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Guaranteed Cost Fuzzy Output Feedback Control Via LMI Method for Re-entry Attitude Dynamics*

Yu-Hui Wang, Qing-Xian Wu, Chang-Sheng Jiang, Guo-Yong Huang
College of Atomation Engineering, Nanjing University of Aeronautics and Astronautics
Nanjing, Jiangsu, P. R. China
xiaodusan@163.com, wuqingxian@nuaa.edu.cn

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Abstract

This paper proposes a new design method of fuzzy robust control, which is used for aerospace vehicle's (ASV's) attitude dynamics during the re-entry phase. For the complicated flight condition during re-entry phase, the attitude T-S fuzzy model with parameter uncertainty is considered, based on which, a guaranteed cost fuzzy controller with disk pole constraints is designed via output parallel-distributed compensation (OPDC) approach, by solving linear matrix inequalities (LMIs) problem, the robust fuzzy controller guarantee the closed-loop system with satisfactory transient and stead-state performances. The simulation results demonstrate the effectiveness of the proposed method.

Keywords: aerospace vehicle, T-S fuzzy, pole constraints, guaranteed cost

1 Introduction

Attitude control plays an important role during the re-entry phase as far as ASV (aerospace vehicle) is concerned. One of the biggest difficulties is that attitude dynamics of an ASV are high complex nonlinear dynamics. Therefore, a non-model-based method is suitable to be considered for design. Fuzzy method is a popular for modeling and control unknown nonlinear system [13]. There are two common inference methods: Mamdani's fuzzy and T-S (Takagi-Sugeno) fuzzy [12]. These two methods are the same in many respects, such as the procedure of fuzzifying the inputs and fuzzy operators, and have been applied in many practical designs. In Ref. [15] and Ref. [16], re-entry vehicle's attitude control was considered by using Mamdani fuzzy. Although Mamdani fuzzy approach is successful to a certain extent, its lack of precise mathematical description makes it difficult to be applied in further general design and fuzzy systems analysis.

T-S fuzzy was first introduced in 1985 [12], which can compensate some shortcomings of Mamdani fuzzy in aspect of mathematical analysis. Furthermore, the advantage of using T-S fuzzy models is that a large class of nonlinear plants can be well represented by local linear models, without the need to modify the original nonlinear dynamics in any significant way [7, 4, 11]. In Ref. [8] and Ref. [10], T-S fuzzy-model-based control was applied to attitude dynamics, and the designed robust fuzzy controllers guaranteed the stability of attitudes. However, stabilization is only a minimum requirement for control systems. In most practical situations, a good controller should also deliver sufficiently fast and well-damped time responses. An efficient way to guarantee satisfactory transient performance is to place the closed-loop poles in a suitable region of the complex plane. In Ref. [3],

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Chilali discussed analysis and synthesis techniques for robust pole placement in LMI (linear matrix inequality) regions, but the proposed method could only be used for linear uncertain systems, as presented in Ref. [6] and Ref. [14].

In this study, pole placement in LMI region is extended to a class of nonlinear systems by using T-S fuzzy technique, which can be used for ASV's re-entry attitude control. For complicated flight conditions and acutely changed parameters during re-entry phase, the attitude dynamics with parameter uncertainty are considered and represented by T-S fuzzy model, based on which, guaranteed cost control is taken to achieve the system's stability under the limited control input, and disk pole placement is considered to achieve fast decay, good damping, and reasonable controller dynamics. Combined the advantages of guaranteed cost control with disk pole constraints, a T-S fuzzy controller is designed via OPDC (output parallel distributed compensation) [2] approach, and it is derived in terms of LMIs with equation constraint. By Lemma 3.2, the equation constraint is eliminated and the design problem is transformed to LMIs problem without equation constrain, which can be solved by using Matlab tool.

This paper is organized as follows. In Section 2, we define the ASV's re-entry attitude dynamics based on T-S fuzzy, and show that the fuzzy plant rule and control rule. Section 3 presents the fuzzy robust control scheme and the proofs of the stability and the problem solutions. In section 4, simulation results are illustrated to confirm the feasibility and superiority of the proposed method. Finally, conclusion remarks are included in Section 5.

2 Problem Formulation

The dynamical equations of rotational motion of ASV in re-entry mode are given:

$$\dot{\omega} = J^{-1}\Omega(\omega)J\omega + J^{-1}G\delta \tag{1}$$

$$\dot{\gamma} = \Xi(\gamma)\omega, \tag{2}$$

where $\omega = [p, q, r]^T$ is the angular rate, J is the inertia, $\gamma = [\phi, \beta, \alpha]^T$ is the attitude angle, $\delta = [\delta_e, \delta_a, \delta_r, \delta_x, \delta_y, \delta_z]^T$ is the control surface deflection, p, q, r, ϕ, β and α are the pitch rate, the roll rate, the yaw rate, the bank angle, the sideslip angle and the attack angle respectively, $\delta_e, \delta_a, \delta_r, \delta_x, \delta_y$ and δ_z are the elevator deflection, the aileron deflection, the rudder deflection, the equivalent control surface deflection of x-axis, y-axis and z axis to the body frame respectively, and

$$\Omega(\omega) = \begin{pmatrix} 0 & r & -q \\ -r & 0 & p \\ q & -p & 0 \end{pmatrix}, \tag{3}$$

$$\Xi(\gamma) = \begin{pmatrix} cos\alpha & 0 & sin\alpha \\ sin\alpha & 0 & -cos\alpha \\ 0 & 1 & 0 \end{pmatrix}, \tag{4}$$

$$G = \begin{pmatrix} g_{p,\delta_e} & g_{p,\delta_a} & g_{p,\delta_r} & g_{p,\delta_x} & 0 & 0 \\ g_{q,\delta_e} & g_{q,\delta_a} & g_{q,\delta_r} & 0 & 0 & g_{q,\delta_z} \\ g_{r,\delta_e} & g_{r,\delta_a} & g_{r,\delta_r} & 0 & g_{r,\delta_y} & 0 \end{pmatrix}. \tag{5}$$

Matrix G is the control allocation of control torque to control surface. Here, only the expression of g_{q,δ_e} as an example is given:

$$g_{q,\delta_e} = \bar{q}S[cC_{m,\delta_e} + X_{cg}(C_{D,\delta_e}sin(\alpha) + C_{L,\delta_e}cos(\alpha))], \tag{6}$$

where $\bar{q}$ is aerodynamic pressure (kg/ms), S is reference area (m^2), c is mean aerodynamic chord (m), X_{cg} is longitudinal distance from momentum reference to vehicle (m). $C_{m,\delta_e}, C_{D,\delta_e}$ and C_{L,δ_e} are pitching moment increment, drag increment coefficient, and lift increment coefficient for left elevon, respectively, and all of them are functions of the angle of attack α, the Mach number $Mach$ and the altitude H.

To design the controller conveniently, the method of backstepping is used to convert the stabilization problem into a regulation problem [8]. Considered equation (2), a virtual control vector is designed as following:

$$\omega_c = -k_1 \Xi^{-1}\gamma. \tag{7}$$

Define the error variable z as:

$$z = \omega - \omega_c = \omega + k_1 \Xi^{-1}\gamma. \tag{8}$$

Then, the differential equations (1) and (2) can be rewritten as

$$\begin{cases} \dot{z} = \left(J^{-1}\Omega(z - k_1\Xi^{-1}\gamma)J + k_1\right) z + k_1\left(-J^{-1}\Omega(z - k_1\Xi^{-1}\gamma)J\Xi^{-1} + (\Xi^{-1})' - k_1\Xi^{-1}\right)\gamma \\ \qquad + J^{-1}G\delta \\ \dot{\gamma} = \Xi z - k_1\gamma, \end{cases} \tag{9}$$

where $(\Xi^{-1})'$ denotes the time derivative of Ξ^{-1}. And define the output of system (9) is:

$$y \triangleq z, \tag{10}$$

the input of system (9) is:

$$u \triangleq \delta, \tag{11}$$

and $x_1 \triangleq z_1, x_2 \triangleq z_2, x_3 \triangleq z_3, x_4 \triangleq \phi, x_5 \triangleq \beta, x_6 \triangleq \alpha, x_z \triangleq [x_1, x_2, x_3]^T, x_\gamma \triangleq [x_4, x_5, x_6]^T$ and $x \triangleq [x_z^T, x_\gamma^T]^T$. Then Eq.(5) is equal to

$$\begin{cases} \dot{x}(t) = f(x)x(t) + g(x)u(t) \\ y(t) = Cx(t), \end{cases} \tag{12}$$

where $f(x), g(x)$ and C are

$$f(x) = \begin{bmatrix} \left(J^{-1}\Omega(z - k_1\Xi^{-1}\gamma)J + k_1\right) & k_1\left(-J^{-1}\Omega(z - k_1\Xi^{-1}\gamma)J\Xi^{-1} + (\Xi^{-1})' - k_1\Xi^{-1}\right) \\ \Xi & -k_1 I_{3\times 3} \end{bmatrix},$$

$$g(x) = \begin{bmatrix} J^{-1}G \\ 0_{3\times 6} \end{bmatrix}, \quad C = \begin{bmatrix} I_{3\times 3} & 0_{3\times 3} \end{bmatrix}.$$

Based on T-S fuzzy, system (12) can be well represented by local linear models. However, because of limited fuzzy rules, modeling error is always existed. Furthermore, during the re-entry phase, the high complicated flight conditions and acutely changed parameters greatly affect the flight performance of ASV. To avoid the above problems, the uncertain system of ASV based on T-S fuzzy model is considered. The ith IF-THEN rule is:

$$\begin{aligned} \texttt{Plant rule i:} \quad & \texttt{IF } y_1(t) \textit{ is } M_{i1} \textit{ and } \cdots\ y_{n_y}(t) \textit{ is } M_{in_y} \\ & \texttt{THEN } \dot{x}(t) = (A_i + \triangle A_i)x(t) + (B + \triangle B_i)u(t) \\ & \qquad y(t) = Cx(t) \qquad i = 1, 2, \cdots, r, \end{aligned} \tag{13}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the input vector, $y(t) \in \mathbb{R}^{n_y}$ is the output vector, for $i = 1, 2, \cdots, r; j = 1, 2, \cdots, n_y$, M_{ij} is the grade of membership of $y_j(t)$ for the ith rule, r is the number of fuzzy rules. Matrices $A_i \in \mathbb{R}^{n\times n}, B_i \in \mathbb{R}^{n\times m}, C \in \mathbb{R}^{n_y\times n}$, $\triangle A_i, \triangle B_i$ are the uncertainties of A_i,B_i respectively, and have the form:

$$[\triangle A_i \ \triangle B_i] = D_i F_i(t)[E_{1i} \ E_{2i}], \tag{14}$$

where D_i, E_{1i}, E_{2i} are constant matrices with proper dimensions which reflect uncertain structure, $F_i(t)$ are unknown matrices containing Lebesgue measurable elements and satisfy $F_i^T(t)F_i(t) < I$.

For the system (12), we design robust fuzzy control law by the approach of OPDC [2], and the *ith* control rule is:

$$\begin{array}{ll} \texttt{Control rule i:} & \texttt{IF } y_1(t) \textit{ is } M_{i1} \textit{ and } \cdots \ y_{n_y}(t) \textit{ is } M_{in_y} \\ & \texttt{THEN } u(t) = N_i y(t) \qquad i = 1, 2, \cdots, r. \end{array} \tag{15}$$

So, the global controller is

$$u(t) = \sum_{i=1}^{r} h_i(y(t)) N_i y(t), \tag{16}$$

and the closed-loop system is

$$\dot{x}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(y)h_j(y)\left(A_i + B_iN_jC + D_iF_i(E_{1i} + E_{2i}N_jC)\right)x(t), \tag{17}$$

where h_i is defined by

$$h_i(y(t)) = \frac{\prod_{j=1}^{n_y} M_{ij}(y_j(t))}{\sum_{i=1}^{r}\prod_{j=1}^{n_y} M_{ij}(y_j(t))}. \tag{18}$$

Note that the normalized weights h_i satisfy $h_i(y(t)) \geq 0$ and $\sum_{i=1}^{r} h_i(y) = 1$ for all $t \geq 0$.

3 Main Results

Guaranteed cost control is a method of synthesizing a closed-loop system, in which the controlled plant has large parameter uncertainty. Here, a linear quadratic cost function is considered as a performance index of the closed-loop fuzzy system (17). Given symmetric positive definite matrices Q and R , the cost function is:

$$J_C = \int_0^\infty \left[x^T(t)Qx(t) + u^T(t)Ru(t)\right] dt. \tag{19}$$

Synchronously, to achieve satisfactory transient performance, disk pole placement is taken to make the system achieve fast decay, good damping, and reasonable controller dynamics. Here, a disk $D(q, r)$ with center at $(-q + j0)$ and radius $r \leq q$ is considered as shown in Fig.1.

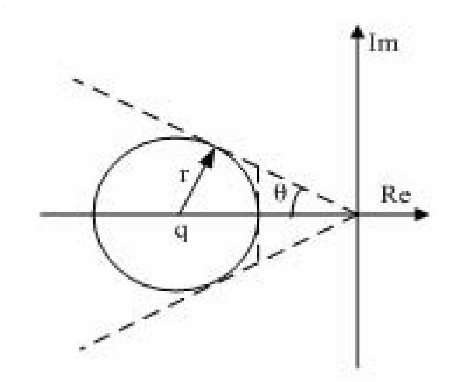

Fig.1 Disk region $D(q, r)$

If $\lambda \in D(q,r)$, it follows that the damping ratio $\xi \geq [1-(\frac{q}{r})^2]^{\frac{1}{2}}$, damped natural frequency $\omega_d \leq r$, the natural frequency $\omega_n \in [q-r, q+r]$ [6].

Next, inspired by the idea of Ref. [6], we associate guaranteed cost control with disk pole constraints, the following definition is proposed.

Definition 3.1 For the given disk and cost function (19), for all $F_i(t)$ which satisfy $F_i^T F_i \leq I$, if symmetric positive definite matrix P satisfy

$$\begin{bmatrix} -P^{-1} & \frac{(A+qI)}{r} \\ \frac{(A+qI)^T}{r} & -P+Q+C^T N_j^T R N_j C \end{bmatrix} < 0, \tag{20}$$

then matrix P is a fuzzy quadratic-D cost matrix of the system (17) under disk pole constraints, where

$$\bar{A} = A_i + B_i N_j C + D_i F_i (E_{1i} + E_{2i} N_j C), i, j = 1, 2, \cdots, r.$$

Remark 3.1: In Ref. [6], the idea is only appropriate for linear systems, based on T-S fuzzy theory, it can be extended to solve the control problems of nonlinear systems.

Noted that the inequalities (20) are not LMIs, for which can be solved by using Matlab tool, the following lemma is given.

Lemma 3.1 ([1]) For matrices Y, D and E with given proper dimensions, where Y is symmetric, then for all F which satisfy

$$F^T F \leq I, Y + DFE + E^T F^T D^T < 0$$

if and only if there exists a $\epsilon > 0$ such that

$$Y + \epsilon D D^T + \epsilon^{-1} E^T E < 0.$$

Under Definition 3.1 and Lemma 3.1, we show the following key theorem.

Theorem 3.1 For the given disk $D(q,r)$ and cost function (19), if there exist real symmetric positive definite matrix V, real matrices W_j and scalar ϵ such that

$$\Psi_{ii} < 0;\ \Psi_{ij} + \Psi_{ji} < 0; CV = MC \qquad i < j \leq r \tag{21}$$

where

$$\Psi_{ij} = \begin{bmatrix} -V & A_i V + B_i W_j C + qV & D_i & 0 & 0 & 0 \\ * & -r^2 V & 0 & rVQ^{\frac{1}{2}} & VE_{1i}^T + C^T W_j^T E_{2i}^T & rC^T W_j^T R^{\frac{1}{2}} \\ * & * & -I & 0 & 0 & 0 \\ * & * & * & -\epsilon I & 0 & 0 \\ * & * & * & * & -I & 0 \\ * & * & * & * & * & -\epsilon I \end{bmatrix}$$

have feasible solutions $(\tilde{\epsilon}, \tilde{V}, \tilde{W}_j)$, then $u(t) = \sum_{i=1}^{r} h_i(y(t)) \tilde{W}_i \tilde{M}^{-1} y(t)$ is the guaranteed cost fuzzy output feedback control law with disk pole constraints and the upper bound of cost function of J_C is $\tilde{J}_C = x_0^T \left(\frac{q}{r^2} P\right) x_0$.(where * denotes the transposed terms for symmetric positions)

Proof. By Definition 3.1, (20) is equivalent to

$$\begin{bmatrix} -P^{-1} & A_i + B_iN_jC + D_iF_i(E_{1i} + E_{2i}N_jC) + qI \\ * & -r^2P + r^2Q + r^2C^TN_j^TRN_jC \end{bmatrix} < 0. \tag{22}$$

And (22) can be rewritten as:

$$\begin{gathered} \begin{bmatrix} -P^{-1} & A_i + B_iN_jC + qI \\ * & -r^2P + r^2Q + r^2C^TN_j^TRN_jC \end{bmatrix} + \\ \begin{bmatrix} D \\ 0 \end{bmatrix} F \begin{bmatrix} 0 & E_{1i} + E_{2i}N_jC \end{bmatrix} + \begin{bmatrix} 0 & E_{1i} + E_{2i}N_jC \end{bmatrix}^T F^T \begin{bmatrix} D \\ 0 \end{bmatrix}^T < 0. \end{gathered} \tag{23}$$

By Lemma 3.1, for all F_i which satisfy $F_i^TF_i \leq I$, the above matrix inequalities hold if and only if there exists an $\epsilon > 0$, such that

$$\begin{gathered} \begin{bmatrix} -P^{-1} & A_i + B_iN_jC + qI \\ * & -r^2P + r^2Q + r^2C^TN_j^TRN_jC \end{bmatrix} + \\ \epsilon^{-1} \begin{bmatrix} D \\ 0 \end{bmatrix} \begin{bmatrix} D^T & 0 \end{bmatrix} + \epsilon \begin{bmatrix} 0 \\ (E_{1i} + E_{2i}N_jC)^T \end{bmatrix} \begin{bmatrix} 0 & E_{1i} + E_{2i}N_jC \end{bmatrix} < 0 \end{gathered} \tag{24}$$

that is

$$\begin{bmatrix} -P^{-1} + \epsilon^{-1}DD^T & A_i + B_iN_jC + qI \\ & r^2(-P + Q + C^TN_j^TRN_jC) \\ * & +\epsilon(E_{1i} + E_{2i}N_jC)^T(E_{1i} + E_{2i}N_jC) \end{bmatrix} < 0. \tag{25}$$

Pre-multiply and post-multiply each side of (25) by using the following matrix,

$$\begin{pmatrix} \sqrt{\epsilon}I & 0 \\ 0 & \sqrt{\epsilon}P^{-1} \end{pmatrix}, \tag{26}$$

then we have

$$\begin{bmatrix} -\epsilon P^{-1} + DD^T & \epsilon(A_i + B_iN_jC + qI)P^{-1} \\ * & U \end{bmatrix} < 0, \tag{27}$$

where

$$U = r^2\epsilon(-P^{-1} + P^{-1}QP^{-1} + P^{-1}C^TN_j^TRN_jCP^{-1}) + \epsilon^2P^{-1}(E_{1i} + E_{2i}N_jC)^T(E_{1i} + E_{2i}N_jC)P^{-1}.$$

Denote $V = \epsilon P^{-1}$, we have

$$\begin{bmatrix} -V + DD^T & (A_i + B_iN_jC + qI)V \\ & r^2(-V + \frac{VQV}{\epsilon} + \frac{VC^TN_j^TRN_jCV}{\epsilon}) \\ * & +V(E_{1i} + E_{2i}N_jC)^T(E_{1i} + E_{2i}N_jC)V \end{bmatrix} < 0. \tag{28}$$

Inequalities (28) are a problem of bilinear matrix inequalities (BMIs) about V and N_j, and it can be transformed to LMI problem by adding equation constraint as $CV = MC$, where M is nonsingular matrix. Then the following inequalities is obtained

$$\begin{bmatrix} -V + DD^T & (A_i + B_iN_jC + qI)V \\ & r^2(-V + \frac{VQV}{\epsilon} + \frac{C^TM^TN_j^TRN_jMC}{\epsilon}) \\ * & +(E_{1i}V + E_{2i}N_jMC)^T(E_{1i}V + E_{2i}N_jMC) \end{bmatrix} < 0. \tag{29}$$

Denote $W_j = N_j M$, we get

$$\begin{bmatrix} -V + DD^T & A_iV + B_iW_jC + qV \\ * & r^2(-V + \frac{VQV}{\epsilon} + \frac{C^TW_j^TRW_jC}{\epsilon}) + (E_{1i}V + E_{2i}W_jC)^T(E_{1i}V + E_{2i}W_jC) \end{bmatrix} < 0. \quad (30)$$

Then by Schur complement, matrix inequalities (21) are obtained.

From the above proof steps, we know if $(\tilde{\epsilon}, \tilde{V})$ is the feasible solutions of (21), then P ($P = \tilde{\epsilon}\tilde{V}^{-1}$) is the fuzzy quadratic-D cost matrix of system (17).

Next we show the upper bound of cost function. By Definition 3.1 and Schur complement, (20) is equal to

$$(\bar{A} + qI)^T P(\bar{A} + qI) - r^2P + r^2Q + r^2C^TN_j^TRN_jC < 0 \quad (31)$$

that is

$$\begin{aligned} \bar{A}^TP + P\bar{A} &< -\frac{1}{q}\bar{A}^TP\bar{A} - \frac{q^2 - r^2}{q}P - \frac{r^2}{q}Q - \frac{r^2}{q}C^TN_j^TRN_jC \\ &< -\frac{r^2}{q}(Q + C^TN_j^TRN_jC). \end{aligned} \quad (32)$$

Consider the Lyapunov function $V(x) = x^TPx$, and its time derivative along the trajectory of system (17) is

$$\dot{V}(x) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_ih_jx^T(P\bar{A} + \bar{A}^TP)x \le -\frac{r^2}{q}x^T\left(\sum_{j=1}^{r} h_j(Q + C^TN_j^TRN_jC)\right)x < 0. \quad (33)$$

Then we have

$$\frac{r^2}{q}x^T\left(\sum_{j=1}^{r} h_j(Q + C^TN_j^TRN_jC)\right)x \le -\dot{V}(x). \quad (34)$$

Integrate both sides of (34) from $t = 0$ to $t = \infty$ and considering the initial condition, we obtain

$$\int_0^\infty x^T\left(\sum_{j=1}^{r}\frac{r^2}{q}h_j(Q + C^TN_j^TRN_jC)\right)xdt \le V(x(0)) = x_0^TPx_0. \quad (35)$$

And the upper bound of J_C is

$$J_C = \int_0^\infty \left[x^T(t)Qx(t) + u^T(t)Ru(t)\right]dt \le \tilde{J}_C = x_0^T\left(\frac{q}{r^2}P\right)x_0. \quad (36)$$

Hence the proof is complete.

Remark 3.2: The problem of (21) is a series of LMIs with equation constraint, and it can not be solved in matlab code. In general case, the following lemma is used to eliminate the equation constraint of LMIs (21).

Lemma 3.2 ([5]) Suppose matrices $L \in \mathbb{R}^{n\times q}, Z \in \mathbb{R}^{n\times q}$ are column full rank, then there exists symmetric positive definite matrix V such that $VL = Z$ if and only if $L^TZ = Z^TL > 0$, while

$$V = Z(L^TZ)^{-1}Z^T + L^\perp X(L^\perp)^T \quad (37)$$

where $X \in \mathbb{R}^{(n-q)\times(n-q)}$ is positive definite matrix, and $L^\perp$ denotes orthogonal complement matrix of L.

The next theorem shows the LMI term without equation constraints.

Theorem 3.2 For the given disk $D(q,r)$ and cost function (19), if there exist real symmetric positive definite matrix X, nonsingular matrix M, real matrices W_j and scalar ϵ such that

$$\Psi_{ii} < 0;\ \Psi_{ij} + \Psi_{ji} < 0 \qquad i < j \le r \tag{38}$$

where

$$\Psi_{ij} = \begin{bmatrix} S_1 & S_2 + B_i W_j C + S_3 & D_i & 0 & 0 & 0 \\ * & S_4 & 0 & S_5 & S_6 + C^T W_j^T E_{2i}^T & rC^T W_j^T R^{\frac{1}{2}} \\ * & * & -I & 0 & 0 & 0 \\ * & * & * & -\epsilon I & 0 & 0 \\ * & * & * & * & -I & 0 \\ * & * & * & * & * & -\epsilon I \end{bmatrix},$$

$$\begin{aligned} S_1 &= -C^T M^T (CC^T M^T)^{-1} MC - C^{T\perp} X (C^{T\perp})^T, \\ S_2 &= A_i C^T M^T (CC^T M^T)^{-1} MC + A_i CT \perp X (C^{T\perp})^T, \\ S_3 &= qC^T M^T (CC^T M^T)^{-1} MC + qC^{T\perp} X (C^{T\perp})^T, \\ S_4 &= -r^2 C^T M^T (CC^T M^T)^{-1} MC - r^2 C^{T\perp} X (C^{T\perp})^T, \\ S_5 &= rC^T M^T (CC^T M^T)^{-1} MCQ^{\frac{1}{2}} + rC^{T\perp} X (C^{T\perp})^T Q^{\frac{1}{2}}, \\ S_6 &= C^T M^T (CC^T M^T)^{-1} MCE_{1i}^T + C^{T\perp} X (C^{T\perp})^T E_{1i}^T \end{aligned}$$

have the feasible solutions $(\tilde{\epsilon}, \tilde{M}, \tilde{X}, \tilde{W}_j)$, then $u(t) = \sum_{i=1}^{r} h_i(y) \tilde{W}_i \tilde{M}^{-1} y(t)$ is the guaranteed cost fuzzy output feedback control law with pole constraints and the cost function of J_C has upper bound $\tilde{J}_C = x_0^T \left(\frac{q}{r^2} P\right) x_0$.

Proof. Since V is symmetric matrix, the equation constraint $CV = MC$ is equal to $VC^T = C^T M^T$. By Lemma 3.2, if denote $L = C^T, Z = C^T M^T$, there has

$$V = C^T M^T (CC^T M^T)^{-1} MC + C^{T\perp} X (C^{T\perp})^T. \tag{39}$$

Then the LMIs (38) are obtained from LMIs (21) with equation constraint by using Eq.(39). And problem (38) can be solved by Matlab LMI toolbox. Then the quadratic-D cost matrix

$$P = \tilde{\epsilon} \left(C^T \tilde{M}^T (CC^T \tilde{M}^T)^{-1} \tilde{M} C + C^{T\perp} \tilde{X} (C^{T\perp})^T \right).$$

Hence the proof is complete.

4 Results

To verify the performance of the proposed method, we give the initial point (that is given the altitude $H = 65km$ and the Mach number $Mach = 20.2$ as the initial states) during the re-entry phase of ASV. And the symmetric, positive definite moment of inertia tensor is given as follows [9]:

$$J = \begin{pmatrix} 554486 & 0 & -23002 \\ 0 & 1136949 & 0 \\ -23002 & 0 & 1376852 \end{pmatrix}.$$

In re-entry mode $y_i \in [-0.5, 0.5], (i = 1, 2, 3)$, and the corresponding membership functions are shown in Fig. 2.

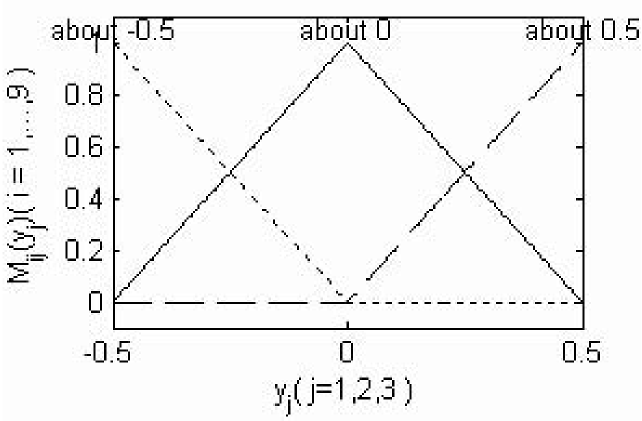

Fig.2 Membership functions of the fuzzy sets $M_{ij}(y_j)$

Then we choose nine operating points: $[y_1, y_2, y_3] = [-0.5, -0.5, -0.5], [0, 0, -0.5], [0.5, 0.5, -0.5], [-0.5, -0.5, 0], [0, 0, 0], [0.5, 0.5, 0], [-0.5, -0.5, 0.5], [0, 0, 0.5], [0.5, 0.5, 0.5]$. Under the membership functions and the nine operating points, nine plant rules and nine control rules can be defined (reference (13) and (15)). And A_i and B_i can be obtained easily by the substitution of each of the nine operating points to $f(x)$, $g(x)$ with $k_1 = 8$. Then we choose $D_i = D = 0.01I_6$, and suppose matrices A_i, B_i exist 30%,50%, parameter perturbation respectively, that is $\triangle A_i = 0.3A_i, \triangle B_i = 0.5B_i$.

By solving LMIs problem (38), the fuzzy robust controller for ASV's attitude dynamics can be designed. The fuzzy output feedback control gains and the fuzzy quadratic-D cost matrix are shown as follows:

$$N_1 = \begin{bmatrix} -1.39 & -3.68 & 1.79 & 0.51 & 1.79 & -1.04 \\ -1.46 & -2.64 & -1.13 & 0.20 & 1.67 & -3.04 \\ -3.28 & -4.29 & -0.39 & 0.23 & 3.62 & -2.01 \end{bmatrix}^T,$$

$$N_2 = \begin{bmatrix} 1.35 & 1.33 & 1.21 & 0.50 & 1.62 & -1.93 \\ 1.71 & 2.83 & 0.73 & 0.31 & 2.04 & -3.08 \\ 0.96 & 2.80 & -1.30 & 0.23 & 3.45 & -1.95 \end{bmatrix}^T,$$

$$N_3 = \begin{bmatrix} -0.99 & 0.52 & -3.30 & 0.51 & 1.60 & -2.83 \\ -1.28 & 1.95 & -5.28 & 0.43 & 2.57 & -3.12 \\ -2.57 & -0.21 & -6.08 & 0.23 & 3.44 & -1.77 \end{bmatrix}^T,$$

$$N_4 = \begin{bmatrix} 2.87 & 4.00 & -0.01 & 0.50 & 2.22 & -1.10 \\ 0.66 & 1.35 & -0.94 & 0.22 & 0.86 & -2.84 \\ 5.10 & 7.42 & -0.27 & 0.29 & 3.60 & -2.29 \end{bmatrix}^T,$$

$$N_5 = \begin{bmatrix} -0.53 & -0.95 & 0.41 & 0.49 & 2.08 & -1.91 \\ -2.27 & -3.89 & 0.85 & 0.30 & 2.05 & -3.01 \\ -0.18 & -0.34 & 0.41 & 0.29 & 3.45 & -1.93 \end{bmatrix}^T,$$

$$N_6 = \begin{bmatrix} -1.03 & -1.72 & 0.41 & 0.50 & 2.12 & -2.58 \\ -2.37 & -4.05 & -0.85 & 0.37 & 3.59 & -3.05 \\ 0.04 & 0.01 & 0.41 & 0.29 & 3.53 & -1.33 \end{bmatrix}^T,$$

$$N_7 = \begin{bmatrix} -3.33 & -4.79 & -0.23 & 0.49 & 2.58 & -1.44 \\ -2.42 & -3.84 & -0.30 & 0.28 & 0.15 & -2.85 \\ -5.41 & -9.68 & 1.66 & 0.35 & 3.48 & -2.47 \end{bmatrix}^T,$$

$$N_8 = \begin{bmatrix} 1.60 & 3.02 & -0.34 & 0.47 & 2.48 & -1.91 \\ 4.37 & 5.24 & 1.20 & 0.29 & 2.01 & -3.01 \\ 3.25 & 2.51 & 3.58 & 0.34 & 3.31 & -1.93 \end{bmatrix}^T,$$

$$N_9 = \begin{bmatrix} 1.35 & 2.87 & 1.97 \\ 2.17 & 2.88 & 1.70 \\ 0.14 & 1.86 & 2.18 \\ 0.49 & 0.32 & 0.35 \\ 2.58 & 4.17 & 3.56 \\ -2.25 & -3.13 & -1.12 \end{bmatrix}, P = \begin{bmatrix} 34567 & 21940 & 24545 & 0 & 0 & 0 \\ 21940 & 34811 & 24001 & 0 & 0 & 0 \\ 24545 & 24001 & 38426 & 0 & 0 & 0 \\ 0 & 0 & 0 & 271090 & -27725 & 7925.6 \\ 0 & 0 & 0 & -27725 & 1719400 & -8073 \\ 0 & 0 & 0 & 7925.6 & -8073 & 4179000 \end{bmatrix}.$$

Next the system response under the proposed controller is simulated. The state response figures with different initial conditions are shown in Fig.3 and Fig.4, respectively.

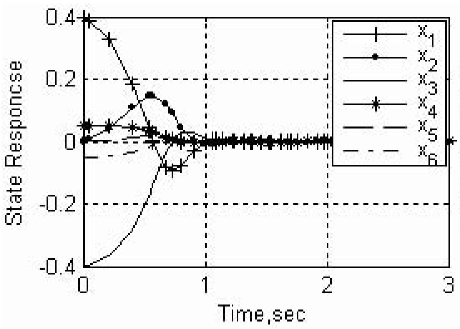

Fig.3 State response with initial conditions $x(0) = [0.4, 0, -0.4, 0.05, 0, -0.05]^T$

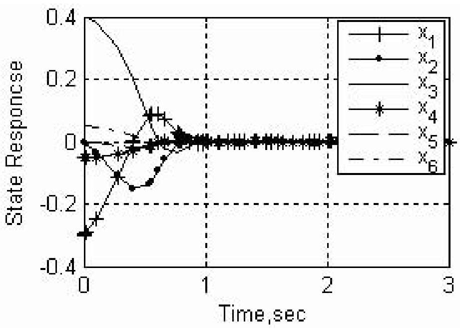

Fig.4 State response with initial conditions $x(0) = [-0.3, 0, 0.4, -0.05, 0, 0.05]^T$

From definition of t_s, $t_s \approx \frac{4}{\xi\omega_n}$, if based on the considered disk, because of $4.8 = q - r \leq \xi\omega_n \leq q + r = 5.2$, t_s should in the region $[0.77, 0.83]s$. However, from the Fig.3 and Fig.4, the settling time is about 1s. The simulation results differ from the theoretical value, which may be mainly caused by two reasons. One of them is model error can not be estimated accurately. Though nine linear subsystems under nine fuzzy rules are applied to approximate the ASV's attitude dynamics, the T-S fuzzy model can not completely describe the complex attitude dynamics without any deviation, it will effect the control results. Another is that the LMIs problem (38) for this example is feasible but not strictly feasible, depending on the 45 inequalities needed to be solved and the pole constraints employed in (38). Despite the simulation results have a little bias, the proposed controller still deliver satisfactory transient and steady-state performance for ASV's attitude dynamics.

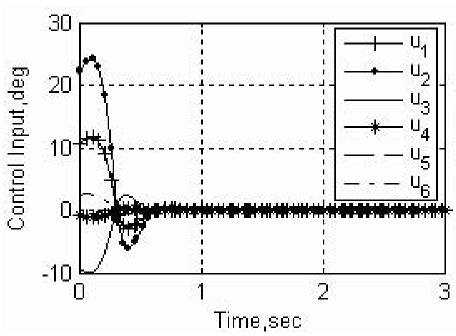

Fig.5 Control input with initial conditions $x(0) = [0.4, 0, -0.4, 0.05, 0, -0.05]^T$

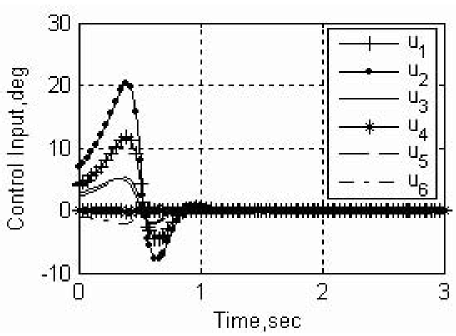

Fig.6 Control input with initial conditions $x(0) = [-0.3, 0, 0.4, -0.05, 0, 0.05]^T$

In general cases of flight control, the control input is a bounded vector, and the surface deflection is required in the region $[-30, 30]deg$. If not in the region, the prospective control effect can not be attained. From Fig.5 and Fig.6, we can see that the control surfaces satisfy the constraint condition. They show that the designed T-S fuzzy controller can guarantee the control input without saturation.

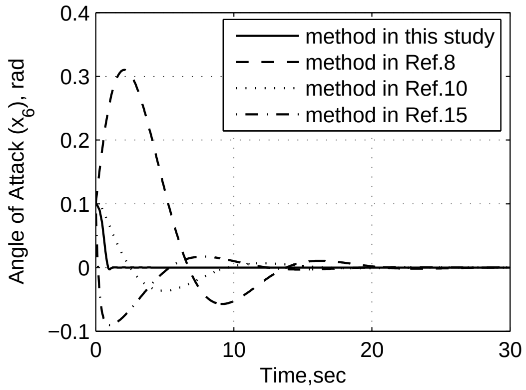

Fig.7 Control input with initial conditions
$x(0) = [0, 0.4, -0.4, 0.1, 0, 0.1]^T$

Fast convergence is always our object, especially for the real time control during the re-entry phase. In Fig.7, we give the response waves of angle of attack (α). Compared with the proposed methods in Ref.[8,10,15], the response time is obviously accelerated. The convergence time in this study is not exceed 1s with no oscillation, which is very important to practical missions. That is due to fast tracking and good robust are the powerful guarantees for missions completion.

5 Conclusion

Although stabilization is considered to be one of the most basal requirements in ASV's attitude control, fast response robust controller is our object, especially for in real time mode. Based on the idea of fuzzy guaranteed control, there has no saturation for control input, by doing this, it guarantee the state's stabilization under the limited input. In addition, disk pole placement is considered to provide the system satisfactory transient performance, such as fast decay, and good damping. The simulation results show that the proposed method can obtain attitude's stabilization during re-entry phase, which also extend a new technique for complex nonlinear systems robust control.

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