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An Application of Possibility Goal Programming to the Time-Cost Trade off Problem

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Abstract

Activity duration is uncertain due to the variations in the real world such as weather, resource availability. Utilization of uncertain planning leads to project scheduling with more stability against environmental variations. This paper presents a new optimal model for time-cost trade-off problem in fuzzy environment. In order to solve this problem, we develop a new solution method for possibility goal programming problems. The significant feature of this model is the determination of optimal duration for each activity in the form of triangular fuzzy numbers. To validate the algorithm developed in this paper, a case study will be presented.

Keywords: time-cost trade off, possibility goal programming, fuzzy sets, fuzzy decision variable

1 Introduction

Since the late 1950s, Critical-Path-Method (CPM) techniques have become widely recognized as valuable tools for the planning and scheduling of projects. But in many cases, project should be implemented before the date calculated by CPM method. To achieve this goal, more sophisticated equipments or employment of more human resources can be used. As a result, the project cost increase. Therefore, for the project to be completed with the least possible amount of time and cost, obtaining a logical trade off between cost and duration of project is necessary. Several mathematical and heuristic models have been developed to solve time-cost trade off problems [1]. These models have mainly focused on deterministic situations. However, during project implementation, many uncertain variables dynamically affect activity duration and the costs could also change accordingly. In this paper, we propose an optimal mathematical model to deal with the time-cost trade off problems in the uncertain environment and also a new approach to solving proposed algorithm.

A. Literature review of time-cost trade off problem

Based upon whether the activity duration is certain or not, time-cost trade off models can be categorized into two types: deterministic scheduling and nondeterministic scheduling. Traditional time-cost trade-off models mostly focus on deterministic situations. Most of these models are heuristic and analytical. Among them are Siemens's model [2] and Moselhi's model [3]. Some researchers used operation research's methods to model and solve time-cost trade off problems [4, 5, 6]. Also some methods were developed based on metaheuristic models such as simulated annealing and genetic algorithm [7, 8]. The above-mentioned time-cost trade-off models mainly focus on deterministic situations. Recently, project managers have paid special attention to uncertain scheduling. Uncertain scheduling models are categorized into two types: probabilistic models and fuzzy models. One of the probabilistic models is Ang's model [9]. In many projects, the required information for estimation of project parameters is not available or is incomplete. Also in many cases the project is done for the first time, this compels us to use expert opinion in forecasting the project parameters. Some authors have claimed that fuzzy set theory is more appropriate to model

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these problems. Guang *et al.* [10] offered a new solution approach for fuzzy time-cost trade-off model based on genetic algorithm. Leu *et al.* [1] proposed a new fuzzy optimal time-cost trade-off method and a GA-based approach to solving it. Arican and gungor [11] presented a fuzzy goal programming model for time-cost trade off problem. Also Hapk and Slowinski [12], Leu *et al.* [13], and Wan [14] developed other methods to deal with fuzzy time-cost trade off problem.

B. Literature review of mathematical programming with fuzzy variable

Generally speaking, in fuzzy linear programming problems, the coefficients of decision variables are fuzzy numbers whereas decision variables are crisp ones. This means that, in an uncertain environment, a crisp decision is made to meet some decision criteria. Initially, Tanaka *et al.* [15] proposed a possibilistic linear programming formulation where the coefficients of decision variables are crisp whereas decision variables are obtained as fuzzy numbers. As an extension of this idea, Guo *et al.* [16] have used linear programming (LP) and quadratic programming (QP) techniques to obtain fuzzy solutions. Tanaka *et al.* [17] dealt with the interactive case in which exponential distribution functions are used. Buckley *et al.* [18] developed a new approach to solve multi objective linear programming problems in which all the parameters and variables are fuzzy numbers. In the other model, Tanaka *et al.* [19] took into consideration three kinds of possibility distribution for decision variables; interval possibility distribution, triangular possibility distribution and exponential possibility distribution. In their approach, possibility distribution of fuzzy parameters and each decision variable must be symmetric. But in the real world, estimations are usually asymmetric. For example, in the time-cost trade off problem, estimation of normal and crash durations mostly are asymmetric. In this paper, we develop a new approach based on fuzzy distance to solve possibility goal programming. In this approach, fuzzy parameters and possibility distribution of decision variables are triangular that can be asymmetric.

This paper is organized as follows. The second section presents some fuzzy concepts. The considered model of time-cost trade off problem in fuzzy environment is described in the third section. In the fourth section, a new approach to solving possibility goal programming will be developed. A case study is offered in fifth section. The final section involves conclusions.

2 Fuzzy Concepts

A fuzzy number $\tilde{M}$ is a convex normalized fuzzy set $\tilde{M}$ of the real line R such that 1. There exists exactly one $X_0 \in R$ with $\mu_{\tilde{M}}(x_0) = 1$ (X_0 is called the mean value of $\tilde{M}$). 2. $\mu_{\tilde{M}}(x)$ is piecewise continuous.

A fuzzy number $\tilde{M}$ is of LR-type if there exist reference function L (for left), R (for right), and scalars $\alpha > 0$, $\beta > 0$ such that

$$\mu_{\tilde{M}}(x_0) = \begin{cases} L\left(\dfrac{m-x}{\alpha}\right), & \text{for} \quad x \le m \\ R\left(\dfrac{x-m}{\beta}\right), & \text{for} \quad x \ge m \end{cases} \tag{1}$$

where m, called the mean value of $\tilde{M}$, is a real number , and α and β are called the left and right spread, respectively[20].

Let $\tilde{A}$ and $\tilde{B}$ be fuzzy numbers, and * denotes any basic fuzzy arithmetic operations such as addition, subtraction and multiplication. Any operations $\tilde{A} * \tilde{B}$ can be defined a fuzzy set on R and expressed in the following form [21]:

$$\mu_{\tilde{A}*\tilde{B}}(z) = \sup_{x*y=z} \{\min[\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(y)]\}. \tag{2}$$

Many ranking methods for fuzzy numbers have been proposed so far. A list of these methods have presented in [22]. However, there is no single approach that can produce a satisfactory result in every situation: some may generate counter-intuitive results and others are not discriminative enough [23]. To overcome such problems, Cheng [24] developed a new distance approach for fuzzy number comparisons based on a calculation of the centroid point

$(\bar{x}_0, \bar{y}_0)$ to obtain the distance index, where $\bar{x}_0$ and $\bar{y}_0$ are centroid values both in the horizontal and vertical axes respectively. A triangular fuzzy number $\tilde{A} = (a_1, a_2, a_3)$ can be expressed as

$$\mu_{\tilde{A}} = \begin{cases} \mu^L_{\tilde{A}}(x), & a_1 \le x \le a_2 \\ \mu^R_{\tilde{A}}(x), & a_2 \le x \le a_3 \end{cases} \tag{3}$$

where $\mu^L_{\tilde{A}} : [a_1, a_2] \to [0,1]$ is the strictly continuous left spread and its corresponding inverse function is denoted by $g^L_{\tilde{A}}(x)$. $\mu^R_{\tilde{A}} : [a_2, a_3] \to [0,1]$ is the strictly continuous right spread and its corresponding inverse function is symbolized by $g^R_{\tilde{A}}(x)$. All the functions can be integrable due to their continuity. Therefore, the centroid point $(\bar{x}_0, \bar{y}_0)$ of a fuzzy number $\tilde{A}$ can be defined in the following form

$$\bar{x}_0(\tilde{A}) = \frac{\int_{a_1}^{a_2} [x\, \mu^L_{\tilde{A}}(x)]dx + \int_{a_2}^{a_3} [x\, \mu^R_{\tilde{A}}(x)]dx}{\int_{a_1}^{a_2} \mu^L_{\tilde{A}}(x)dx + \int_{a_2}^{a_3} \mu^R_{\tilde{A}}(x)dx},$$
$$\bar{y}_0(\tilde{A}) = \frac{\int_0^1 [y\, g^L_{\tilde{A}}(y)]dy + \int_0^1 [y\, g^R_{\tilde{A}}(y)]dy}{\int_0^1 g^L_{\tilde{A}}(y)dy + \int_0^1 g^R_{\tilde{A}}(y)dy}. \tag{4}$$

The ranking index can be expressed as

$$R(\tilde{A}) = \sqrt{(\bar{x}_0)^2 + (\bar{y}_0)^2}. \tag{5}$$

For triangular fuzzy numbers, formulation (4) of calculating the centroid point can be simplified as

$$\bar{x}_0 = \frac{a_3^2 - a_1^2 + a_2 \times a_3 - a_2 \times a_1}{3 \times (a_3 - a_1)},$$
$$\bar{y}_0 = \frac{a_1 + 4 \times a_2 + a_3}{3 \times (a_1 + 2 \times a_2 + a_3)}. \tag{6}$$

For any fuzzy number $\tilde{A}_i$ and $\tilde{A}_j$, ranking fuzzy number has the following properties:

$$\begin{aligned} &1)\ \text{If}\quad R(\tilde{A}_i) > R(\tilde{A}_j),\quad \text{then}\quad \tilde{A}_i > \tilde{A}_j. \\ &2)\ \text{If}\quad R(\tilde{A}_i) = R(\tilde{A}_j),\quad \text{then}\quad \tilde{A}_i = \tilde{A}_j. \\ &3)\ \text{If}\quad R(\tilde{A}_i) < (\tilde{A}_j),\quad \text{then}\quad \tilde{A}_i < \tilde{A}_j. \end{aligned} \tag{7}$$

3 Optimum Time-cost Trade Off Model

In this section, a new optimal model for time-cost trade off problem is developed in fuzzy environment. The assumptions of the time-cost trade off model presented in this paper are: 1) Normal and crash durations are uncertain and their values are denoted in the form of triangular fuzzy numbers. 2) Value of crashing cost is crisp. 3) In the project network, activities are shown on the arcs and events are shown on nodes. The fuzzy time-cost trade off model is shown as follows:

$$\min \sum_{i=1}^{n-1} \sum_{j \in S_i} C_{ij} \times (\tilde{T}^n_{ij} - \tilde{t}_{ij}) \tag{8}$$

$$\min \ \tilde{t}_n \tag{9}$$

$$s.t.$$

$$\tilde{t}_i + \tilde{t}_{ij} \le \tilde{t}_j, \qquad i = 1,\ldots,n-1; j \in S_i \tag{10}$$

$$\tilde{T}^r_{ij} \le \tilde{t}_{ij} \le \tilde{T}^n_{ij}, \qquad i = 1,\ldots,n-1; j \in S_i \tag{11}$$

where n is the number of nodes; S_i is the set of activities that begin with event i; $\tilde{T}^n_{ij}$ is the normal duration of activity i, j that is denoted in the form of triangular fuzzy number; $\tilde{T}^r_{ij}$ is the crash duration of activity i,j that is denoted in the form of triangular fuzzy number; $\tilde{t}_{ij}$ is the duration of activity i, j; $\tilde{t}_i$ is the occurrence time of event i. Also the normal and crash durations can be considered as other forms of the fuzzy numbers such as trapezoidal fuzzy numbers, doing so, in the solution method, the formula of centroid point $(\bar{x}_0, \bar{y}_0)$ have to be calculated by Eq. (4) instead of Eq. (6).

The objective function (8) minimizes crashing costs. Objective function (9) minimizes makespan of project execution. Eq. (10) shows the precedence relationship between activities. Eq. (11) restricts duration of each activity to the interval between normal and crash times. In the first objective function, taking into account that $C_{ij} \times \tilde{T}_{ij}^n$ is constant, therefore this term can be removed. Thus the fist objective function can be rewritten as

$$max \quad \sum_{i=1}^{n} \sum_{j \in S_i} C_{ij} \times \tilde{t}_{ij} \,. \tag{12}$$

If values $\tilde{T}_{ij}^n$ and $\tilde{T}_{ij}^r$ are symmetric triangular fuzzy numbers, then the model presented by Tanaka *et al.*[19] can be used to solve time-cost trade off problem, but if normal and crash durations of activities are asymmetric triangular fuzzy numbers then there is no solution method to solve time-cost trade off model. Therefore, in the next section we develop a new approach to solving possibility goal programming that can be used to solve time-cost trade off problem with asymmetric parameters and variables.

4 Possibility Goal Programming

Since there is no solution for time–cost trade off problem presented in the previous section, we develop a new approach to solving possibility goal programming models. Consider the multiple objective problem (13) where the coefficients of decision variables are crisp whereas decision variables are obtained as fuzzy numbers. The main object of solving this model is to determine the decision vector $\tilde{X} = [\tilde{x}_1, \tilde{x}_2, ..., \tilde{x}_n]$ so that constraints reach the maximum of satisfaction and $\tilde{X}$ be the acceptable solution for all objective functions simultaneously. In model (13), each element $\tilde{x}_j$ of decision vector $\tilde{X}$ is defined by a triangular possibility distribution and denoted by $\tilde{x}_j = (x_{j1}, x_{j2}, x_{j3})$.

$$\begin{aligned} & min \ \tilde{Z} = C\tilde{X} \\ & s.t. \\ & \quad A\tilde{X} \geq \tilde{b} \\ & \quad \tilde{X} \geq 0 \end{aligned} \tag{13}$$

where $C = [c_{ij}]$ for $i = 1,2,...,p$, $j = 1,2,...,n$, and $A = [a_{ij}]$, $i = 1,2,...,m$, $j = 1,2,...,n$ are crisp coefficients matrixes and each element of right hand side vector $\tilde{b} = [\tilde{b}_i], i = 1,2,...,m$ is a triangular fuzzy number denoted by $\tilde{b}_i = (b_{i1}, b_{i2}, b_{i3})$. Vector $\tilde{Z} = [\tilde{z}_i], i = 1,2,..., p$ is the vector of objective functions. Due to the fact that each element of decision vector $\tilde{X}$ has a triangular possibility distribution, so we can denote each objective function by $\tilde{z}_i = (z_{i1}, z_{i2}, z_{i3})$. The algorithm for solving problem (13) is described as follows:

Step1: Obtain an aspiration level $\tilde{s}_i$ for each objective function $\tilde{z}_i$. This step could be done in two ways. In the first way you can ask decision maker to give aspiration level for each objective function. Another way is to reduce model (13) to three independent multi objective linear programming models such as (14). Solving these three models by common approaches for multiple objective decision making (MODM) problems, three values for each objective function are obtained.

$$\begin{aligned} &\min \ z_{ir} = \sum_{j=1}^{n} c_{ij} x_{jr}, \qquad i = 1,2,...,p \\ &s.t. \\ &\quad \sum_{j=1}^{n} a_{ij} x_{jr} \geq b_{ir}, \qquad i = 1,2,...,m \\ &\quad x_{jr} \geq 0, \qquad j = 1,...,n \end{aligned} \tag{14}$$

where r=1, 2, 3. It is demonstrable that $z_{i1} \leq z_{i2} \leq z_{i3}$ is always true. Now we have an aspiration level (z_{i1}, z_{i2}, z_{i3}) for each objective function $\tilde{Z}_i$. We can offer these aspiration levels to decision maker for more modifications. In this manner aspiration level vector $\tilde{S} = [\tilde{s}_i], i = 1,2,..., p$ is generated.

Step2: After generating aspiration level vector $\tilde{S}$, we can rewrite model (13) as follows

$$\begin{aligned} &find \quad \tilde{X} \\ &s.t. \\ &\quad C\tilde{X} \leq \tilde{S} \\ &\quad A\tilde{X} \geq \tilde{b} \\ &\quad \tilde{X} \geq 0. \end{aligned} \tag{15}$$

We can rewrite model (15) in a short form as follows

$$\begin{aligned} &find \quad \tilde{X} \\ &s.t. \\ &\quad K\tilde{X} \leq \tilde{h} \\ &\quad \tilde{X} \geq 0 \end{aligned} \tag{16}$$

where $K = \begin{pmatrix} C \\ -A \end{pmatrix}$, and $\tilde{h} = \begin{pmatrix} \tilde{S} \\ -\tilde{b} \end{pmatrix}$.

Step3: All constraints of model (16) are fuzzy inequalities that their right hand side is a triangular fuzzy number and their left hand side is a combination of fuzzy variables with triangular possibility distribution. Therefore decision vector $\tilde{X}$ must be determined in a way that constraints reach the maximum satisfaction. To achieve this goal, we use ranking fuzzy numbers which discussed in previous section. In the other word, to satisfy constraints of model (16), the following equation must meet.

$$R\left(\sum_{j=1}^{n} k_{ij} \tilde{x}_j\right) \leq R(\tilde{h}_i), \quad i = 1,2,..., p + m. \tag{17}$$

Define d_i as follows

$$d_i = R(\tilde{h}_i) - R\left(\sum_{j=1}^{n} k_{ij} \tilde{x}_j\right), \quad i = 1,2,..., p + m. \tag{18}$$

Therefore the objective function must be maximization of the least amount of d_i that can be described as follows:

$$D = \min_i \{ d_i \}. \tag{19}$$

Now we can rewrite model (13) as follows

$$
\begin{aligned}
& \max \quad D \\
& s.t. \\
& \quad D \le d_i, && i = 1,2,\ldots,p+m \\
& \quad d_i \ge 0, && i = 1,2,\ldots,p+m \\
& \quad x_{j2} \ge x_{j1}, && j = 1,2,\ldots,n \\
& \quad x_{j3} \ge x_{j2}, && j = 1,2,\ldots,n \\
& \quad x_{j2} - x_{j1} \le q_{j1} \times x_{j2}, && j = 1,2,\ldots,n \\
& \quad x_{j3} - x_{j2} \le q_{j2} \times x_{j2}, && j = 1,2,\ldots,n \\
& \quad x_{j1}, x_{j2}, x_{j3} \ge 0, && j = 1,2,\ldots,n
\end{aligned} \tag{20}
$$

where $q_{j1} \in [0,1]$. The third and fourth constraints in model (20) are set to guarantee obtained values for decision variables to be in standard format of triangular fuzzy numbers. Also q_{j1} and q_{j2} determine the ambiguity of decision variables (in fact determine the support set of decision variables). These values are presented by decision maker.

5 Case Study

If fuzzy decision variables $\tilde{t}_{ij}$ and $\tilde{t}_i$ are in crisp form, the model (8)-(11) can be solved by common fuzzy multiple objective linear programming solutions.

In the case study presented in this section, first, decision variables are considered in fuzzy form and then they are considered in crisp form.

The precedence relationships network of a project with 10 activities is depicted in Figure 1. Information of activities is given in Table 1. Because of the fact that values $\tilde{T}_{ij}^n$ and $\tilde{T}_{ij}^r$ are asymmetric triangular fuzzy numbers, possibility goal programming method presented in the fourth section, should be used to solve this problem. Values q_{j1} and q_{j2}, $j = 1,2,\ldots,n$, are considered 0.3 and 0.2 respectively. Also the aspiration levels for the first and second objective functions are considered (1543, 1875.5, 2208) and (35, 40, 60) respectively. The solution results are shown in Tables 2 and 3.

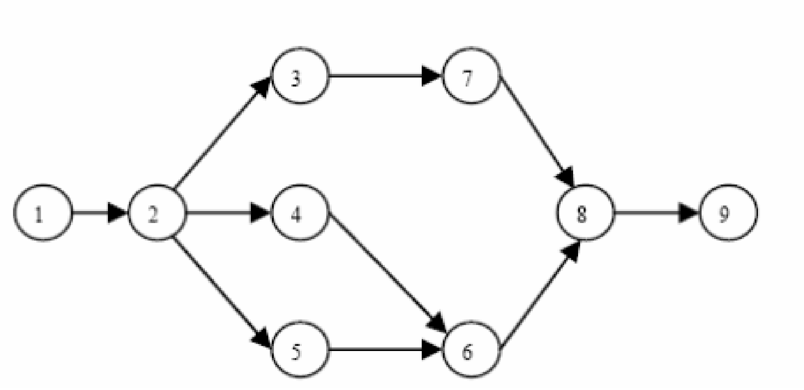

Figure 1. Precedence relationship network of case study

Table 1. Information of activities

Activity index	Normal duration	Crash duration	Crash cost
1-2	(5,8,9)	(1,1,2)	15
2-3	(5,6,6)	(2,2,2)	25
2-4	(5,7,9)	(1,2,3)	23
2-5	(8,10,12)	(3,4,5)	35
5-6	(7,7,9)	(3,4,4)	18
4-6	(5,7,7)	(3,4,5)	32
3-7	(6,8,10)	(2,3,4)	20
7-8	(7,7,7)	(2,3,4)	17
6-8	(6,7,8)	(2,4,6)	30
8-9	(9,10,13)	(3,4,6)	27

Table 2. Fuzzy and crisp duration of each activity

Activity index	Fuzzy duration of activity	Crisp duration of activity
1-2	(6.85,6.85,8.22)	8.32
2-3	(5.29,5.29,7.85)	6
2-4	(6.54,6.54,7.85)	7.64
2-5	(9.35,9.35,11.22)	10.64
5-6	(7.17,7.17,8.6)	7.64
4-6	(5.91,5.91,7.1)	7
3-7	(7.48,7.48,8.98)	8.64
7-8	(6.54,6.54,7.85)	7
6-8	(6.54,6.54,7.85)	7.32
8-9	(10,10,12)	10.97

Table 3. Fuzzy and crisp occurrence time of each event

Node index	Fuzzy occurrence time of event	Crisp occurrence time of event
1	(0,0,0)	0
2	(5.85,8.26,8.26)	8.32
3	(16.04,20.85,20.85)	18.29
4	(12.21,17.09,17.09)	19.61
5	(15.85,21.28,21.53)	18.97
6	(21.03,30.05,30.83)	26.61
7	(21.24,27.31,30.34)	26.94
8	(26.85,38.04,38.04)	33.94
9	(41.8,42.31,50.77)	44.91

In the case where decision variables are fuzzy, total crash cost and makespan of project are (1758, 1758, 2110) and (41.8, 42.31, 50.77), respectively. In the case where decision variables are crisp, these values are 1992.87 and 44.91 respectively.

As depicted in Figure 2, crashing cost obtained from time-cost trade off model with crisp decision variables is covered by value obtained from time-cost trade off model with fuzzy decision variables. Also we expect that obtained amount for makespan resulted from model with fuzzy decision variable covers makespan resulted from model with crisp decision variables (depicted in Figure 3).

As it is shown in tables 2 and 3, values obtained from proposed model in this paper, cover values obtained from model with crisp decision variables. However, with regard to uncertainty in real world, utilizing proposed model will help us to attain better project scheduling with more stability against environmental variations.

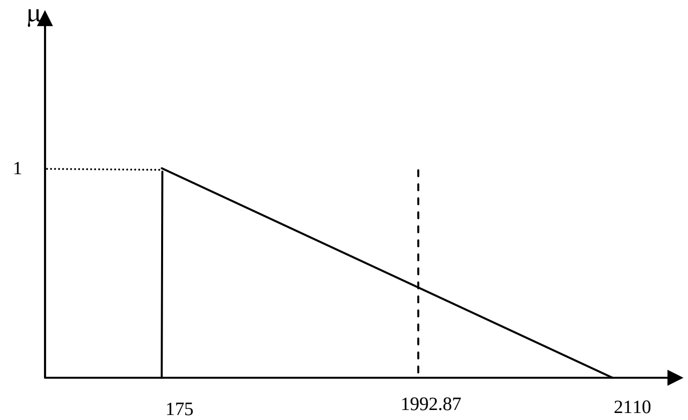

Figure 2. Comparison between crisp and fuzzy crash cost

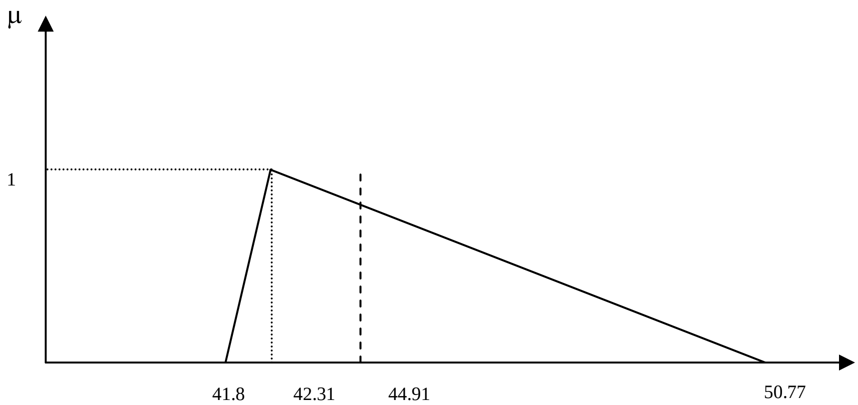

Figure 3. Comparison between crisp and fuzzy makespan

6 Conclusion

Time-cost trade off problem is one of the main aspects of project scheduling. Due to variations in the real world, usually, risks in estimation of project parameters are considerably high. Therefore, it is necessary to use of uncertain models (capable of formulating vagueness in the real world) to solve time-cost trade off problems, and give a scheduling with more stability against environmental variations. On the other hand, crisp decision making in uncertain environment causes loss of some parts of information. In this paper we developed a new optimal approach to model time-cost trade off problem in the fuzzy environment. Also to solve the proposed model, a new approach to solving possibility goal programming was developed.

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