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Projective Synchronization of a New Chaotic System

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Abstract

This paper mainly concerns projective synchronization (PS) of a new chaotic system. PS with both identical and different scaling factors between two identical chaotic systems is realized. In addition, the PS of new chaotic system with unknown parameters including the unknown coefficients of nonlinear terms is studied by using adaptive control. Numerical simulations are presented to show the effectiveness of the proposed chaos synchronization scheme.

Keywords: a new chaotic system, projective synchronization, adaptive control

1 Introduction

Since the pioneering work of Fujisaka and Yamada [1] and Pecora and Carroll [2], chaos synchronization has become an active research subject in nonlinear science because of its many potential applications in physics, secure communication, chemical reactor, biological networks, economics, and artificial neural networks. Especially, several theoretical studies and laboratory experimentations about chaos synchronization have been applied to the secure communication. Generally speaking, the idea of synchronization is to use the output of a drive system to control a response system so that the response of the latter follows the output of the drive system asymptotically. Up to now, various schemes of synchrony such as complete synchronization [3], phase synchronization [4], lag synchronization [5], and generalized synchronization [6]. have been described and studied.

In recent years, projective synchronization, which has been first reported by Mainieri and Rehacek [7] in partially linear systems and developed by many authors [8-11], is the most noticeable one. More recently, a new synchronization method called 'Modified projective synchronization' is proposed in [12] where the chaotic systems can synchronize up to a constant scaling matrix. Modified projective synchronization in two chaotic systems with unknown parameters is realized by using adaptive control [12-15]. Furthermore, the PS has been used in the research of secure communication [16] due to the unpredictability of the scaling factor.

This paper addresses projective synchronization of a new chaotic system. PS with both identical and different scaling factors between two identical chaotic systems is achieved. We also present an effective scheme for PS in two chaotic systems with uncertainties rendered by the unknown coefficients of nonlinear terms, however, the current study mainly take into account chaotic system with uncertain linear terms coefficients [12-15].

At present, we constructed a new chaotic system [17], which is described by

$$\begin{cases} \dot{x} = a(y - x) \\ \dot{y} = bx + cy - xz \\ \dot{z} = x^2 - hz \end{cases} \tag{1}$$

where a, b, c and h are constants. When parameters a=20，b=14, c=10.6 and h=2.8, the system (1) shows chaotic behavior. For more detailed analysis of the complex dynamics of the system, please see relative reference [17].

The organization of this paper is as follows. In Section 2, we achieve projective synchronization in the new chaotic system with both identical and different scaling factors. In Section 3, by employing adaptive control theory, we obtain a sufficient condition for projective synchronization in the new chaotic system with unknown parameters. Conclusion is obtained in the final section.

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2 Projective Synchronization between Two Chaotic Systems

In this section, projective synchronization in the new chaotic system with both identical and different scaling factors is achieved via the Lyapunov stability theory and Barblat's lemma.

2.1 PS with identical scaling factor

In this subsection, by using the Lyapunov stability theory and Barblat's lemma, we obtain the condition for projective synchronization between two chaotic systems.

From (1), the drive system is as follows

$$\begin{cases} \dot{x}_1 = a(x_2 - x_1) \\ \dot{x}_2 = bx_1 + cx_2 - x_1x_3 \\ \dot{x}_3 = x_1^2 - hx_3, \end{cases} \tag{2}$$

and the response system with control input reads

$$\begin{cases} \dot{y}_1 = a(y_2 - y_1) + u_1 \\ \dot{y}_2 = by_1 + cy_2 - y_1y_3 + u_2 \\ \dot{y}_3 = y_1^2 - hy_3 + u_3 \end{cases} \tag{3}$$

where $u_i (i = 1, 2, 3)$ are the nonlinear control laws such that two chaotic systems can be synchronized with a scaling factor α. Define the error signals as $e_i = x_i - \alpha y_i \ (i = 1, 2, 3)$.

We have the following error dynamics

$$\begin{cases} \dot{e}_1 = ae_2 - ae_1 - \alpha u_1 \\ \dot{e}_2 = be_1 + ce_2 - x_1x_3 + \alpha y_1y_3 - \alpha u_2 \\ \dot{e}_3 = -he_3 + x_1^2 - \alpha y_1^2 - \alpha u_3. \end{cases} \tag{4}$$

For two identical chaotic systems without $(u_i = 0)$, if the initial condition of two systems is different, the trajectories of the two identical systems will quickly separate each other and become irrelevant. However, for the two controlled chaotic systems, the two systems will approach synchronization for any initial condition by appropriate control laws. For this goal, the following laws for the system (3) are designed

$$\begin{cases} u_1 = \dfrac{1}{\alpha}\left[ae_2 + (k_1 - a)e_1\right] \\ u_2 = \dfrac{1}{\alpha}[be_1 - x_1x_3 + \alpha y_1y_3 + (k_2 + c)e_2] \\ u_3 = \dfrac{1}{\alpha}[x_1^2 - \alpha y_1^2 + (k_3 - h)e_3] \end{cases} \tag{5}$$

where k_i (i=1, 2, 3) are the control gains of positive scalars.

Then, we have the following theorem

Theorem 1 *For given nonzero scalar* α *, the PS between two systems (2) and (3) will occur by the adaptive control laws (5).*

Proof. Choose the following Lyapunov function

$$V = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2\right).$$

The time derivative of the Lyapunov function along the trajectory of error system (4) is

$$\begin{aligned} \dot{V} &= \dot{e}_1e_1 + \dot{e}_2e_2 + \dot{e}_3e_3 \\ &= e_1\left[ae_2 - ae_1 - \alpha u_1\right] + e_2\left[be_1 + ce_2 - x_1x_3 - \alpha y_1y_3 - \alpha u_2\right] + e_3[-he_3 + x_1^2 - \alpha y_1^2 - \alpha u_3]. \end{aligned} \tag{6}$$

By substituting Eq.(5) into Eq.(6), we have $\dot{V} = -k_1e_1^2 - k_2e_2^2 - k_3e_3^2 \le 0$, where e^T=(e_1, e_2, e_3), p= $diag$(k_1, k_2, k_3).

Since $\dot{V}$ is negative semi-definite, we can not immediately obtain that the origin of error system (4) is asymptotically stable. In fact, as $\dot{V} \le 0$, then $e_1, e_2, e_3 \in L_\infty$. From the error system (4), we have $\dot{e}_1, \dot{e}_2, \dot{e}_3 \in L_\infty$. Since $\dot{V} = -e^T pe$ and p is a positive-definite matrix, we obtain

$$\int_0^t \lambda_{\min}(p)\|e\|^2 dt \le \int_0^t e^T pedt = \int_0^t -\dot{V}dt = V(0) - V(t) \le V(0),$$

where $\lambda_{\min}(p)$ is the minimum eigenvalue of positive matrix p .Then $e_1, e_2, e_3 \in L_2$. According to the Barbalat's lemma, we have $\lim_{t\to\infty}\|e(t)\| = 0$. Therefore, the response system (3) synchronizes the drive system (2). This completes the proof.

Numerical simulations are given to show the feasibility and effectiveness of the controllers (5). Choose the scaling factor $\alpha=-2$ and the control gains k_1=4, k_2=8, k_3=4. The fourth-order Runge-Kutta method is used to solve the systems with time step size 0.001. The initial conditions of the drive system and the response system are ($x_1(0)$, $x_2(0)$, $x_3(0)$) = (-1, 2, 2) and ($y_1(0)$, $y_2(0)$, $y_3(0)$) = (-2, -2, -2). Hence the error system has initial values ($e_1(0)$, $e_2(0)$, $e_3(0)$) = (-5, -2, -2), synchronization of the system (2) and (3) via adaptive control laws (5) are shown in Fig.1 and Fig.2. Fig.1 displays the time response of the projective synchronization errors $e_1, e_2, e_3 \to 0$, as $t \to \infty$ implying that all the state variables tend to be synchronized in a proportional. Fig.2 depicts the projection of the synchronized attractors of the drive system (2) (dotted line) and the response system (3) (solid line), which illustrates a projective synchronization with $\alpha = -2$.

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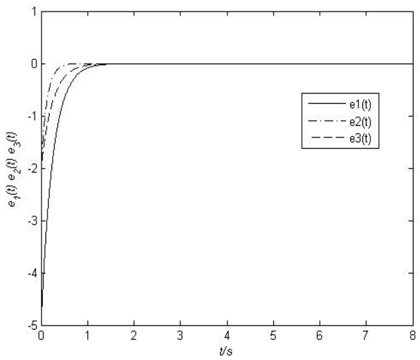


Fig. 1. Error signals between drive and response systems

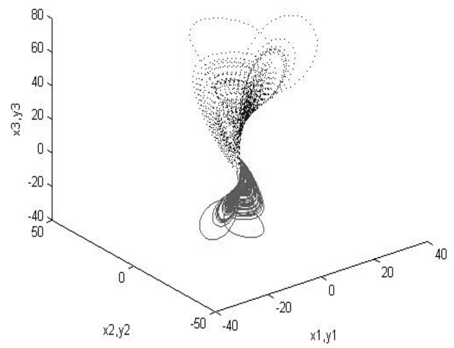


Fig. 2. Chaotic attractors when α=-2.

Specially, when $\alpha = 1$ or $\alpha = -1$, it will achieve complete synchronization or anti-synchronization

2.2 Projective Synchronization with Different Scaling Factors

In the former section, we realize projective synchronization in chaotic system with the same scaling factor α . Now, the study will achieve projective synchronization with different scaling factors which implies that the three state variables of the drive system are in proportion to that of the response system with three different nonzero scaling factors α_1, α_2, α_3, respectively. This synchronization form called 'modified projective synchronization' in [12], has been considered recently in chaotic systems [12-15]. In this subsection, we will focus on this type of synchronization between two new chaotic systems.

We define the error vectors as $e_i=x_i-\alpha_i y_i$ (i=1, 2, 3). Hence the error system is

$$\begin{cases} \dot{e}_1 = a(x_2 - x_1) - \alpha_1 a(y_2 - y_1) - \alpha_1 u_1 \\ \dot{e}_2 = bx_1 + cx_2 - x_1x_3 - \alpha_2 by_1 - \alpha_2 cy_2 + \alpha_2 y_1 y_3 - \alpha_2 u_2 \\ \dot{e}_3 = x_1^2 - hx_3 - \alpha_3 y_1^2 + \alpha_3 hy_3 - \alpha_3 u_3. \end{cases} \tag{7}$$

Our objective is to design effective controllers to achieve projective synchronization with different scaling factors between the systems (2) and (3). For this goal, the following control laws for the systems are designed

$$\begin{cases} u_1 = \dfrac{1}{\alpha_1}[a(\alpha_2-\alpha_1)y_2 + ae_2 + (k_1-a)e_1] \\ u_2 = \dfrac{1}{\alpha_2}\left[b(\alpha_1-\alpha_2)y_1 - x_1x_3 + \alpha_2 y_1 y_3 + be_1 + (k_2+c)e_2\right] \\ u_3 = \dfrac{1}{\alpha_3}\left[x_1^2 - \alpha_3 y_1^2 + (k_3-h)e_3\right] \end{cases} \quad (8)$$

where k_i (i=1, 2, 3) are positive scalars.

Then we obtain the following theorem.

Theorem 2. *Given the controllers (8), the system (2) and (3) can asymptotically achieve projective synchronization with different scaling factors.*

Proof. Define a Lyapunov function

$$V = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2\right).$$

The time derivative of the Lyapunov function along the trajectory of error system (7) is

$$\begin{aligned} \dot{V} &= \dot{e}_1 e_1 + \dot{e}_2 e_2 + \dot{e}_3 e_3 = e_1\left[a(x_2-x_1) - \alpha_1 a\left(y_2-y_1\right) - \alpha_1 u_1\right] \\ &+ e_2\left[bx_1 + cx_2 - x_1x_3 - \alpha_2 by_1 - \alpha_2 cy_2 + \alpha_2 y_1 y_3 - \alpha_2 u_2\right] \\ &+ e_3\left[x_1^2 - hx_3 - \alpha_3 y_1^2 + \alpha_3 hy_3 - \alpha_3 u_3\right]. \end{aligned} \quad (9)$$

By substituting Eq.(8) into Eq.(9), we have $\dot{V} = -k_1e_1^2 - k_2e_2^2 - k_3e_3^2 \le 0$.

Since V is a positive decreasing function and $\dot{V}$ is negative semi-define, this implies that the origin of the error system (7) is asymptotically stable. Therefore, the response system (3) synchronizes the drive system (2). We can approach projective synchronization with different scaling factors asymptotically with the controllers (8).

Numerical simulations are given to verify the effectiveness of the controllers (8). Choose the following scaling factors α_1=-2, α_2=2, α_3=-3 and the control gains k_1=4, k_2=8, k_3=4. The fourth-order Runge-Kutta method is used to solve the systems with time step size 0.001. We assume that the initial condition ($x_1(0)$, $x_2(0)$, $x_3(0)$) = (-1, 2, 2) and ($y_1(0)$, $y_2(0)$, $y_3(0)$) = (-2, -2, -2) are employed. Hence the error system has initial values ($e_1(0)$, $e_2(0)$, $e_3(0)$) = (-5, 6, -4), We can observe that the drive system (2) and the response system (3) achieve projective synchronization immediately (see Fig.3 and Fig.4) after the control is activated although the initial condition are different. Fig.3 displays the time response of the projective synchronization errors $e_1, e_2, e_3 \to 0$, as $t \to \infty$ implying that all the state variables tend to be synchronized in a proportional. Fig.4 depicts the projection of synchronized attractors of the drive system (2) (dotted line) and the response system (3) (solid line) with α_1= -2, α_2=2, α_3= -3.

It is worth mentioning that the convergence rate of error signals can be adjusted by the control gains k_i (i=1, 2, 3).

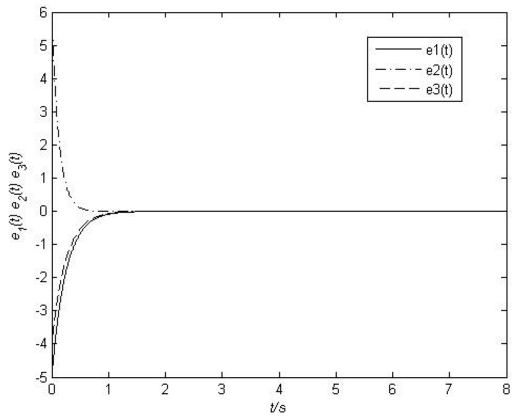


Fig. 3. Error signals between drive and response systems

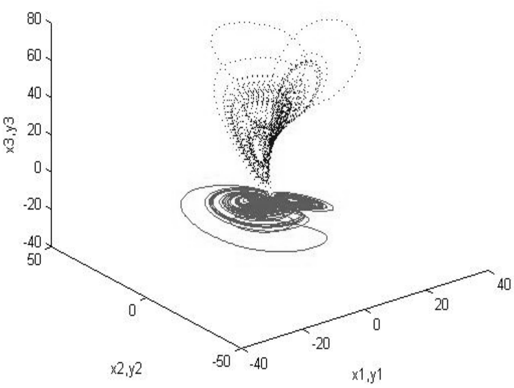


Fig. 4. Chaotic attractors when α_1=-2, α_2=2, α_3=-3.

3 Projective Synchronization with Different Scaling Dactors Between Two Uncertain Chaotic Systems

In References [12-15], projective synchronization between chaotic systems with unknown parameters is achieved, when the coefficients of linear terms are unknown. However, for any physical system, it is more important to know the nonlinear terms. Therefore, in this section, we study projective synchronization in the new chaotic system with unknown coefficients of nonlinear terms.

For the chaotic system (1), assume the parameters *a*, *b*, *c*, *h* and the coefficients of two nonlinear terms (denote them as *m*, *n*) are unknown. So the system (1) can be written as

$$\begin{cases} \dot{x}_1 = a\left(x_2 - x_1\right) \\ \dot{x}_2 = bx_1 + cx_2 + mx_1x_3 \\ \dot{x}_3 = nx_1^2 - hx_3. \end{cases} \tag{10}$$

The response system with control has the following form

$$\begin{cases} \dot{y}_1 = a\left(y_2 - y_1\right) + u_1 \\ \dot{y}_2 = by_1 + cy_2 + my_1y_3 + u_2 \\ \dot{y}_3 = ny_1^2 - hy_3 + u_3. \end{cases} \tag{11}$$

Define the error vectors as $e_i = x_i - \alpha_i y_i$ (i=1, 2, 3), the error system is

$$\begin{cases} \dot{e}_1 = a(x_2 - x_1) - \alpha_1 a\left(y_2 - y_1\right) - \alpha_1 u_1 \\ \dot{e}_2 = bx_1 + cx_2 + mx_1x_3 - \alpha_2 by_1 - \alpha_2 cy_2 - \alpha_2 my_1y_3 - \alpha_2 u_2 \\ \dot{e}_3 = nx_1^2 - hx_3 - \alpha_3 ny_1^2 + \alpha_3 hy_3 - \alpha_3 u_3. \end{cases} \tag{12}$$

The following control laws and update laws for system (10) are designed

$$\begin{cases} u_1 = \dfrac{1}{\alpha_1}[\hat{a}(e_2 - e_1 + (\alpha_2 - \alpha_1)y_2) + e_1] \\ u_2 = \dfrac{1}{\alpha_2}[\hat{b}(e_1 + (\alpha_1 - \alpha_2)y_1) + \hat{m}x_1x_3 - \alpha_2\hat{m}y_1y_3 + (1+\hat{c})e_2] \\ u_3 = \dfrac{1}{\alpha_3}[\hat{n}x_1^2 - \alpha_3\hat{n}y_1^2 + (1-\hat{h})e_3], \end{cases} \tag{13}$$

and

$$\begin{cases} \dot{\hat{a}} = (e_2 - e_1 + (\alpha_2 - \alpha_1)y_2)e_1 - \tilde{a} \\ \dot{\hat{b}} = (e_1 + (\alpha_1 - \alpha_2)y_1)e_2 - \tilde{b} \\ \dot{\hat{c}} = e_2^2 - \tilde{c} \\ \dot{\hat{h}} = -e_3^2 - \tilde{h} \\ \dot{\hat{m}} = x_1x_3e_2 - \alpha_2 y_1y_3e_2 - \tilde{m} \\ \dot{\hat{n}} = x_1^2e_3 - \alpha_3 y_1^2e_3 - \tilde{n} \end{cases} \tag{14}$$

where $\tilde{a} = \hat{a} - a, \tilde{b} = \hat{b} - b, \tilde{c} = \hat{c} - c, \tilde{h} = \hat{h} - h, \tilde{m} = \hat{m} - m, \tilde{n} = \hat{n} - n . \hat{a}, \hat{b}, \hat{c}, \hat{h}, \hat{m}$ and $\hat{n}$ are estimated parameters of unknown parameters *a*, *b*, *c*, *h*, *m*, *n*, respectively.

Then we obtain the following theorem.

Theorem 3. *For given nonzero scalar* α_i *(i=1, 2, 3), projective synchronization between two systems (10) and (11) will occur by the adaptive control laws (13) and update laws (14).*

Proof. Define a Lyapunov function

$$V = \frac{1}{2}\left(e_1^{\,2} + e_2^{\,2} + e_3^{\,2} + \tilde{a}^2 + \tilde{b}^2 + \tilde{c}^2 + \tilde{h}^2 + \tilde{m}^2 + \tilde{n}^2\right).$$

The time derivative of the Lyapunov function along the trajectory of error system (12) is

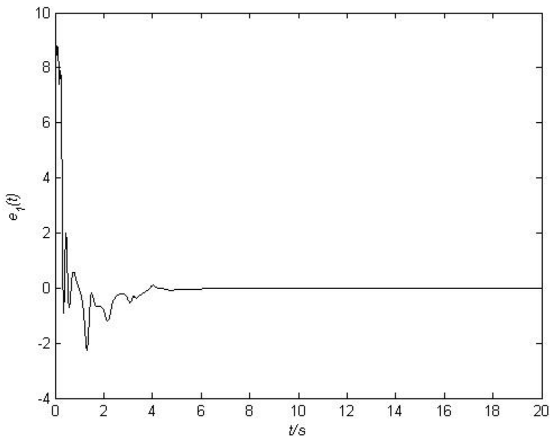
$$\begin{aligned}\dot{V} &= \dot{e}_1 e_1 + \dot{e}_2 e_2 + \dot{e}_3 e_3 + \dot{\tilde{a}}\tilde{a} + \dot{\tilde{b}}\tilde{b} + \dot{\tilde{c}}\tilde{c} + \dot{\tilde{h}}\tilde{h} + \dot{\tilde{m}}\tilde{m} + \dot{\tilde{n}}\tilde{n} \\ &= e_1(a(x_2 - x_1) - \alpha_1 a(y_2 - y_1) - \alpha_1 u_1) \\ &\quad + e_2(bx_1 + cx_2 + mx_1x_3 - \alpha_2 by_1 - \alpha_2 cy_2 - \alpha_2 my_1y_3 - \alpha_2 u_2) \\ &\quad + e_3(nx_1^{\,2} - hx_3 - \alpha_3 ny_1^{\,2} + \alpha_3 hy_3 - \alpha_3 u_3) + \dot{\tilde{a}}\tilde{a} + \dot{\tilde{b}}\tilde{b} + \dot{\tilde{c}}\tilde{c} + \dot{\tilde{h}}\tilde{h} + \dot{\tilde{m}}\tilde{m} + \dot{\tilde{n}}\tilde{n}\,. \end{aligned} \tag{15}$$

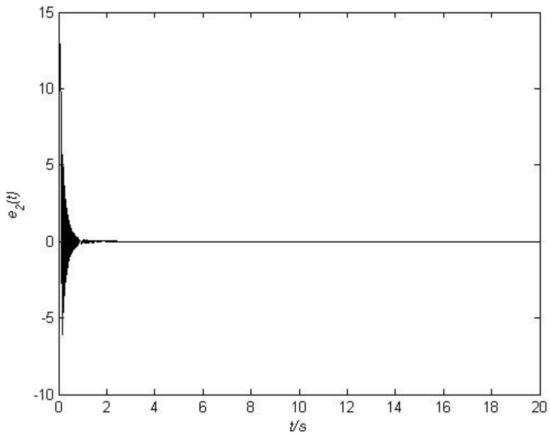
By substituting Eqs.(13) and(14) into Eq.(15), we have

$$\dot{V} = -e_1^{\,2} - e_2^{\,2} - e_3^{\,2} - \tilde{a}^2 - \tilde{b}^2 - \tilde{c}^2 - \tilde{h}^2 - \tilde{m}^2 - \tilde{n}^2\,.$$

Since the Lyapunov function V is a positive definite and its derivative $\dot{V}$ is negative definite in the neighborhood of the zero solution for the system (12). Based on the Lyapunov stability theory, the error dynamical system can converge to the origin asymptotically. Therefore, the response system (11) synchronizes the drive system (10). We can approach projective synchronization with different scaling factors asymptotically with the controllers (13) and update laws (14).

In the numerical simulations, the fourth-order Runge-Kutta method is used to solve the systems with time step size 0.001. We assume that the initial condition $(x_1(0), x_2(0), x_3(0))=(2, 2, 2)$ and $(y_1(0), y_2(0), y_3(0))=(-2, -2, -2)$, and $\alpha_1=3$, $\alpha_2=4$, $\alpha_3=-2$, are employed. Hence the error system has the initial values $(e_1(0), e_2(0), e_3(0))=(8, 10, -2)$.The six unknown parameters are chosen as $a=20$, $b=14$, $c=10.6$, $h=2.8$, $m=-1$ and $n=1$ in simulations so that the new system exhibits a chaotic behavior. Synchronization of the system (10) and (11) via adaptive control laws (13) and (14) with the initial estimated parameters $\hat{a}=10$, $\hat{b}=6$, $\hat{c}=5$, $\hat{h}=5$, $\hat{m}=1$ and $\hat{n}=-1$ are shown in Figs. 5, 6 and 7, respectively. Fig. 5. displays the synchronization errors of system (10) and (11). Fig. 6 depicts chaotic attractors of the drive system (10) and the response system (11) when $\alpha_1=3$, $\alpha_2=4$, $\alpha_3=-2$. Fig. 7 shows that the estimations $\hat{a}(t)$, $\hat{b}(t)$, $\hat{c}(t)$, $\hat{h}(t)$, $\hat{m}(t)$, $\hat{n}(t)$ of the unknown parameters converge to $a=20$, $b=14$, $c=10.6$, $h=2.8$, $m=-1$ and $n=1$, as $t\to\infty$.





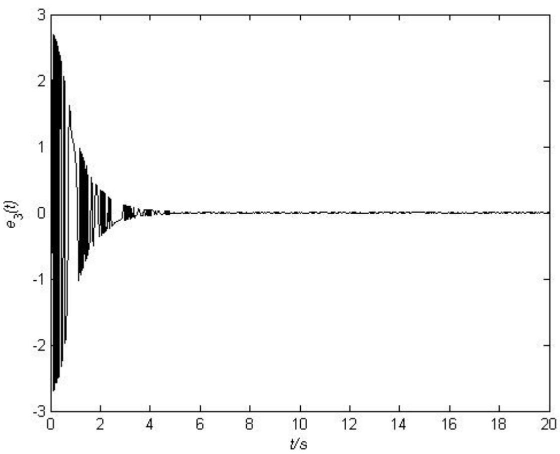


Fig. 5. Error signals between drive and response systems

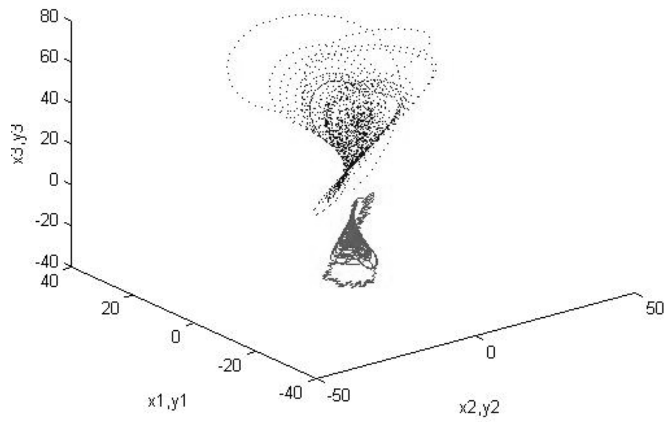


Fig. 6. Chaotic attractors of the drive system (8) (dotted line) and the response system (9) (solid line) when α_1=3, α_2=4, α_3=-2.

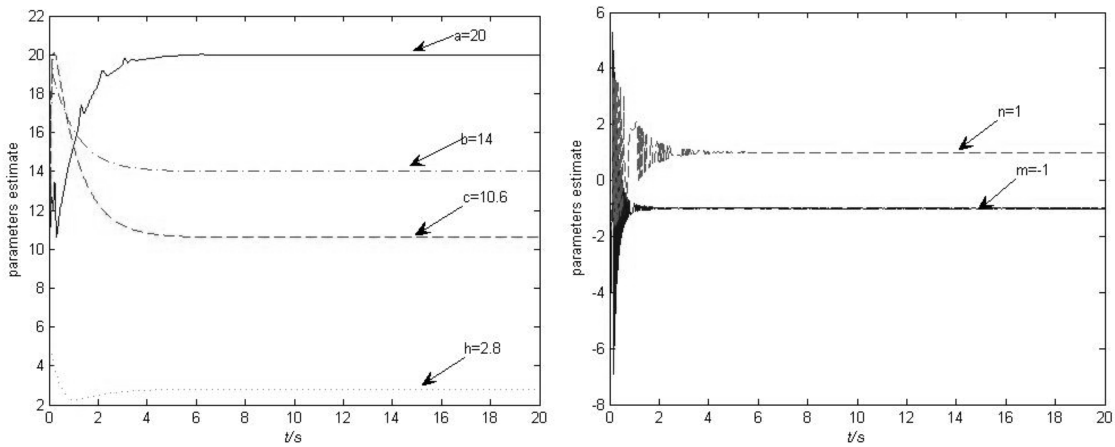


Fig. 7. Estimated values for unknown parameters as $t\rightarrow\infty$.

4 Conclusion

In this paper, based on the Lyapunov stability theory and Barblat's lemma, we achieved the projective synchronization of a new chaotic system with identical or different scaling factors. The PS of the new chaotic system with uncertainties including the coefficients of nonlinear terms was obtained via adaptive control. Numerical simulations showed the effectiveness of the analytical results.

Acknowledgements

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