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A New Heuristic Model for Fully Fuzzy Project Scheduling

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Abstract

Resource constrained project scheduling (RCPS) problems are the most common problems that project managers in each project might face with. In addition, the dynamic conditions and variations of real world lead to uncertain data, information and knowledge. In these situations, deterministic models are inefficient, so utilizing these models leads to an increase in the risk of project. Therefore the development of a new method to deal with RCPS problems in context of uncertain data and environment is one of the most necessary needs of project managers. In this paper a novel method to RCPS problems in fuzzy environment is presented. In the proposed model, duration of each activity, resource availability and demands of each activity on resources are uncertain so ranking fuzzy numbers is used to generate priority the list. Three dimensional Guntt chart is introduced to graphically depict project scheduling results for the first time in the literature. To illustrate the algorithm developed in this paper, a numerical example will be presented

Keywords: resource constrained project scheduling, ranking fuzzy numbers, heuristic models, priority list, Guntt chart, uncertain scheduling

1 Introduction

Resource constrained project scheduling has been the subject of academic research for nearly 40 years since Kelley [13] and Wiest [24] raised this problem in 1960s and has been widely used in many areas. One of the most common goals is to minimize the project completion time under resource constraints and precedence relations. RCPS models can be categorized into two types: deterministic scheduling and nondeterministic scheduling. RCPS models presented so far, mostly focus on deterministic scheduling. Most of these models are heuristic and analytical. Comprehensive review of deterministic models can be found in [3].

Recently, project managers have paid special attention to uncertain scheduling. Because during project implementation, many uncertain variables dynamically affect project parameters such as time, resource and cost. Uncertain scheduling models are categorized into two types: probabilistic models and fuzzy models. Among of the probabilistic models are Ahuja [1] and Padilla [19].

On the other hand, it has been claimed that fuzzy set theory is more appropriate to model uncertainty that is associated with parameters such as time, resource availability. In practice, due to a sheer lack of information about activities, the values of project variables are often estimated by experts. Many of the values are defined based upon fuzzy and/or incomplete information. This type of information might be best modeled by fuzzy set techniques instead of probabilistic ones. Previous models that have been developed to solve fuzzy RCPS problems can be categorized into three types; mathematical optimization models, heuristic models and metaheuristic models. The model presented by Pan and Yeh [22] is one of the mathematical optimization models. They modeled fuzzy multi mode resource constrained project scheduling problems as a multi objective model. Lai and Li [14] and Hussein and Abosina [11]

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utilized fuzzy dynamic programming to model fuzzy RCPS problem. Fu and Wang [7] and Mjelde [18] proposed single objective and multi objective models respectively to model resource allocation problems in fuzzy environment.

Considering the order of complexity in RCPS problems, it has been proved that these problems belong to the class of the NP hard problems [12,16], so in optimization models, by increasing the number of project's activity, the required time to solve sharply increase. Therefore the heuristic and metaheuristic models can be used efficiently to tackle these problems. One of the metaheuristic models that was presented to solve fuzzy RCPS problems is Pan and Yeh [20]. They used simulated annealing algorithm to solve these problems. Also Leu *et al.* [17] and Pan and Yeh [21] solved fuzzy RCPS problems by using the genetic algorithm. Hapke *et al.* [9] proposed an algorithm based on Pareto simulated annealing (PSA) to solve fuzzy RCPS problems.

The third type of methods that have been used to solve fuzzy RCPS problems is heuristic models. In addition to the simplicity, obtaining suitable results in shorter time is also another advantage of these models in comparison with other types. By combining priority rules and ranking fuzzy numbers methods, Hapke and Slowinski [10] proposed a heuristic model to deal with fuzzy RCPS problems. In this model, Hapke *et al.* considered duration of each activity as uncertain whereas resource availability and resource demand of each activity as crisp. But in many projects in the real world, these two parameters: resource availability and resource demand of each activity are uncertain too. The research and development projects are one of the cases in which the two mentioned parameters are widely uncertain.

In this paper, a new heuristic method for fuzzy resource constrained project scheduling is developed and the three dimensional Guntt chart is introduced for the first time in the literature. The developed method here can be considered as uncertain decision making problem in fuzzy environment. For the first time in the literature this concept was introduced to model time-cost trade off problems in uncertain environment by Ghazanfari *et al.* [8].

This paper is organized as follows. The second section presents some introductions on related definitions and concepts of fuzzy sets, some operations on fuzzy numbers and ranking of fuzzy numbers. In the third section, the new heuristic model for fuzzy RCPS problems is presented. In the next section, three dimensional Guntt chart with a numerical example will be presented to describe and illustrate the efficiency of proposed model. Finally in the last section, conclusion will be remarked.

2 Basic Concepts and Definitions

A fuzzy number $\tilde{M}$ is a convex normalized fuzzy set $\tilde{M}$ of the real line R such that 1. There exists exactly one $X_0 \in R$ with $\mu_{\tilde{M}}(x_0) = 1$ (X_0 is called the mean value of $\tilde{M}$). 2. $\mu_{\tilde{M}}(x)$ is piecewise continuous.

A fuzzy number $\tilde{M}$ is of LR-type if there exist reference function L (for left), R (for right), and scalars $\alpha > 0, \beta > 0$ such that

$$\mu_{\tilde{M}}(x) = \begin{cases} L\left(\dfrac{m-x}{\alpha}\right), & \text{for} \quad x \le m \\ R\left(\dfrac{x-m}{\beta}\right), & \text{for} \quad x \ge m \end{cases} \tag{1}$$

where m, called the mean value of $\tilde{M}$, is a real number , and α and β are called the left and right spread [25], respectively.

Let $\tilde{A}$ and $\tilde{B}$ be fuzzy numbers, and * denotes any basic fuzzy arithmetic operations such as addition, subtraction and multiplication. Any operations $\tilde{A} * \tilde{B}$ can be defined as a fuzzy set on R and expressed in the following form [23]:

$$\mu_{\tilde{A}*\tilde{B}}(z) = \sup_{z=x*y}\{\min[\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(y)]\}\ . \tag{2}$$

Ranking of fuzzy numbers is another important issue in the fuzzy project scheduling. Many ranking methods for fuzzy numbers have been proposed so far. A list of these methods have presented in [4, 15]. However, there is no single approach that can produce a satisfactory result in every situation: some generate counter-intuitive results and others are not discriminative enough [2]. To overcome such problems, Cheng [5] developed a new distance approach for fuzzy number comparisons based on a calculation of the centroid point $(\bar{x}_0, \bar{y}_0)$ to obtain the distance index, where $\bar{x}_0$ and $\bar{y}_0$ are centroid values both in the horizontal and vertical axes respectively. A triangular fuzzy number $\tilde{A} = (a_1, a_2, a_3)$ can be expressed as

$$\mu_{\tilde{A}} = \begin{cases} \mu_{\tilde{A}}^L(x), & \text{for } a_1 \le x \le a_2 \\ \mu_{\tilde{A}}^R(x), & \text{for } a_2 \le x \le a_3 \end{cases} \tag{3}$$

where $\mu_{\tilde{A}}^L : [a_1, a_2] \to [0,1]$ is the strictly continuous left spread and its corresponding inverse function is denoted by $g_{\tilde{A}}^L(x)$. $\mu_{\tilde{A}}^R : [a_2, a_3] \to [0,1]$ is the strictly continuous right spread and its corresponding inverse function is symbolized by $g_{\tilde{A}}^R(x)$. All the functions can be integrable due to their continuity. Therefore, the centroid point $(\bar{x}_0, \bar{y}_0)$ of a fuzzy number $\tilde{A}$ can be defined in the following form

$$\begin{aligned} \bar{x}_0(\tilde{A}) &= \frac{\int_{a_1}^{a_2} [x\, \mu_{\tilde{A}}^L(x)]dx + \int_{a_2}^{a_3} [x\, \mu_{\tilde{A}}^R(x)]dx}{\int_{a_1}^{a_2} \mu_{\tilde{A}}^L(x)dx + \int_{a_2}^{a_3} \mu_{\tilde{A}}^R(x)dx}, \\ \bar{y}_0(\tilde{A}) &= \frac{\int_0^1 [y\, g_{\tilde{A}}^L(y)]dy + \int_0^1 [y\, g_{\tilde{A}}^R(y)]dy}{\int_0^1 g_{\tilde{A}}^L(y)dy + \int_0^1 g_{\tilde{A}}^R(y)dy}. \end{aligned} \tag{4}$$

The ranking index can be expressed as

$$R(\tilde{A}) = \sqrt{(\bar{x}_0)^2 + (\bar{y}_0)^2}\ . \tag{5}$$

For triangular fuzzy numbers, formulation (4) of calculating the centroid point can be simplified as

$$\begin{aligned} \bar{x}_0 &= \frac{a_3^2 - a_1^2 + a_2 \times a_3 - a_2 \times a_1}{3 \times (a_3 - a_1)}, \\ \bar{y}_0 &= \frac{a_1 + 4 \times a_2 + a_3}{3 \times (a_1 + 2 \times a_2 + a_3)}. \end{aligned} \tag{6}$$

For any fuzzy number $\tilde{A}_i$ and $\tilde{A}_j$, ranking fuzzy number has the following properties

$$\begin{aligned} &1)\ \text{If}\quad R(\tilde{A}_i) > R(\tilde{A}_j), \quad \text{then} \quad \tilde{A}_i > \tilde{A}_j. \\ &2)\ \text{If}\quad R(\tilde{A}_i) = R(\tilde{A}_j), \quad \text{then} \quad \tilde{A}_i = \tilde{A}_j. \\ &3)\ \text{If}\quad R(\tilde{A}_i) < (\tilde{A}_j), \quad \text{then} \quad \tilde{A}_i < \tilde{A}_j. \end{aligned} \tag{7}$$

3 New Heuristic Model for Fuzzy Resource Constrained Project Scheduling

The heuristic methods for solving RCPS problems can be generally divided into serial and parallel. The parallel approach considers in one moment all the activities which could be scheduled in that moment based upon available resources and the order in which the activities are considered for scheduling is defined by a priority list. But in the serial approach, activities based upon defined priority list, are considered sequentially to obtain a feasible solution for project [6]. Numerous heuristic methods have been proposed to generate a priority list and they can be found in [6]. Here, by combining the shortest process time (SPT) and Cheng's ranking fuzzy numbers methods a new heuristic model to solve RCPS problems in fuzzy environment is developed. The main assumptions of the proposed methods are as follows.

a) Duration of each activity is uncertain and denoted in the form of triangular fuzzy numbers.
b) The amount of resource availability is uncertain and denoted in the form of triangular fuzzy numbers.
c) The demands of each activity on resources are uncertain and denoted in the form of triangular fuzzy numbers.
d) When each activity starts, it can not be stopped until it is finished.

Proposed algorithm includes seven steps as follows.

Step 1. $\tilde{t}_0 = (0,0,0)$: $\tilde{t}_j = (j,1,1)$ **.**

Here, each point of time has been shown as symmetric triangular fuzzy numbers with 1 deviation from left and right side and its concept is "approximately jth time period". For example $\tilde{t}_3 = (3,1,1)$ shows approximately third time period of project. The start time of project is certain and can be shown as $\tilde{t}_0 = (0,0,0)$.

Step 2. Generate priority list of activities by using SPT algorithm. In other words, between activities that can be started, the activity which has the shortest duration should be done as first activity. If two activities have the same duration, activity that have lower number should be done earlier. For comparing activities' durations, Cheng's fuzzy ranking method mentioned in section 2 can be used. In this paper, activities priority list denoted by R, and S is a set of under-processing activities. It's clear that, at first $S = \varnothing$.

Step 3. $j = 0$.

Step 4. Consider activities from set R that their precedence have been completed. Add these activities to the set S based upon priority list and resources availability. Set start time of these activities equal to $\tilde{t}_j$. As mentioned in previous sections, resource availability and amount of required resource for each activity are fuzzy numbers, so, use mentioned ranking fuzzy numbers method in section 2 for comparisons.

Step 5. Let $\widetilde{Ft}_k = \tilde{t}_j + \tilde{d}_k$, $k \in S$, is k^{th} activity finishing time of set S. Compare $\widetilde{Ft}_k$ with $\tilde{t}_j$; If $\widetilde{Ft}_k < \tilde{t}_j$, then k^{th} activity is completed and this activity should be eliminated from set S and its used resources should release.

Step 6. If $R = \phi$ and $S = \phi$, then stop, and the scheduling is finished; otherwise go to step 7.

Step 7. If $j = 0$, then set $\tilde{t}_1 = (1,1,1)$; else set $\tilde{t}_j = (j+1,1,1)$ and go to step 4.

The presented algorithm, proposed for parallel scheduling and it can be obtained for serial scheduling. In the next section a numerical example is presented to describe proposed concepts and algorithm in the same way.

4 Numerical Example

In this section a numerical example is presented to clarify and illustrate proposed model. Activities network of a project is shown in Figure 1. The type of network is activity on node (AON) and the activities 1 and 6 are dummy activities. Duration and amount of resource requirement of each activity are given in Table 1.

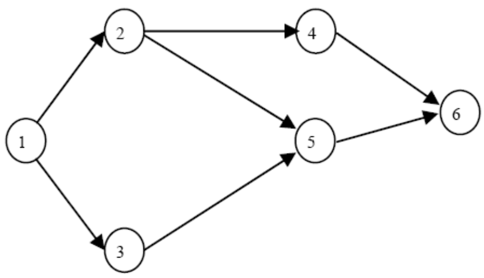

Figure 1. Precedence network diagram

For simplicity, one resource has been considered to all activities and the amount of resource availability is (6.1.1).

Table 1. Duration and resource requirement information of each activity

Activity No.	Resource requirement ($\tilde{r}_j$)	Activity duration ($\tilde{d}_j$)
2	(5,1,0)	(6,1,1)
3	(2,1,1)	(3,1,1)
4	(6,1,1)	(4,1,0)
5	(3,1,1)	(7,1,1)

By doing step 2 of the proposed algorithm, activities priority list obtained as R= {3, 2, 4, 5}.

Doing other steps of the proposed algorithm leads to scheduling presented in Table 2.

Table 2. Scheduling results

Activity No.	Activity start time ($\widetilde{St}_j$)	Activity finish time ($\widetilde{Ft}_j$)
2	(4,1,1)	(10,2,2)
3	(0,0,0)	(3,1,1)
4	(11,1,1)	(15,2,1)
5	(15,1,1)	(22,2,2)

As mentioned in abstract and introduction, three dimensional Guntt chart is introduced to graphically depict this example. This Guntt chart is as follows:

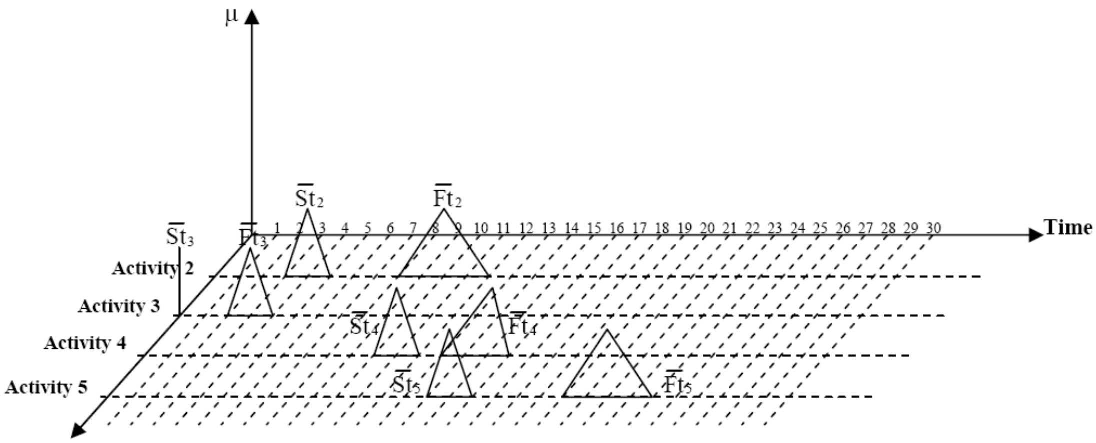

Figure 2. Three dimensional Guntt chart

Three dimensional depiction of project scheduling in fuzzy environment shows the uncertainty of decision making in the best way. It is the main advantage of the developed Gunnt chart.

5 Conclusion

Project parameters are uncertain due to the variations in the real world such as weather, resource availability. Due to variations in the real world, utilization of deterministic methods to project scheduling result in increasing risk of scheduling, therefore, uncertain project scheduling has more stability against environmental variations. In this paper, a novel heuristic method has been developed to solve resource constrained project scheduling problem in which duration of each activity, the resource availability and the demand of each activity on resources are uncertain. Three dimensional Guntt charts was developed to show scheduling result. The obtained solution of proposed model can be used in optimization models as a primary solution.

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