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# Random Fuzzy Variable Modeling on Repairable System

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**Abstract**

Repairable system analysis is in nature an evaluation of repair effects. Recent tendency in reliability engineering literature was imposing repair regimes and estimate system repair effects or linking repair to certain covariate to extract repair impacts. Hinted by engineering tune up exercises, we propose a repair model in terms of random variable distribution with a fuzzy parameter because fuzziness reflects the evolution of system dynamic rule changes according to its design specifications. In this paper, we develop an average chance distribution for random fuzzy lifetimes based on the foundational work of self-dual fuzzy credibility measure theory proposed by Liu (2004) and the traditional probability measure theory. We further propose a maximum average chance principle for data-assimilated parameter estimation, which will lead to two empirical distributions – an average chance empirical distribution and an empirical probability distribution with the expected fuzzy parameter as the point estimate for its parameter. The differences between the two filtered lifetimes will facilitate the repair effects. 

## 1. Introduction

Repairable system analysis is in nature an evaluation of repair effects. Recent tendency in reliability engineering literature was imposing repair regimes and estimate system repair effects or linking repair to certain covariate to extract repair impacts. Hinted by engineering tune up exercises, we propose a repair model in terms of random variable distribution with a fuzzy parameter because fuzziness reflects the evolution of system dynamic rule changes according to its design specifications. In this paper, we develop an average chance distribution for random fuzzy lifetimes and further propose a maximum average chance principle for data-assimilated parameter estimation, which will lead to two empirical distributions – an average chance empirical distribution and an empirical probability distribution with the expected fuzzy parameter as the point estimate for its parameter. The differences between the two filtered lifetimes are expected to facilitate the repair effects.

## 2. Random Fuzzy Lifetimes

### 2.1 Concept of random fuzzy lifetime and average chance distribution

Liu (2004) defined random fuzzy variable in a very formal way. However, it might be difficult for the reliability engineers. Therefore, we will give an intuitive and constructive definition.

Liu (2004) defined random fuzzy variable in a very formal way. However, it might be difficult for the reliability engineers. Therefore, we will give an intuitive and constructive definition.

**Definition 2.1**: A random fuzzy lifetime, denoted as $\xi = \{X_{\beta(\theta)}, \theta \in \Theta\}$, is a collection of positive real-valued random variables $X_\beta$>0 defined on the common probability space ($\Omega$,$\mathfrak{A}$,Pr) and indexed by a fuzzy variable $\beta(\theta)$ defined on the credibility space ($\Theta$, $2^\Theta$,Cr).

A random fuzzy lifetime, denoted by $\xi$, is a special case of random fuzzy variable. In other words, random fuzzy lifetime is a bivariate mapping from ($\Omega \times \Theta$, $\mathfrak{A} \times 2^\Theta$) to the space ($\mathbb{R}^+$, $\mathfrak{B}(\mathbb{R}^+)$).

**Definition 2.2**: (Liu and Liu, 2002) Let $\xi$ be a random fuzzy variable, then the *average* chance measure denoted by ch{$\cdot$}, of a random fuzzy event {$\xi \le x$}, is:

$$\mathrm{ch}\left\{\xi \le x\right\} = \int_0^1 \mathrm{Cr}\left\{\theta \in \Theta \middle| \Pr\left\{\xi\left(\theta\right) \le x\right\} \ge \alpha\right\} \mathrm{d}\,\alpha \tag{1}$$

Then function $\Phi(\cdot)$ is called as *average* chance distribution if and only if:

$$\Phi(x) = \mathrm{ch}\{\xi \le x\} \tag{2}$$

The nonnegative real-valued function $\phi_\xi(\cdot)$ is called average chance density for a random fuzzy variable $\xi$ if for $\phi_\xi$(x)≥0, $x \in \mathbb{R}$ such that:

$$\Phi_\xi(x) = \int_{-\infty}^{x} \phi_\xi(u)\,\mathrm{d}\,u \tag{3}$$

Liu (2004) mentioned an exponentially distributed random fuzzy variable $\xi$ has a density function:

$$f(x) = \begin{cases} \dfrac{1}{\beta}\exp\left(-\dfrac{x}{\beta}\right) & \text{if } x \ge 0 \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

if the value of $\beta$ is provided as a fuzzy variable, then $\xi$ is a random fuzzy variable. Accordingly, let fuzzy parameter $\beta$ be defined by a membership function, $\mu_\beta(\cdot)$, and the probability density is defined by Equation (4), then the random fuzzy variable $\xi$ is said to be exponentially distributed. This example hints a constructive definition for specifying or a random fuzzy variable and consequently the average chance distribution for the random fuzzy variable. We know that a probability distribution $F_X$ can define a random variable $X$, then the parameter of $F_X$ will index random variable $X$ too. If we allow the parameter of the distribution $F_X(\cdot\,;\beta)$ to be a fuzzy variable $\beta$ then without any doubts, $\beta$ also indexes the random variable $X$, denoted as $X_\beta$. Conversely, the fuzzy variable $\beta$ indexes random variable $X$, will index the corresponding distribution $F_X$ too. We state such an equivalence as a lemma.

**Lemma 2.4**: Let {$F(x;\beta(\theta)),\theta \in \Theta$ } be a family of probability distributions on the probability space ($\Omega$,$\mathfrak{A}$,Pr) with a common fuzzy variable (parameter) $\beta$ on the credibility space ($\Theta$, $2^\Theta$,Cr), which induces a membership function, $\mu_\beta(\cdot)$, then $\left\{F\left(\cdot\,;\beta\left(\theta\right)\right), \theta \in \Theta\right\}$ is equivalent to $\left\{X_{\beta(\theta)}, \theta \in \Theta\right\}$ and therefore defines a random fuzzy variable $\xi$.

The emphasis on the equivalence between probability distribution group and random variable group is not the something new and hence is not the unique creation for random fuzzy variable theory. In homogeneous continuous Markov chain theory, the (conservative) stochastic semi-group $\{P_t, t \geq 0\}$ and the random variable group $\{X_t, t \geq 0\}$ are equivalent in constructing the process. The next theorem extends the Lemma 2.4 further. A function $\Lambda$ is defined as a 1-1 mapping of probability distribution $F_X$. The familiar examples of $\Lambda$ are hazard function, survival function, and moment generating function of a random variable *X*. Function $\Lambda$ in genera can also be used to construct the group equivalent to $\left\{X_{\beta(\theta)}, \theta \in \Theta\right\}$.

**Theorem 2.5**: Let $\xi$ be a continuous random fuzzy lifetime having probability distribution function $F(t;\beta(\theta))$, where the fuzzy parameter $\beta$ is defined on the credibility space ($\Theta$, $2^{\Theta}$,Cr) with membership $\mu_\beta(\cdot)$. Then function $\Pi(\cdot)$ can uniquely define the random fuzzy lifetime $\xi$ if the operator or function $\Lambda$ such that $F(t;\beta)=\Lambda(\Pi(t;\beta))$.

The proof of the theorem is a straight application of Definition 2.1 and Lemma 2.4.

### 2.2 Continuous random fuzzy lifetime models

In statistical lifetime modeling and analysis, the elementary lifetime models are exponential, Weibull, Log-normal, gamma, and bathtub, etc.. These are essential for the construction of random fuzzy lifetimes. Table 1 lists these models. Whenever dealing with random fuzzy lifetime data analysis, we can construct the appropriate model for the data in terms of Theorem 2.5.

Table 1: Commonly used distributional lifetime models

| Name | Probability density & hazard function | |
|---|---|---|
| Exponential | density | $\beta \exp(-\beta t)$ |
| | hazard | $\beta$ |
| Weibull | density | $(\beta/\eta)(t/\eta)^{\beta-1} \exp(-(t/\eta))^{\beta}$ |
| | hazard | $(\beta/\eta)(t/\eta)^{\beta-1}$ |
| Extreme - value | density | $(1/u)\exp((t-b)/u)\exp(-\exp((t-b)/u))$ |
| | hazard | $(1/u)\exp((t-b)/u)$ |
| Log-Normal | density | $\left(1/\left(\sqrt{2\pi}\sigma t\right)\right)\exp\left(-(\ln t-\mu)^2/2\sigma^2\right)$ |
| | hazard | $\left(\left(1/\left(\sqrt{2\pi}\sigma t\right)\right)\exp\left(-(\ln t-\mu)^2/2\sigma^2\right)\right)/(1-\Phi((\ln t-\mu)/\sigma))$ |
| Gamma | density | $\left(\lambda(\lambda t)^{\beta-1}/\Gamma(\beta)\right)e^{-\lambda t}$ |
| | hazard | $\left(\lambda(\lambda t)^{\beta-1}/\Gamma(\beta)\right)e^{-\lambda t}/(1-I(\beta,\lambda t))$ |
| Bathtub | density | $(\beta/\eta)(t/\eta)^{\beta-1} \exp\left((t/\eta)^{\beta-1}\right)\exp\left(-\exp\left((t/\eta)^{\beta-1}\right)\right)$ |
| | hazard | $(\beta/\eta)(t/\eta)^{\beta-1} \exp\left((t/\eta)^{\beta-1}\right)$ |

In Table 2, $I(\beta,\lambda t)$ denotes the incomplete gamma function of the first-type and $\Phi(\cdot)$ represents the cumulative distribution of a standard normal variable.

### 2.3 Accelerated life testing models

Accelerated life testing is an important methodology in new product design and warrantee policy decision making. The basic assumption is that a change in stress factors only alters the scale, only the shape, of the failure time distribution.

**Definition 3.6**: (Accelerated random fuzzy life model) Let $F_0(t;\beta)$ be the baseline failure time distribution function for a random fuzzy lifetime $\xi$ having a fuzzy parameter $\beta$ defined on the credibility space ($\Theta$, $2^{\Theta}$, Cr) with membership $\mu_\beta(\cdot)$, then the accelerated random fuzzy life model specifies the probability distribution for the random fuzzy failure time under time-independent stress variable $z$ as:

$$F(t;\beta,z)=F_0(t\varsigma(z);\beta) \tag{5}$$

where function of stress variable $\varsigma:\mathbb{R}\rightarrow\mathbb{R}^{+}$.

The average chance distribution with stress variable $z$ is therefore:

$$\Phi(t;z)=\int_0^1 \mathrm{Cr}\{\theta : F_0(t\varsigma(z);\beta(\theta))\geq\alpha\}\,\mathrm{d}\,\alpha \tag{6}$$

where stress variable $z$ may be assumed to be either fuzzy or deterministic. The function, $\varsigma:\mathbb{R}\rightarrow\mathbb{R}^{+}$, is usually defined in terms of the relationship between the parameter of lifetime distribution and stress variable(s).

Well-known accelerated life models are power rule model:

$$\beta=\frac{\lambda}{z^{c}},\ \ c>0 \tag{7}$$

$$\beta=\lambda\exp\left(\frac{\delta}{T}\right),\ \ T>0 \tag{8}$$

Combined power rule and Arrhenius reaction rate model:

$$\beta=\lambda z^{-c}\exp\left(\frac{\delta}{T}\right),\ \ T>0,\ \ c>0 \tag{9}$$

Jurkov's model

$$\beta=\lambda\exp\left(\frac{\delta-Cz}{T}\right),\ \ \ T>0 \tag{10}$$

Generalized Eyring model:

$$\beta=\lambda T\exp\left(\frac{\delta}{T}\right)\exp\left(Cz+\frac{dz}{T}\right),\ \ \ T>0 \tag{11}$$

and others.

### 2.4 Proportional hazards model

Except accelerated testing model, another covariate model playing very important roles in lifetime analysis is Cox's (1972) proportional hazards (abbreviated as PH) model:

$$h(t;\beta,\gamma)=h_0(t;\beta)\varsigma(\gamma^{T}y) \tag{12}$$

where $h_0(t;\beta)$ is called the baseline hazard function having a fuzzy parameter $\beta$ defined on the credibility space $(\Theta, 2^{\Theta}, \widetilde{\mathrm{Cr}})$ with membership $\mu_\beta(\cdot)$, while $\varsigma:\mathbb{R}\to\mathbb{R}^+$ with:

$$\gamma^T y = \gamma_0 + \gamma_1 y_1 + ... + \gamma_p y_p \tag{13}$$

where $y=(1,y_1,\ldots,y_p)^T$ is covariate vector and $\gamma=(\gamma_0,\gamma_1,\ldots,\gamma_p)^T$ is covariate effect parameter vector. A typically function of $\varsigma:\mathbb{R}\to\mathbb{R}^+$ used is the exponential function $\varsigma(x)=\exp(x)$. It is easy to show that the accumulated hazard if covariate $y$ is not time-dependent is:

$$H(t;\beta,\gamma) = H_0(t;\beta)e^{\gamma^T y} \tag{14}$$

And therefore the average chance distribution with covariate $y$ is:

$$\Phi(t,y) = \sum_0^1 \mathrm{Cr}\left\{(\theta_1,\theta_2) : H_0(t;\beta(\theta_1))e^{\gamma^T y(\theta_2)} \ge -\ln(1-\alpha)\right\} d\alpha \tag{15}$$

where covariate $y$ is assumed to fuzzy but parameter $\gamma$ is assumed to be deterministic. Other options are also possible to be formulated.

It is necessary to mention here, the two types of covariate models are not only powerful in product reliability design, analysis but also useful in repairable system maintenance optimal planning and analysis. In probabilistic reliability literature, researchers have many useful developments. Therefore, in random fuzzy repairable system analysis it is necessary to bring them in.

## 3. Exponential Random Fuzzy Failure Times

From the Subsection 2.2, it is easy to see that exponential random fuzzy failure times are probably the simplest model to handle.

Let us use exponentially distributed random fuzzy lifetime which has probability density:

$$f(t;\beta) = \begin{cases} 0 & t \le 0 \\ \beta e^{-\beta t} & t > 0 \end{cases} \tag{16}$$

### 3.1 Exponential random fuzzy failure time with trapezoidal membership function

The fuzzy parameter $\beta$ needs four parameters for its specification.

$$\mu_\beta(x) = \begin{cases} \dfrac{x-a}{b-a} & a < x \le b \\ 1 & b < x \le c \\ \dfrac{d-x}{d-c} & c < x \le d \\ 0 & otherwise \end{cases} \tag{17}$$

Note that:

$$\Pr\{\xi(\theta) \le t\} = 1 - e^{\beta t} \tag{18}$$

Therefore event $\{\theta$: $\Pr\{\xi(\theta)\le t\}\ge\alpha\}$ is a fuzzy event and is equivalent to the fuzzy event $\{\theta$: $\beta(\theta)\ge -\ln(1-\alpha)/t\}$, now we need to find the credibility measure $\widetilde{\mathrm{Cr}}\{\theta: \beta(\theta)\ge -\ln(1-\alpha)/t\}$.

For trapezoidal fuzzy variable $\beta$, the credibility measure of event $\{\theta: \beta(\theta)\leq x\}$:

$$\tilde{\mathrm{Cr}}\{\theta:\beta(\theta)\leq x\}=\begin{cases} 0 & x\leq a \\ \dfrac{x-a}{2(b-a)} & a<x\leq b \\ \dfrac{1}{2} & b<x\leq c \\ \dfrac{x+d-2c}{2(d-c)} & c<x\leq d \\ 1 & x>d \end{cases} \tag{19}$$

Therefore the credibility measure of the complement event $\{\theta: \beta(\theta)\geq x\}$ is:

$$\tilde{\mathrm{Cr}}\{\theta:\beta(\theta)\geq x\}=\begin{cases} 1 & x\leq a \\ \dfrac{2b-x-a}{2(b-a)} & a<x\leq b \\ \dfrac{1}{2} & b<x\leq c \\ \dfrac{d-x}{2(d-c)} & c<x\leq d \\ 0 & x>d \end{cases} \tag{20}$$

Accordingly, the range for integration with $\alpha$ can be determined as shown in Table 2.

Table 2: Range analysis for $\alpha$

| $x=-\dfrac{\ln(1-\alpha)}{t}$ | Range for $\alpha$ | Expression for $\tilde{\mathrm{Cr}}\{\theta:\beta(\theta)\geq -\ln(1-\alpha)/t\}$ |
|---|---|---|
| $-\infty<x\leq a$ | $0\leq\alpha\leq 1-e^{-at}$ | $1$ |
| $a<x\leq b$ | $1-e^{-at}<\alpha\leq 1-e^{-bt}$ | $1-\dfrac{x-a}{2(b-a)}$ |
| $b<x\leq c$ | $1-e^{-bt}<\alpha\leq 1-e^{-ct}$ | $\dfrac{1}{2}$ |
| $c<x\leq d$ | $1-e^{-ct}\leq\alpha\leq 1-e^{-dt}$ | $1-\dfrac{x+d-2c}{2(d-c)}$ |
| $x>d$ | $1-e^{-dt}<\alpha\leq 1$ | $0$ |

The average chance distribution for the exponentially distributed random fuzzy lifetime is then

$$\Phi_{\xi}(\mathrm{t})=\int_{0}^{1}\tilde{\mathrm{Cr}}\left\{\theta:\beta(\theta)\geq-\ln(1-\alpha)/t\right\}\mathrm{d}\alpha \tag{21}$$

Note that the expression of $x=-\ln(1-\alpha)/t$ appears in Equation (21), which facilitates the link between intermediate variable $\alpha$ and average chance measure. The average chance distribution for the exponentially distributed random fuzzy lifetime is then derived by splitting the integration into five terms according to the range of $\alpha$ and the corresponding mathematical expression for the credibility measure $\tilde{\mathrm{Cr}}\left\{\theta:\beta(\theta)\geq-\ln(1-\alpha)/t\right\}$, which is detailed in Table 2. Then the exponential random fuzzy lifetime has an average chance distribution function:

$$\Phi_{\xi}(\mathrm{t})=1+\frac{e^{-bt}-e^{-at}}{2(b-a)t}+\frac{e^{-dt}-e^{-ct}}{2(d-c)t} \tag{22}$$

The average chance density for the exponentially distributed random fuzzy lifetime is then the derivative with respect to $t$:

$$\phi_{\xi}(\mathrm{t})=\frac{e^{-at}-e^{-bt}}{2(b-a)t^{2}}+\frac{be^{-bt}-ae^{-at}}{2(b-a)t}+\frac{e^{-ct}-e^{-dt}}{2(d-c)t^{2}}+\frac{ce^{-ct}-de^{-dt}}{2(d-c)t} \tag{23}$$

and the average chance reliability is $R_{\xi}(t)=1-\Phi_{\xi}(t)$, i.e.,

$$R_{\xi}(t)=\frac{e^{-at}-e^{-bt}}{2(b-a)t}+\frac{e^{-ct}-e^{-dt}}{2(d-c)t} \tag{24}$$

With an obvious reason, we work out all the technical details in step-by-step manner for explaining the insight of the average chance distribution of an exponential random fuzzy failure time. The other forms following this subsection are merely special cases of the trapezoidal membership function.

### 3.2 Exponential random fuzzy failure time with triangular membership function

A triangular membership function can be regarded as the special case when the parameter $b$ and parameter $c$ equal each other. In this way, the triangular membership takes the form

$$\mu_{\beta}(x)=\begin{cases}\dfrac{x-a}{b-a}, & a<x\leq b\\ \dfrac{d-x}{d-b}, & b<x\leq d\\ 0, & \text{Otherwise}\end{cases} \tag{25}$$

Similar to the analysis for $\alpha$ in Subsection 3.1, the average chance distribution for the exponentially distributed random fuzzy lifetime is then derived by splitting the integration into five terms according to the range of $\alpha$ and the corresponding mathematical expression for the credibility measure $\tilde{\mathrm{Cr}}\left\{\theta:\beta(\theta)\geq-\ln(1-\alpha)/t\right\}$, and thus the average chance distribution is:

$$\Phi_{\xi}\left(\mathrm{t}\right)=1+\frac{e^{-bt}-e^{-at}}{2\left(b-a\right)t}+\frac{e^{-dt}-e^{-bt}}{2\left(d-b\right)t} \tag{26}$$

The average chance density is

$$\phi_{\xi}\left(\mathrm{t}\right)=\frac{e^{-at}-e^{-bt}}{2\left(b-a\right)t^{2}}+\frac{be^{-bt}-ae^{-at}}{2\left(b-a\right)t}+\frac{e^{-bt}-e^{-dt}}{2\left(d-b\right)t^{2}}+\frac{be^{-bt}-ce^{-dt}}{2\left(d-b\right)t} \tag{27}$$

The average chance reliability is

$$R_{\xi}\left(t\right)=\frac{e^{-at}-e^{-bt}}{2\left(b-a\right)t}+\frac{e^{-bt}-e^{-dt}}{2\left(d-b\right)t} \tag{28}$$

## 4. The Revelation of Intrinsic Dynamics of a Repairable System

To facilitate a repairable system modeling in terms of random fuzzy failure models comes from the bivariate mapping nature.

### 4.1 A philosophical understanding of fuzzy logic

The idea to use random fuzzy failure time models for facilitating repairable system modeling roots from our basic understanding on fuzzy phenomenon, described by fuzzy set in earlier fuzzy mathematics – possibility measure based theoretical framework or by fuzzy variable under fuzzy credibility measure theory framework, particularly, the fuzzy parameter for specifying the system failure model. Logically speaking, randomness and fuzziness are two different types of uncertainty. Randomness is logically the break down of the law of causality because of the lack of some conditions under which the event occurrence is inevitable. This is traditionally a well-received formalization of uncertainty in terms of the usage of probability calculus by science and engineering. Fuzziness is logically the break down of the law of excluding the middle, but this is less well known and is often ignored by the communities of engineering and management, particularly, reliability engineering. Evolution appears in all aspects around us, no matter in natural world, social phenomena, or engineering practices. Any system contains many factors, many strata, and many intermediaries. These interconnections, interactions, and within the system strata must have intermediate links. Therefore holding on the intermediate strata of the system structure is a necessary step to understanding system underlying dynamics in the way of entirety. Fuzzy membership function is the appropriate mathematical mechanism reflecting the evolving system state from one strum to another.

### 4.2 Random fuzzy model reveals the system state intermediate evolution

In standard statistical lifetime modeling and analysis reliability function of a system reveals the system functioning (probabilistic) behavior. Similarly, the average chance reliability function reveals the system co-existing random and fuzzy behavior in general. It is necessary to emphasize here that the average chance reliability function reveals also the system state intermediate evolution pattern.

In order to gain an intuitive perceptions on the average chance reliability function, let us assume that the triangular membership function defined by $a=0.1$, $b=(c=)0.25$, and $d=0.30$. Recall that if we treat the system obeying an exponential probability law, for example, the exponential random lifetime has a parameter $m_{\beta}=E(\beta)=0.225$.

Figure 1 gives a comparison between $R_{\xi}(t;\beta)$ and $R(t;0.225)$. It is easy to see that replacing a fixed value parameter, for the case in Figure 1, 0.225 by the fuzzy parameter $\beta$ with triangular membership (0.1, 0.25, 0.30) enables the system state evolution to be revealed. In other words, in case of system failure observations, denoted by $\{t_1, t_2, \cdots, t_n\}$, is from a exponential random fuzzy system, however, we use an exponential random model to analyze it, the system reliability with exponential random modeling will underestimate the “true” reliability, which will lead us into wrong maintenance decision on the system.

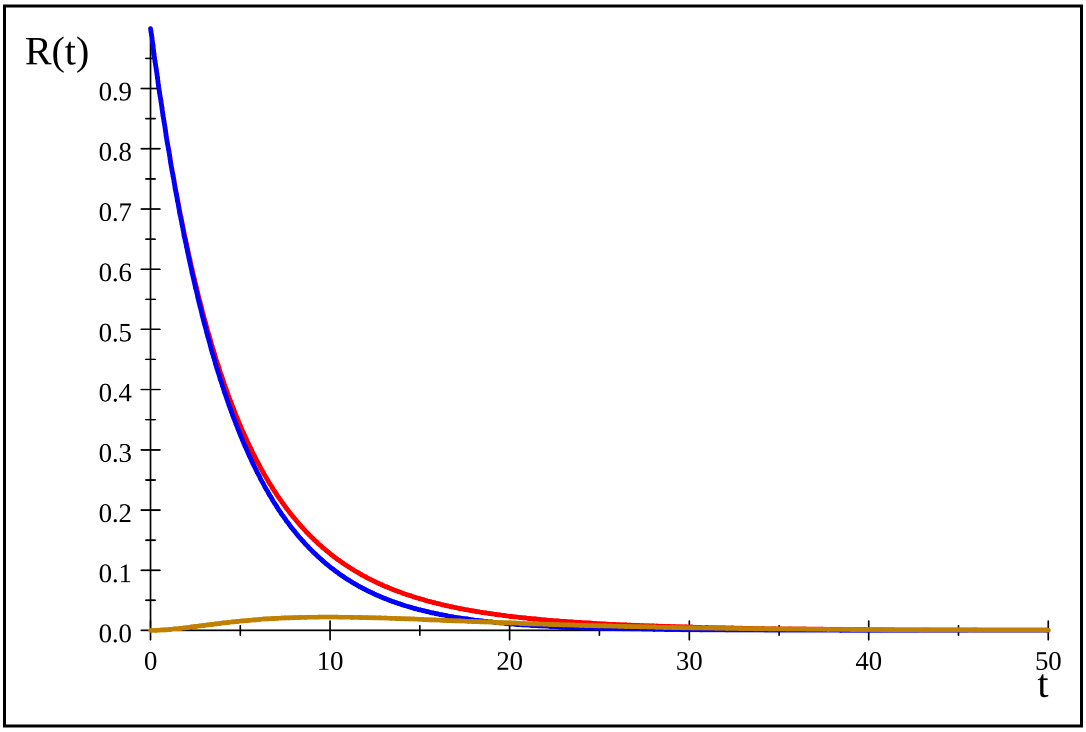


Figure 1: Average chance reliability $R_\xi(t;\beta)$ (Red) of an exponential random fuzzy failure time with triangular parameter $(0.1,\ 0.25,\ 0.30)$, corresponding exponential lifetime reliability $R(t;0.225)$ (Blue), and the difference function $d\left(R_\xi(t;\beta), R(t;0.225)\right)$ (Sienna)

**4.3 Maintenance is one of the root the causes for system state intermediate evolution**

What are the root causes which may trigger the system state intermediate evolution? To address this problem, we need to examine a system operating environment and system functioning characters.

Because a complex system is constituted by relevant hard subsystems and soft subsystems, the content of reliability analysis and computation of the system operating behavior inevitably involves the reliability of every subsystem (no matter soft or hard one) and relationships between subsystems. In this context, the fuzzy problem comes from the uncertainty and non-describable knowledge on the operating reliability of each individual subsystem and the operating coordination of the subsystems.

1. Time impacts on reliability of individual subsystems. Time fact affect subsystem reliability can be analyzed from two angles: materials constituting of the subsystems are wearing our and downgrading according to specifications in long term and the shape and strength of materials are changed associated with movements in short term.

2. Operating environmental impacts on reliability of individual subsystem. Operation environment involves hard side, say, temperature, humidity, dust, light etc and soft side, say, work floor culture and in general company and local social culture

environment. It is worth to stress here, the environmental factors interact with time factors and such relationships are difficult to evaluate and therefore a fuzzy issue appears here for consideration.

3. Human behaviour impacts on reliability of individual subsystem. In today's globalization environment, more and more complex systems are international-made. Inevitably the human factors affect system reliability directly and indirectly during the system design, manufacturing, shipping and the end-usage operating. Even making the focus narrow to system operating, it is obvious the human and system (machine) interaction is often too complicated to describe. This will be a fuzzy problem again. However, more and more complex system with automatic control subsystems and human (operator) restricted overriding function can not use simple models to handle them.

4. System design impacts on reliability of individual system. Today's quality starts at design stage. The allocation of reliability to individual subsystem is not known completely. As a matter of fact, the system operating behaviour is unknown in principle before the system being manufacturing and putting into functioning. Therefore, the investigation and analysis of the complex system reliability is enabling the system information from fuzzy state, i.e., an intermediate evolution.

5. System maintenance impacts on reliability of individual system. It is generally acknowledged that maintenance should improve the system reliability and thus the performance. However, in real-world, the opposite often occurs, i.e., maintenance may damage the system.

In summary, the fuzzy problems appeared in complex system reliability analysis lie in shorting of system structural clarity, shorting of the underlying mechanism of the interaction between subsystems and shorting of overall information of the system as a whole. Accordingly, identifying the root causes affecting the system behaviour will be very difficult.

Mathematically, the positive impacts which help system improvements and the negative impacts which cause system deteriorations can be appropriately modelled by random fuzzy failure time model since the system state intermediate evolution is a reflection of fuzzy behaviour.

Therefore, appropriate maintenance will improve the system reliability and becomes the cause of system state intermediate links evolving toward higher reliability.

## 5. Mean Square Error Filtering

Filtering is an important approach to fit the failure time dynamics under certain optimal criterion. With the fitted failure times under different model assumptions, we may evaluate the system improvement or damage impacts from maintenance work.

It is noticed that in linear model theory, we may assume the system failure times $\{t_1, t_2, \cdots, t_n\}$ operated under mean square error criterion:

$$J = \frac{1}{n}\sum_{i=1}^{n} w_i \left(t_i - \hat{t}_i\right)^2 \tag{29}$$

where $\{\hat{t}_1, \hat{t}_2, \cdots, \hat{t}_n\}$ are fitted or filtered time corresponding to $\{t_1, t_2, \cdots, t_n\}$ respectively, and $\{w_1, w_2, \cdots, w_n\}$ are the weight valued in Equation (29). If the data $\{t_1, t_2, \cdots, t_n\}$ are *i.i.d.* from exponential

### 5.1 Mean square error filtering with exponential density as weight function

If the data $\{t_1, t_2, \cdots, t_n\}$ are assumed to sampled *i.i.d.* from exponential distribution with fixed parameter $\beta_0$, then let:

$$w_i = \beta_0 \exp(-\beta_0 t_i),\ i = 1, 2, \cdots, n \tag{30}$$

We will have:

$$\lim_{n\to\infty} J = E\left[\left(T - \hat{T}\right)^2\right] \tag{31}$$

However, we notice that $\beta_0$ is not available and can be only estimated from data, denoted the estimate as $\hat{\beta}_0$, the weight should be replaced by:

$$\hat{w}_i = \hat{\beta}_0 \exp\left(-\hat{\beta}_0 t_i\right),\ i = 1, 2, \cdots, n \tag{32}$$

where $\hat{\beta}_0$ is a maximum likelihood estimator for parameter $\beta_0$. Then, we can use the genetic algorithm search the filtered failure time, $\{\hat{t}_1^r, \hat{t}_2^r, \cdots, \hat{t}_n^r\}$ such that the object function

$$\hat{J} = \frac{1}{n}\sum_{i=1}^{n} \hat{w}_i \left(t_i - \hat{t}_i^r\right)^2 \tag{33}$$

is minimized, where the superscript $r$ indicates the exponential random failure model.

### 5.2 Mean square error filtering with random fuzzy exponential density as weight function

If the data $\{t_1, t_2, \cdots, t_n\}$ are assumed to sampled *i.i.d.* from exponential distribution with a fuzzy parameter $\beta$, then let:

$$w_i = \phi_\xi(t_i),\ i = 1, 2, \cdots, n \tag{34}$$

We will have:

$$\lim_{n\to\infty} J = E\left[\left(T - \hat{T}\right)^2\right] \tag{35}$$

However, we notice that the parameters for specifying fuzzy vaiable $\beta$ is not available and can be only estimated from data, for example, in terms of maximum average chance principle, denoted the estimate as $\hat{\theta} = \left(\hat{\theta}_1, \hat{\theta}_2, \cdots, \hat{\theta}_p\right)$, the weight should be replaced by:

$$\hat{w}_i = \phi_\xi\left(t_i; \hat{\theta}\right),\ i = 1, 2, \cdots, n \tag{36}$$

We can also use the genetic algorithm search the filtered failure time, $\{\hat{t}_1^{rf}, \hat{t}_2^{rf}, \cdots, \hat{t}_n^{rf}\}$ such that

$$\hat{J} = \frac{1}{n}\sum_{i=1}^{n} \hat{w}_i \left(t_i - \hat{t}_i^{rf}\right)^2 \tag{37}$$

is minimized, where the superscript $rf$ indicates the exponential random fuzzy failure time model.

### 5.3 Grey differential equation filtering

Grey differential equation model is a small sample based approach without imposing distributional assumptions. Guo (2007) give a systematic discussion on grey differential equation filtering and it might be very easy to be implemented.

### 5.4 Repair effects

Once the filtering sequences are calculated, the repair effect, denoted by $r_i$ at time $t_i$, $i = 1, 2, \cdots, n$, is defined by:

$$r_i = t_i^r - t_i^{rf}, \ i = 1, 2, \cdots, n \tag{38}$$

(i) If $r_i > 0$, we say that the maintenance is adequate to cover the wear out and shock damages so that system gets improved;

(ii) If $r_i < 0$, we say that the maintenance is not adequate to cover system wear out or shock damages such that the system is improved;

(iii) If $r_i = 0$, the maintenance and the wear out or shock damages are balanced so that system remains the same.

## 6. Conclusions

In this paper, we define the random fuzzy failure time in an intuitive and constructive manner and then engage the discussion on the average chance distribution for the random fuzzy failure time. An exponential random fuzzy failure time with trapezoidal membership example is detailed developed for illustration purpose. Accelerated testing model and proportional hazards model are also reviewed with the intention to facilitate a repairable system model counting for all relevant covariate information. Finally, we explore the two filtering model so that the maintenance effects are estimated. Note here, the maintenance effect estimated here is the difference between the “true” maintenance improvement effect and system wear out effect or shock damage effects.

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