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Chaos Synchronization of a New Chaotic System via Nonlinear Control

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Abstract

This paper studies chaos synchronization of a new chaotic system via nonlinear control. Based on Lyapunov stability theory, using a nonlinear controller can synchronize two identical systems. By the linear control theory, nonlinear controllers are designed to synchronize two different chaotic systems. Numerical simulations show the effectiveness and feasibility of the proposed methods.

Keywords: Chaotic system, synchronization, nonlinear controller

1. Introduction

During the last two decades, synchronization in chaotic dynamic systems has received a great deal of interest among scientists from various research fields since Pecora and Carroll[1] introduced a method to synchronize two identical chaotic systems with different initial conditions. Chaos synchronization has many potential applications in laser physics, chemical reactor, secure communication, biomedical and so on. Many techniques for chaos control and synchronization have been developed, such as linear feedback method, active control approach, adaptive technique, time-delay feedback approach, and back steeping method [2-8]. In general, it is known that nonlinear control is an effective method for making two identical or different chaotic systems be synchronized.

This letter mainly investigates the chaos synchronization of a new chaotic system which was introduced by Cai et al.[9]. Based on Lyapunov stability theory, a nonlinear controller is proposed to realize two identical systems synchronization. Compared with the work in Ref. [10], where three controllers were used to achieve the asymptotical synchronization, our controllers can synchronize systems in less time. Then, based on linear control theory, a class of nonlinear control scheme is designed to realize the synchronization between the new chaotic system and the Liu chaotic system. Finally, numerical simulations are also provided for illustration and verification.

This article is organized as follows. In Section 2, a new chaotic system is introduced. Its chaos synchronization of two identical chaotic systems is considered in Section 3. In Section 4, we discuss the synchronization problem between the new chaotic system and the Liu chaotic system. Two numerical examples are given to demonstrate the effectiveness of the proposed methods in Section 5. Finally the conclusion is given in Section 6.

2. The New Chaotic System

Cai et al [9] introduced a new chaotic system of three-dimensional quadratic autonomous ordinary differential equations. This system is described as

$$\begin{cases} \dot{x} = a(y-x) \\ \dot{y} = bx + cy - xz \\ \dot{z} = x^2 - hz \end{cases} \tag{1}$$

where a, b, c and h are real constants and x, y, z are state variables.

This system has many interesting complex dynamical behaviors, and it has potential applications in secure communication. When a=20, b=14, c=10.6 and h=2.8, it has a chaotic attractor, as shown in Fig. 1.

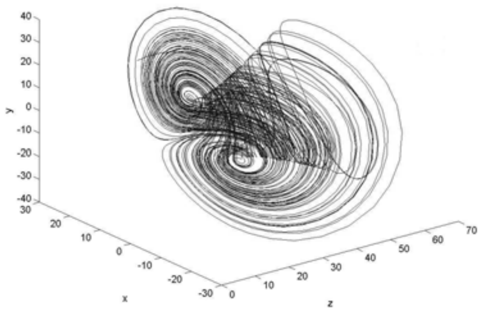

Fig. 1 the new attractor

3 Synchronization of Two Identical Chaotic Systems

For the chaotic system (1), the drive and response systems can be defined as follows:

$$\begin{cases} \dot{x}_1 = a(y_1 - x_1) \\ \dot{y}_1 = bx_1 + cy_1 - x_1 z_1 \\ \dot{z}_1 = x_1^2 - hz_1 \end{cases} \tag{2}$$

and

$$\begin{cases} \dot{x}_2 = a(y_2 - x_2) \\ \dot{y}_2 = bx_2 + cy_2 - x_2 z_2 + u_1 \\ \dot{z}_2 = x_2^2 - hz_2 + u_2 \end{cases} \tag{3}$$

where a=20, b=14, c=10.6 and h=2.8 and u_1 and u_2 are control inputs to be designed. The aim of this section is to determine the control laws u_i for the global synchronization of two identical systems.
Let the error variables be

$$\begin{cases} e_1 = x_2 - x_1 \\ e_2 = y_2 - y_1 \\ e_3 = z_2 - z_1. \end{cases} \tag{4}$$

Substitute Eqs. (2) and (3) into (4), we have the following error dynamics:

$$\begin{cases} \dot{e}_1 = a(e_2 - e_1) \\ \dot{e}_2 = be_1 + ce_2 - x_2 z_2 + x_1 z_1 + u_1 \\ \dot{e}_3 = e_1^2 + 2e_1 x_1 - he_3 + u_2. \end{cases} \tag{5}$$

For the two identical chaotic systems without control (u_i=0, i=1, 2), the trajectories of the two identical systems will quickly separate and become irrelevant on the condition that initial values $(x_1(0),y_1(0),z_1(0)) \neq (x_2(0),y_2(0),z_2(0))$. However, with appropriate control schemes, the two systems will approach synchronization for any initial values.

Theorem 1. Systems (2) and (3) will approach global asymptotical synchronization for any initial condition with the following control law:

$$\begin{cases} u_1 = x_2 z_2 - x_1 z_1 - 2be_1 - 2ce_2 \\ u_2 = -e_1^2 - 2e_1 x_1. \end{cases} \tag{6}$$

Proof. Construct a Lyapunov function

$$V(t) = e^{\mathrm{T}} Pe \tag{7}$$

where $P = \begin{pmatrix} 0.7 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ is positive definite. So $V(t) > 0$.

With Eq. (6), the time derivative of the Lyapunov function along the trajectories of system (5) is

$$\dot{V}(t) = \dot{e}^{\mathrm{T}}Pe + e^{\mathrm{T}}P\dot{e} = -28e_1^2 - 21.2e_2^2 - 5.6e_3^2 = -e^{\mathrm{T}}Qe$$

where

$$Q = \begin{pmatrix} 28 & 0 & 0 \\ 0 & 21.2 & 0 \\ 0 & 0 & 5.6 \end{pmatrix}.$$

Obviously Q is positive definite. Hence, we have $\dot{V}(t) < 0$. By Lyapunov stability theory, the origin of error asymptotically stable. That is to say, systems (2) and (3) achieve global asymptotical synchronization.

4. Synchronization between the new chaotic system and Liu system

In order to observe the chaos synchronization behavior in the new chaotic system and Liu system [11], it is assumed that the new chaotic system drive Liu system. Thus, the drive and response systems are as follows:

$$\begin{cases} \dot{x}_1 = 20(y_1 - x_1) \\ \dot{y}_1 = 14x_1 + 10.6y_1 - x_1 z_1 \\ \dot{z}_1 = x_1^2 - 2.8z_1 \end{cases} \tag{8}$$

and

$$\begin{cases} \dot{x}_3 = 10(y_3 - x_3) + u_1 \\ \dot{y}_3 = 40x_3 - x_3 z_3 + u_2 \\ \dot{z}_3 = 4x_3^2 - 2.5z_3 + u_3. \end{cases} \tag{9}$$

For chaotic synchronization of above different systems, the error dynamical system is described by

$$\begin{cases} \dot{e}_1 = 10(e_2 - e_1) + 10(x_1 - y_1) + u_1 \\ \dot{e}_2 = 40e_1 + 26x_1 - 10.6y_1 - x_3 z_3 + x_1 z_1 + u_2 \\ \dot{e}_3 = 4x_3^2 - x_1^2 - 2.5e_3 + 0.3z_1 + u_3 \end{cases} \tag{10}$$

where

$$\begin{cases} e_1 = x_3 - x_1 \\ e_2 = y_3 - y_1 \\ e_3 = z_3 - z_1. \end{cases} \tag{11}$$

Now we definite the control functions u_1, u_2 and u_3 as follows:

$$\begin{cases} u_1 = -10(x_1 - y_1) \\ u_2 = -40e_1 - 26x_1 + 10.6y_1 + x_3 z_3 - x_1 z_1 - e_2 \\ u_3 = x_1^2 - 4x_3^2 - 0.3z_1 \end{cases} \tag{12}$$

Hence the error system (10) becomes

$$\begin{cases} \dot{e}_1 = -10e_1 + 10e_2 \\ \dot{e}_2 = -e_2 \\ \dot{e}_3 = -2.5e_3 \end{cases} \tag{13}$$

The error system (13) is a linear system of the form, $\dot{w} = Aw$. Thus by linear control theory, if the system matrix A is Hurwitz, the system is asymptotically stable. Hence the error system (13) with

$$A = \begin{pmatrix} -10 & 10 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2.5 \end{pmatrix}$$

has all eigenvalues with negative real parts. This guarantees the asymptotic stability of the system (13), which implies that the Liu system (9) synchronize the system (8).

5. Numerical simulations

In this section, to verify and demonstrate the effectiveness of the proposed methods, we consider two numerical examples. In the numerical simulations, the fourth-order Runge-Kutta method is used to solve the systems with time step size 0.001.

Example 1. Consider the systems given in (2) and (3). The initial values of the drive system and response system are taken as ($x_1(0)$, $y_1(0)$, $z_1(0)$)=(2,4,0), ($x_2(0)$, $y_2(0)$, $z_2(0)$)=(-1,-3,6) respectively. Thus, the initial errors are $e_1(0)$= -3, $e_2(0)$= -7 and $e_3(0)$=6. The simulation results are illustrated in Figs. 2 and 3. As we respect, one can observe that response system starts to trace drive system and finally becomes the same after $t \geq 2$.

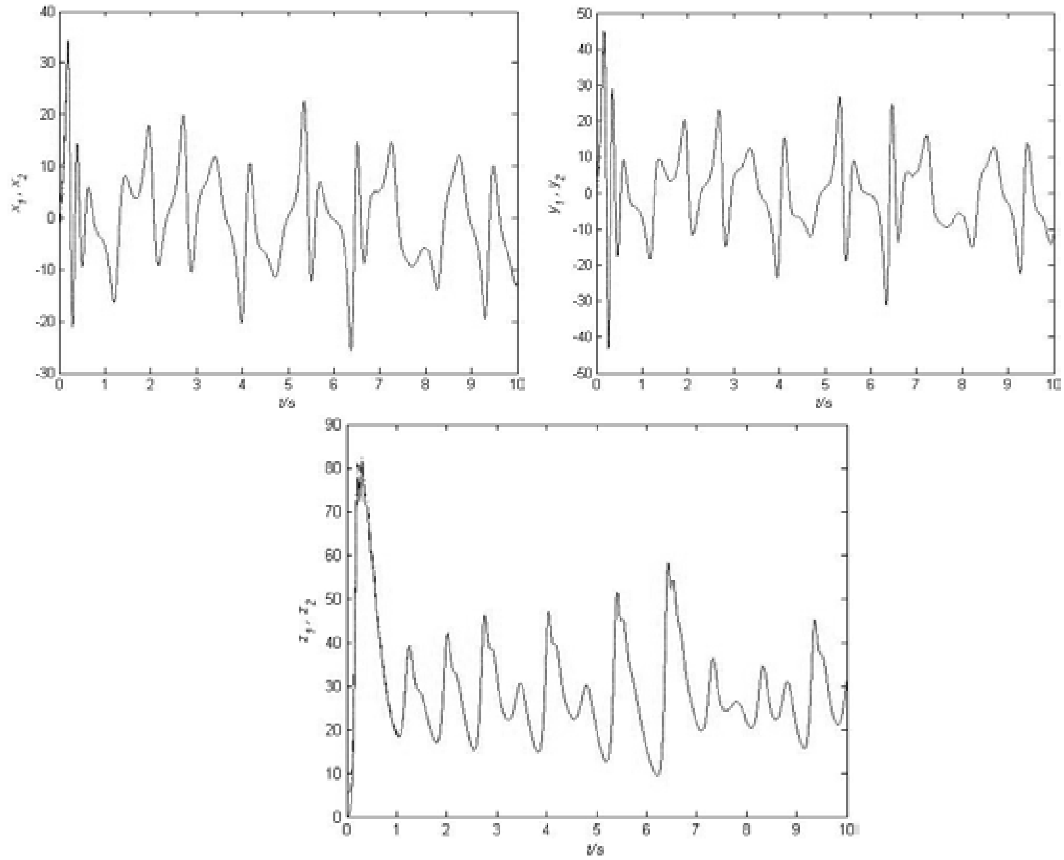

Fig. 2 State trajectories. drive system states (-), response system states(-.)

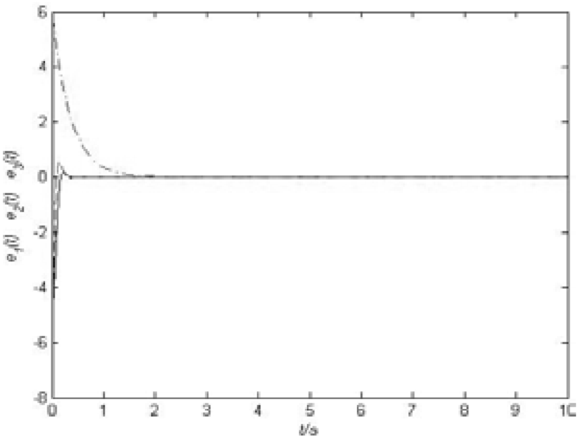

Fig. 3 Synchronization errors e_1(-),e_2(-.),e_3(--)

Example 2. Consider the systems given in (8) and (9). The initial values of the drive system and response system are taken as $(x_1(0), y_1(0), z_1(0))=(10,9,1)$, $(x_3(0), y_3(0), z_3(0))=(2,2,8)$ respectively. Thus, the initial errors are $(e_1(0), e_2(0), e_3(0))=(-8,-7,7)$. The simulation results are illustrated in Figs. 4 and 5. From the figures, it can be seen that the synchronization errors converge to zero and two different systems are indeed achieving chaos synchronization.

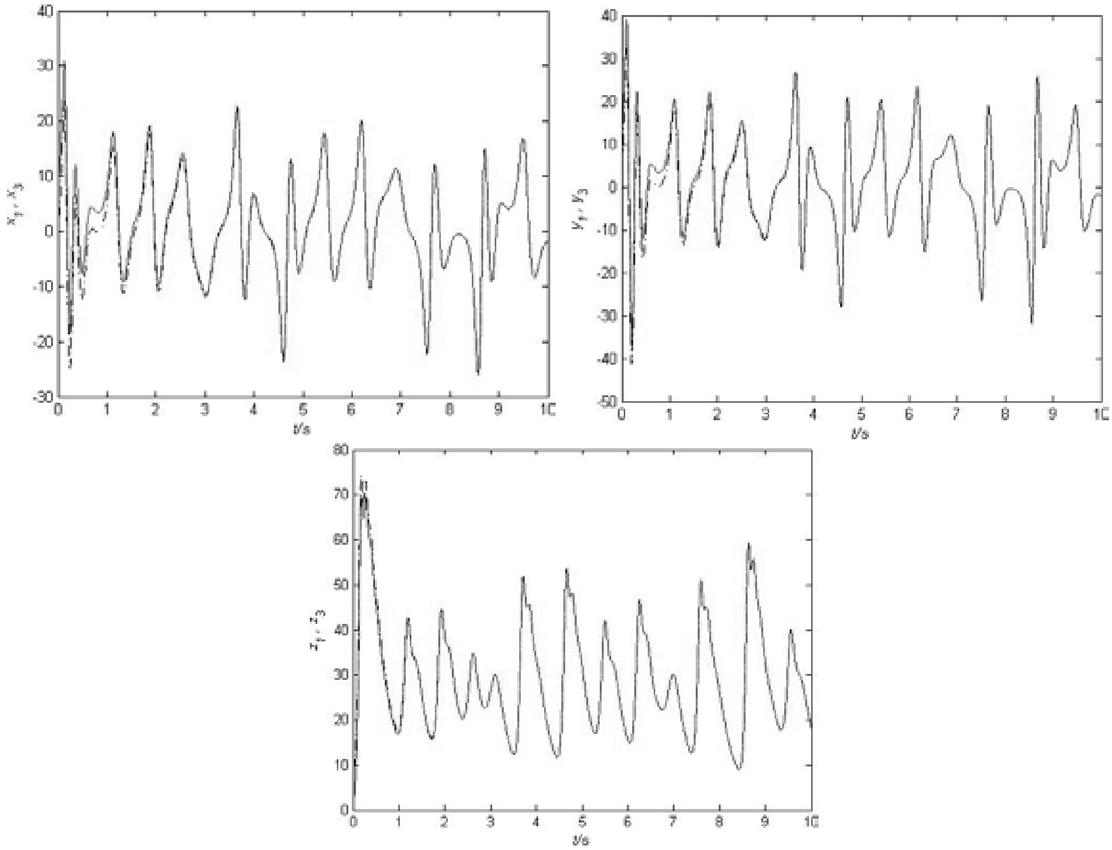

Fig. 4 State trajectories. drive system states (-), response system states (-.)

6. Conclusion

Nonlinear control is an effective method for making two identical or different chaotic systems be synchronized. In this paper, using nonlinear control, we have investigated the synchronization of a new chaotic system which was introduced in Ref. [9]. Numerical simulations are also provided to show the effectiveness and feasibility of the method.

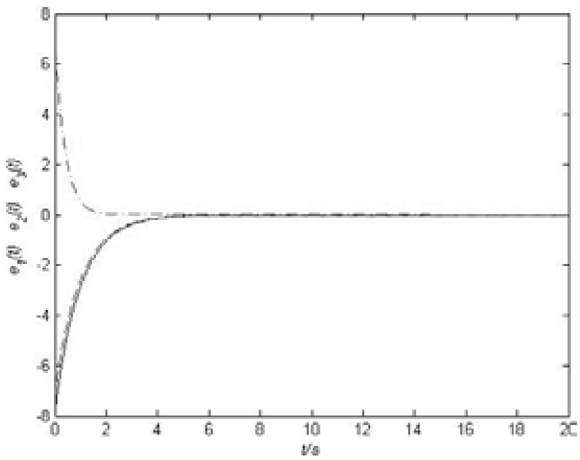

Fig. 5 Synchronization errors e_1(-),e_2(-.),e_3(--)

Acknowledgements

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