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Designing a Supply Chain Network Model with Uncertain Demands and Lead Times

Mohammad Saeed Jabal Ameli[1], Nader Azad [1,*], Amir Rastpour[2]
[1]*Department of Industrial Engineering, Iran University of Science and Technology*
P.C.16846113114, Tehran, Iran
[2]*Industrial Engineering group, University of Tabriz, P.C. 5166616471, Tabriz, Iran*

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Abstract

In this paper, we present an integrated stochastic supply chain design model that optimizes location, inventory and transportation decisions, simultaneously. Customers' demands and distribution centers' lead times are generated randomly and the objective is to minimize the location costs as well as inventory costs at distribution centers, and transportation costs in the supply chain. We assume each distribution center maintains a certain amount of safety stock in order to achieve a certain service level for the customers it services. We show that this problem can be formulated as a non-linear integer-programming model. A Tabu search based procedure is developed for solving the problem. We comprise the Tabu search solution with the optimal solution, and a simulated annealing algorithm. The results indicate that the method is efficient for a wide variety of problem sizes.

Keywords: facility location, inventory, integrated supply chain design, uncertain demand, uncertain lead-time

1 Introduction

All companies that aim to be competitive on the market have to pay attention to their organizations related to the entire supply chain. In particular, companies have to analyze the supply chain in order to improve the customer's service level without an uncontrolled growth of cost. In few words, companies have to increase the efficiency of their logistics operations.

Facility location problems (FLP), which are typically used to design distribution networks, involve determining the sites to install resources, as well as the assignment of potential consumers to those resources. One example of FLP, is the location of manufacturing plants, the assignment of warehouses to these plants and finally the assignment of retailers to each warehouse.

For a thorough review of facility location problem see Hamacher and Drezner [6], Barmel and Simchi-Levi [1], and Daskin [2].

There are some papers in the literature, which consider the facility location problem in distribution network design. Klose and Drexl [8] propose a classification for distribution network design problem. Geoffrion and Graves [5] propose a Bender's decomposition approach to solve a capacitated single source, multi-commodity network flow problem. Tragantalerngsak *et al.* [17] considered a two-echelon facility location problem in which the facilities in the first echelon are uncapacitated and the facilities in the second echelon are capacitated. Melo *et al.* [10] present a dynamic multi-commodity capacitated distribution network design problem but they did not propose a solution algorithm for solving the problem. Lu and Bostel [9] present a facility location model for reveres logistics system and propose an algorithm based on lagrangian relaxation for solving the model.

Recently some authors have incorporated inventory control decisions into the facility location problem, and they show that cost savings can be obtained by considering location and inventory decisions simultaneously. Simchi-Levi [16] considers a hierarchical planning model for stochastic distribution system, in which the locations and demand of customers are determined according to some probability distribution. Jayaraman [7] incorporates the inventory costs into a facility location problem, assuming fixed lot sizes and deterministic demands. Nozick and Turnquist [12]

* Corresponding author. Email: n.azad@aut.ac.ir, ndr_azad@yahoo.com (N. Azad).

incorporate inventory costs assuming the demands to arrive in a Poisson manner and a base stock inventory policy. Shen [13], studied the joint location-inventory model in which location, shipment and non-linear safety stock inventory costs are included in the same model. Erlebacher and Meller [4] formulate a non-linear integer location-inventory model. They use a continuous approximation for solving the problem. Daskin *et al.* [3] apply lagrangian relaxation to solve the location-inventory model. Shen *et al.* [15] present a location-inventory model that is similar to the model of Shen [13] and use column generation for solving the problem. Miranda and Garrido [11] present a location-inventory model that is similar to the model of Daskin *et al.* [3], and they apply lagrangian relaxation method for solving the model. Shen [14] propose a multi-commodity location-inventory model and use column generation for solving the problem.

As we mentioned in the previous, significant cost savings can be obtained by considering location and inventory decisions simultaneously instead of the sequential approach, where location decisions are made before the inventory decisions. In addition, the demands and lead times in the supply chain system are usually uncertain. In this paper, we present an integrated stochastic supply chain design model that optimizes location, inventory and transportation decisions simultaneously. We assume each customer has uncertain demand that follows a normal distribution, and we assume each distribution center has uncertain lead-time that follows a normal distribution. The goal of our model is to choose a set of distribution centers to serve the customers and allocate customers to each open distribution center, and to determine the inventory policy based on the information of customers demands in order to minimize the total expected cost of locating distribution centers, shipment, and inventory costs.

A Tabu search based procedure is used for solving the problem. The reminder of this paper is organized as follows. In section 2, mathematical formulation of the problem and analysis of the model are presented. Section 3, discusses the solution approach for solving the problem. Section 4, discusses some computational results. Finally, section 5 contains some conclusions of the paper.

2 Problem Description & Formulation

In our model, we assume that each customer has uncertain demand that follows a normal probability distribution, and customers' demands are independent. In addition, we assume that each distribution center has uncertain lead time that follows a normal probability distribution, and distribution centers' lead times are independent. The distribution centers are assumed to follow a (Q, R) inventory policy.

Now in the following we present how the problem can be formulated as a mixed integer nonlinear program in Section 2.1, then, in Section 2.2 we present some analysis for solving and simplifying the model.

2.1 Formulation

Before presenting the model, let us introduce the notations that will be used throughout the paper:

Index sets:

K: Set of customers;

J: Set of potential distribution centers.

Parameters and notations:

μ_k: Mean daily demand at customer k ($\forall k \in K$);

σ_k^2: Variance of daily demand at customer k ($\forall k \in K$);

F_j: Fixed cost for opening and operating distribution center j ($\forall j \in J$);

b_j: Capacity for the potential distribution center j ($\forall j \in J$);

h_j: Inventory holding cost per unit of product per year at distribution center j ($\forall j \in J$);

p_j: Fixed cost per order placed to the supplier by distribution center j ($\forall j \in J$);

L_j: Mean lead time for distribution center j in days, ($\forall j \in J$);

S_j^2: Variance lead time for distribution center j in days ($\forall j \in J$);

g_j: Fixed cost per shipment from the supplier to distribution center j ($\forall j \in J$);

a_j: Cost per unit of a shipment from the supplier to potential distribution center j ($\forall j \in J$);

c_{jk}: Cost per unit of shipment from distribution center j to customer k ($\forall j \in J$, $\forall k \in K$);

α : Desired percentage of customer orders satisfied (fill rate);
z_α : α -percentile of the standard normal random variable *Z*, i.e. $p(Z \le z_\alpha) = \alpha$;
χ : Days per year.

Decision variables:

$$Y_{jk} = \begin{cases} 1 & \text{if customer } k \text{ is assigned to distribution center } j \\ 0 & \text{otherwise} \end{cases} \quad (\forall k \in K, \forall j \in J);$$

$$U_j = \begin{cases} 1 & \text{if distribution center } j \text{ is opened} \\ 0 & \text{otherwise} \end{cases} \quad (\forall j \in J);$$

Q_j : Order size at distribution center j ($\forall j \in J$).

Objective function:

The objective function minimizes the following costs:

1. The fixed cost of locating distribution centers, given by term $\sum_{j\in J} F_j U_j$.
2. The annual shipment cost from distribution centers to the customers, given by term $\sum_{j\in J}\sum_{k\in K} \chi c_{jk}\, \mu_k\, Y_{jk}$.
3. The expected annual inventory cost: is the sum of working inventory and safety stock costs. Working inventory representing product that has been ordered from the supplier but not requested by the customers and safety stock cost inventory intended to buffer the system against stock outs during ordering lead times. Working inventory cost includes the fixed costs of placing orders as well as the shipment costs from the supplier to the distribution centers as well a the holding costs of working inventory. Let D_j denote the total annual demand going through distribution center *j* ($D_j = \sum_k \chi \mu_k Y_{jk}$). Then the total annual cost of ordering inventory from the supplier to distribution center *j* is given by

$$p_j \frac{D_j}{Q_j} + \left(g_j + a_j Q_j\right)\frac{D_j}{Q_j} + h_j \frac{Q_j}{2}.$$

The first term, represents the total fixed cost of placing orders per year. The second term represents the delivery cost from the supplier to distribution center *j* per year, and the third term is the cost of average working inventory.

We assume that cost of shipping an order of size *M* form the supplier to distribution center *j* is given by the term of $(g_j + a_j \times M)$.

The yearly safety stock cost at distribution center *j* is given by $h_j\, z_\alpha\, \sigma_j$ and $\sigma_j = \sqrt{\sum_{k\in K}\left(L_j \sigma_k^2 + \mu_k^2 S_j^2\right) Y_{jk}}$.

The safety stock at distribution center *j* is $h_j\, z_\alpha \sqrt{\sum_{k\in K}\left(L_j \sigma_k^2 + \mu_k^2 S_j^2\right) Y_{jk}}$

The problem formulating is as follows:

$$\begin{aligned} \min : & \sum_{j\in J} F_j U_j + \sum_{j\in J}\sum_{k\in K} \chi c_{jk}\, \mu_k\, Y_{jk} \\ & + \sum_{j\in J}\left[\left(p_j + g_j\right)\frac{\sum_{k\in K}\chi\mu_k Y_{jk}}{Q_j} + a_j \sum_{k\in K}\chi\mu_k Y_{jk} + \frac{h_j Q_j}{2} + h_j\, z_\alpha \sqrt{\sum_{k\in K}\left(L_j\sigma_k^2 + \mu_k^2 S_j^2\right)Y_{jk}} \right] \end{aligned} \tag{1}$$

Subject to:

$$\sum_{j\in J} Y_{jk} = 1, \forall k \in K \tag{2}$$

$$\sum_{k\in K} \chi\, \mu_k Y_{jk} \le b_j U_j, \forall j \in J \tag{3}$$

$$Y_{jk} \in \{0,1\}, U_j \in \{0,1\}, \forall j \in J, \forall k \in \mathrm{K} \tag{4}$$

$$Q_j \ge 0, \forall j \in J. \tag{5}$$

The model minimizes the total expected costs made of: the fixed cost for opening distribution centers, the annual shipment cost from distribution centers to the customers, and the expected annual inventory cost. Constraints (2) ensure that each customer is assigned to exactly one distribution center. Constraints (3) are the capacity constraints associated with the distribution centers. Constraints (4) enforce the integrality restrictions on the binary variables and finally constraints (5) enforce the non-negative restrictions on the corresponding decision variable.

2.2 Analysis of the Model

In the above problem, the decision variable Q_j, only has appeared in the objective function and was not used in the constraints. Therefore, we can take the derivative of the objective function with respect to Q_j, and so we obtain the optimal value of Q_j. By taking the derivative of objective function with respect to Q_j, we obtain the optimal value of Q_j is given by:

$$Q_j = \sqrt{\frac{2\left(p_j + g_j\right)\sum_{k\in K}\chi\mu_k Y_{jk}}{h_j}}. \tag{6}$$

By substituting (6) into the objective function (1), the problem can be formulated as follows:

$$\begin{aligned} \min: & \sum_{j\in J} F_j U_j + \sum_{j\in J}\sum_{k\in K} \chi c_{jk}\, \mu_k\, Y_{jk} \\ & + \sum_{j\in J}\left[\sqrt{2\, h_j\, (P_j + g_j)\sum_{k\in K}\chi\mu_k Y_{jk}} + a_j \sum_{k\in K}\chi\mu_k Y_{jk} + h_j\, z_\alpha \sqrt{\sum_{k\in K}\left(L_j\sigma_k^2 + \mu_k^2 S_j^2\right)Y_{jk}}\right] \end{aligned} \tag{7}$$

Subject to: (2)-(4).

Therefore, we can eliminate the Q_j form this model.

3 Solution Approach

In this section, we propose two approaches for solving the problem: optimal solution and a heuristic method.

3.1 Finding the Optimal Solution

The two non-linear terms of the objective function (7) (the annual working inventory cost and the holding cost for safety stock) are the concave terms, and the other terms of the objective function are linear, so the objective function (7) is absolutely concave and the problem we studied in the previous section is a concave integer programming model. Therefore, if we solve the model with branch and bound methods, the resulting solutions are global optima.

3.2 Use of Tabu Search Algorithm for Solving the Problem

The facility location problem has been shown to be non-polynomial (NP)-hard problem. Since this problem is more complex than facility location problem, so this problem belongs to the class of NP-hard problems. Therefore, we use Tabu search algorithm for solving the problem.

The overall approach in Tabu search algorithm is to avoid entrainment in cycles by forbidding or penalizing moves, which take the solution, in the next iteration, to points in the solution space, previously visited. Tabu search uses a local or neighborhood search procedure to iteratively move from a solution X to a solution X_n in the neighborhood of X, until some stopping criterion has been satisfied. To explore regions of the search space that would be left unexplored by the local search procedure, tabu search modifies the neighborhood structure of each solution as the search progresses. The solutions admitted to $N^*(X)$, the new neighborhood, are determined using special memory structures. The search then progresses by iteratively moving from a solution X to a solution X_n in $N^*(X)$.

Perhaps the most important type of short-term memory to determine the solutions in $N^*(X)$; also, the one that gives its name to tabu search, is the use of a tabu list. Tabu list contains the solutions that have been visited in the recent past (less than *m* moves ago, where *m* is the lenth of the tabu list).

The steps of tabu search are shown in Fig.1

The Tabu search parameters are as follows:

G: Number of accepted solutions;
r: Counter for the number of accepted solutions;
m: Lenth of the tabu list;
X: A feasible solution;
$C(X)$: The value of objective function for *X*.

$X_{best} = \phi$
Select an initial solution X_0
$X_{best} = X_0$, $X = X_0$
$r = 0$
While $(r < G)$ Do
Generate solution X_n in the neighborhood of X,
$\Delta C = C(X_n) - C(X)$
If the candidate move is in the tabu list, then
If $C(X_n) < C(X_{best})$ then
$X = X_n$, $r = r + 1$
$X_{best} = X_n$, update the tabu list
Else
Generate another solution X_n in the neighborhood of X
End If
Else
If $\Delta C \le 0$ then
$X = X_n$, $r = r + 1$
Update the tabu list
If $C(X_n) < C(X_{best})$ then
$X_{best} = X_n$
End If
Else
Generate another solution X_n in the neighborhood of X
End If
End If
End While

Fig.1: Tabu search algorithm

In the following sections, we describe the Tabu search algorithm which we use for solving the problem.

3.2.1 Initial Solution Construction

For obtaining the initial solution, we assign customers to the distribution centers, randomly. The procedure for obtaining the initial solution is as follows.

Step1: Put customers into a set *K*.
Step2: 1- Select a customer from *K* randomly. 2- Delete the customer from *K*.
Step3: Select a distribution center randomly.
Step4: If remaining capacity of the distribution center is greater than the demand of the customer then assign the customer to the distribution center and go to Step 5 otherwise go to Step 3 for selecting another distribution center.
Step5: Is *K* empty? If yes, stop, otherwise go to Step 2.

3.2.2 Improving the Initial Solution

In this phase, the main objective is to improve the initial solution. We apply four different types of move for generating a candidate move: mov1, mov2, mov3, mov4. We generate a candidate move (from X_0 to the candidate solution X_n) using one of the four moves randomly.

Mov1: Randomly, one of the opened distribution centers (a_j) is closed and all of the customers are reallocated among the remaining opened distribution centers. The procedure of mov1 is as follows:

Step1: Select an open distribution center randomly (a_j). Let D_j be the set of customers that assigned to a_j.

Step2: Select a customer (*k*) from D_j, randomly.

Step3: Randomly select one of the opened distribution centers that have enough remaining capacity for demand of customer *k*, and assign customer *k* to this distribution center.

Step4: Delete the customer *k* from D_j.

Step5: Is D_j empty? If yes, close a_j and stop, otherwise go to Step 2.

Mov2: In this move we select two open distribution centers randomly, (a_i , a_j), and exchange a_i and a_j. In this move capacities of a_i and a_j are checked for serving the customers.

Mov3: One of the opened distribution centers (a_i) is closed randomly, and a closed distribution center (a_j) is opened randomly Then we assign all of the customers corresponding to the eliminated distribution center (a_i) to the new opened distribution center (a_j). In this move the capacity of a_j is checked for serving the customers.

Mov4: Select two open distribution centers, randomly, (v_i , v_j). Then randomly select a customer (c_i) in v_i and a customer (c_j) in v_j and exchange c_i and c_j. In this move, we must check the capacities of distribution centers.

When the candidate move is generated, this feasible solution is applied to the Tabu search algorithm procedure. This process is repeated until $r < G$.

4 Computational Results

The computational experiments described in this section were designed to evaluate the performance of our overall solution procedure with respect to a series of test problems.

It was coded in visual basic 6 and run on a Pentium 4 with 3 GB processor. The daily demand of the customers was drawn from a uniform distribution between 3 and 7. The variances of daily demands of customers were drawn from a uniform distribution between 1 and 3. In addition, we use the following parameter values:

h_j is uniformly drawn from [2, 4];

p_j is uniformly drawn from [15, 20];

L_j is uniformly drawn from [6, 10];

S_j^2 is uniformly drawn from [1, 3];

g_j is uniformly drawn from [15, 20];

a_j is uniformly drawn from [2, 5];

c_{jk} is uniformly drawn from [2, 5];

χ = 250, z_α = 1.96 (97.5% service level).

4.1 Comparison of Optimal and Tabu Search Solutions

For evaluating the proposed Tabu search algorithm, fifteen problems are solved by LINGO 8 software (Table 1). According to Section 3.1, the solution, which we obtain from LINGO software, is global optima, and we can compare our heuristic solution by optimal value, which is obtained by LINGO software. For each problem, the tuning of the parameters is done by carrying out random experiments.

Table 1: Comparison of optimal and Tabu search solutions

			Optimal Value		Tabu Search Solution		
NO.	# Customers	# DCs	Cost	CPU time	Cost	CPU time	Gap(%)
1	4	2	52364.9	2	52364.9	1	0.00
2	6	3	59871.3	5	59871.3	2	0.00
3	8	4	68327.5	10	68327.5	6	0.00
4	20	6	111236.2	80	111236.2	22	0.00
5	30	8	148632.8	143	148632.8	29	0.00
6	40	12	193541.7	224	193856.7	41	0.16
7	50	15	251356.2	376	252896.2	55	0.61
8	60	17	303254.1	765	305985.6	68	0.90
9	70	19	361596.5	1253	365386.4	81	1.05
10	80	21	414313.7	2011	419359.3	96	1.22
11	90	23	455827.3	3363	461241.5	113	1.19
12	100	25	511538.2	5127	518061.9	130	1.28
13	120	30	603791.4	10657	611718.8	164	1.31
14	150	35	769478.2	3 hours limit	761293.4	213	
15	180	38	927695.8	3 hours limit	904243.6	262	

Gap: Gap from optimal solution (%).

It can be seen that the solutions of the Tabu search algorithm are optimal (or near optimal) in different problems (Table 1). The average CPU time are less than or equal to 262 seconds for Tabu search algorithm (CPU times are in the seconds). However, the maximal average CPU time for obtaining the optimal solutions is equal to 10657 seconds, and for instances 14 and 15 by a 3 hours time limit, LINGO can not find the optimal solution, and the Tabu search algorithm in this instance is better than the best solutions that are obtained by LINGO.

4.2 Comparison of Tabu Search based Approach and Simulated Annealing based Approach

In this section, we compare our Tabu search method with simulated annealing (SA) method. The procedure for obtaining initial solution and candidate move, we use in SA method, are the same to the procedure of obtaining initial solution and candidate move in Tabu search algorithm. The comparison of Tabu search algorithm and SA algorithm are shown in Table 2. It can be seen that the solution quality in Tabu search algorithm is better than the solution quality in SA algorithm.

Table 2: Comparison of Tabu search algorithm and SA algorithm

			Tabu search algorithm		SA algorithm	
NO.	# Customers	# DCs	Cost	CPU Time	Cost	CPU Time
1	30	8	148632.8	29	148632.8	28
2	40	12	193856.7	41	195736.4	39
3	50	15	252896.2	55	254543.7	53
4	60	17	305985.6	68	309262.9	64
5	70	19	365386.4	81	370641.3	79
6	80	21	419359.3	96	423343.2	91
7	90	23	461241.5	113	467197.4	100
8	100	25	518061.9	130	527149.8	128
9	120	30	611718.8	164	623597.2	158
10	150	35	761293.4	213	773602.5	206
11	180	38	904243.6	262	919549.1	253
12	200	40	986213.6	294	1005062.4	283
13	230	43	1129367.2	350	1152513.6	336
14	250	45	1213483.9	391	1239314.9	374

5 Conclusions

In this paper, we have outlined an integrated stochastic supply chain design model that optimizes location, inventory and transportation decisions simultaneously. We assumed each customer has uncertain demand with a normal distribution, also, we assumed each distribution center has uncertain lead time with a normal distribution. A Tabu search based procedure was developed for solving the problem. We comprised the Tabu search based procedure with the optimal solution, and a Simulated annealing algorithm. The results of extensive computational tests indicate that the method is efficient for a wide variety of problem sizes and structures.

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