positive definite and $\dot{V}$ is a negative definite in the neighborhood of the zero solution for the system (2); Therefore, based on the Lyapunov stability theory, the controlled system (3) can asymptotically converge to the unstable equilibrium E_0 (0, 0, 0, 0) with the controllers (4) and the parameters estimation update law (5). Fig.10 shows the time responses of the four states of the controlled system (3). The controllers ($\mu_1, \mu_2, \mu_3, \mu_4$) action is shown in Fig. 11. We can see the system is driven to the origin immediately.

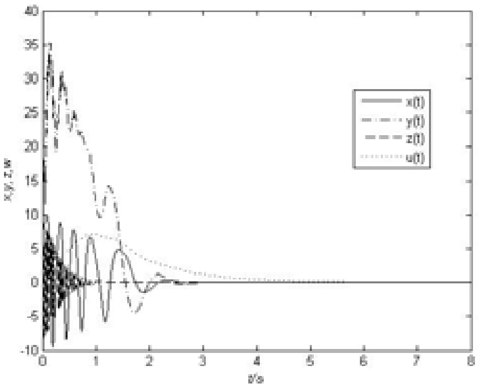


Fig.11: The time response of states(*x*, *y*, *z*, *w*): stabilizing the equilibrium E_0.

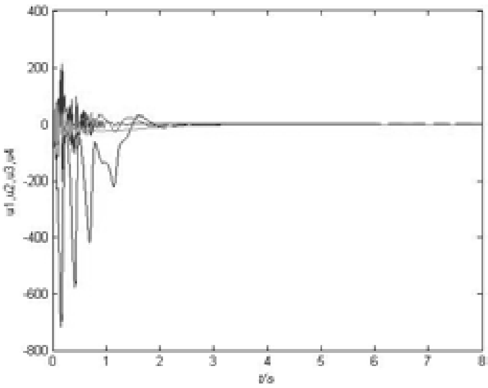


Fig.12. Control actions $\mu_1(t), \mu_2(t), \mu_3(t), \mu_4(t)$

5 Conclusion

In this paper, a new hyperchaotic system is introduced. There are abundant and complex dynamical behaviors in new hyperchaotic system (2). This new hyperchaotic attractor is different from the hyperchaotic Lorenz attractor, hyperchaotic Chen attractor as well as hyperchaotic system proposed by Liu Congxin and Wang Faqiang, etc. Further, adaptive controller has been derived for controlling hyperchaos to unstable equilibrium point of the uncontrolled system with unknown parameters. These new hyperchaotic attractors and their forming mechanism need further study and exploration. A great deal of achievements will be obtained in the near future.

Acknowledgements

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