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A Survey of Entropy of Fuzzy Variables*

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Abstract

Given a fuzzy variable, what is the degree of difficulty of predicting the specified value that the fuzzy variable will take? Entropy provides a measure to characterize the fuzziness. Given some constraints, there are usually multiple compatible membership functions. Which membership function shall we take? The maximum entropy principle attempts to select the membership function that maximizes the value of entropy. This paper provides a personal (not balanced) overview of entropy of fuzzy variables and fuzzy maximum entropy principle. Three basic requirements for entropy of fuzzy variables are also introduced. Finally, the cross-entropy and quadratic entropy of fuzzy variables are documented.
Keywords: fuzzy variable, credibility measure, entropy, maximum entropy principle

1 Introduction

The concept of fuzzy set was initiated by Zadeh [25] via membership function in 1965. In order to measure a fuzzy event, Zadeh [26] proposed the concept of possibility measure in 1978. Since then possibility theory has been studied by many researchers. Although possibility measure has been widely used, it has no self-duality property. However, a self-dual measure is absolutely needed in both theory and practice. In order to define a self-dual measure, Liu and Liu [15] introduced the concept of credibility measure in 2002. Li and Liu [11] gave a sufficient and necessary condition for credibility measure in 2006.

Credibility theory was founded by Liu in 2004 [17] and refined by Liu [19] in 2007 as a branch of mathematics for studying the behavior of fuzzy phenomena. Credibility theory is deduced from the normality, increasing, self-duality, and maximality axioms. Then we have some basic concepts such as fuzzy variable, membership function, credibility distribution, expected value, variance, critical value, entropy, distance, and fundamental theorems such as credibility subadditivity theorem, credibility extension theorem, credibility semicontinuity law, product credibility theorem, and credibility inversion theorem. A detailed survey may be found in Liu [18].

Inspired by the Shannon entropy of random variables (Shannon [22]), fuzzy entropy is a measure of fuzziness and has been studied by many researchers such as De Luca and Termini [3], Kaufmann [8], Yager [23], Kosko [9], Pal and Pal [20], Bhandari and Pal [1], and Pal and Bezdek [21]. The above definitions of entropy characterize the uncertainty resulting primarily from the linguistic vagueness rather than resulting from information deficiency, and vanishes when the fuzzy variable is an equipossible one.

However, we hope that an entropy of fuzzy variables meets the following three basic requirements. *Minimum:* the entropy of a crisp number is minimum, i.e., 0. *Maximum:* the entropy of an

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equipossible fuzzy variable is maximum. *Universality:* the entropy is applicable not only to finite and infinite cases but also to discrete and continuous cases.

In order to meet these requirements, Li and Liu [10] provided a new definition of fuzzy entropy to characterize the uncertainty resulting from information deficiency which is caused by the impossibility to predict the specified value that a fuzzy variable takes.

In many real problems, there is only partial information about fuzzy variable such as expected value and variance. In this case, there is an infinity of membership functions consistent with the given information. Which membership function shall we take? Following the idea of *maximum entropy principle* by Jaynes [7], we will select the membership function that maximizes the value of entropy and satisfies the prescribed constraints. Li and Liu [12] proved some maximum entropy theorems for fuzzy variables.

The purpose of this paper is to provide a personal overview of entropy of fuzzy variables rather than a balanced survey of all activities in the area. The paper is organized as follows. Section 2 introduces the five axioms and the concepts of credibility measure, credibility space, fuzzy variable, membership function, expected value, variance, and moments. Sections 3 and 4 provided the definition of entropy of fuzzy variables as well as it basic properties. Section 5 deals with maximum entropy principle. Section 6 provides the concept of cross-entropy of fuzzy variables. Section 7 is devoted to the quadratic entropy. The paper is concluded in Section 8.

2 Credibility Theory

Let Θ be a nonempty set, and let $\mathcal{P}$ be the power set of Θ (i.e., all subsets of Θ). Each element in $\mathcal{P}$ is called an event. In order to present an axiomatic definition of credibility, we accept the following four axioms:

Axiom 1. *(Normality)* $\text{Cr}\{\Theta\} = 1$.

Axiom 2. *(Monotonicity)* $\text{Cr}\{A\} \le \text{Cr}\{B\}$ *whenever* $A \subset B$.

Axiom 3. *(Self-Duality)* $\text{Cr}\{A\} + \text{Cr}\{A^c\} = 1$ *for any* $A \in \mathcal{P}$.

Axiom 4. *(Maximality)* $\text{Cr}\{\cup_i A_i\} = \sup_i \text{Cr}\{A_i\}$ *for any events* $\{A_i\}$ *with* $\sup_i \text{Cr}\{A_i\} < 0.5$.

Definition 1 *(Liu and Liu [15]) The set function* Cr *on the power set* $\mathcal{P}$ *is called a credibility measure if it satisfies the four axioms.*

It is easy to verify that $\text{Cr}\{\emptyset\} = 0$ and the credibility measure takes values between 0 and 1.

Let $\Theta = \{\theta_1, \theta_2\}$. For this case, there are only four events: $\emptyset, \{\theta_1\}, \{\theta_2\}, \Theta$. Define $\text{Cr}\{\emptyset\} = 0$, $\text{Cr}\{\theta_1\} = 0.7$, $\text{Cr}\{\theta_2\} = 0.3$, and $\text{Cr}\{\Theta\} = 1$. Then the set function Cr is a credibility measure.

Let Θ be a nonempty set. Define $\text{Cr}\{\emptyset\} = 0$, $\text{Cr}\{\Theta\} = 1$ and $\text{Cr}\{A\} = 1/2$ for any subset A (excluding $\emptyset$ and Θ). Then the set function Cr is a credibility measure.

Suppose that the credibility of each singleton set is given. Is the credibility measure fully and uniquely determined? The following theorem will answer the question.

Theorem 1 *(Li and Liu [11], Credibility Extension Theorem) Suppose that* Θ *is a nonempty set, and* $\text{Cr}\{\theta\}$ *is a nonnegative function on* Θ *satisfying the credibility extension condition*

$$\begin{gathered}\sup_{\theta\in\Theta} \text{Cr}\{\theta\} \ge 0.5, \\ \text{Cr}\{\theta^*\} + \sup_{\theta\ne\theta^*} \text{Cr}\{\theta\} = 1 \textit{ if } \text{Cr}\{\theta^*\} \ge 0.5.\end{gathered} \tag{1}$$

Then $\mathrm{Cr}\{\theta\}$ *has a unique extension to a credibility measure on* $\mathcal{P}$ *as follows,*

$$\mathrm{Cr}\{A\} = \begin{cases} \sup\limits_{\theta\in A} \mathrm{Cr}\{\theta\}, & \textit{if } \sup\limits_{\theta\in A} \mathrm{Cr}\{\theta\} < 0.5 \\ 1 - \sup\limits_{\theta\in A^c} \mathrm{Cr}\{\theta\}, & \textit{if } \sup\limits_{\theta\in A} \mathrm{Cr}\{\theta\} \ge 0.5. \end{cases} \tag{2}$$

Definition 2 *Let* Θ *be a nonempty set,* $\mathcal{P}$ *the power set of* Θ*, and* Cr *a credibility measure. Then the triplet* $(\Theta, \mathcal{P}, \mathrm{Cr})$ *is called a credibility space.*

Product credibility measure is also needed for fuzzy arithmetic, and may be defined in multiple ways. In order to make it uniquely, we need the following axiom.

Axiom 5. *(Product Credibility Axiom) Let* Θ_k *be nonempty sets on which* Cr_k *are credibility measures,* $k = 1, 2, \cdots, n$*, respectively, and* $\Theta = \Theta_1 \times \Theta_2 \times \cdots \times \Theta_n$*. Then*

$$\mathrm{Cr}\{(\theta_1, \theta_2, \cdots, \theta_n)\} = \mathrm{Cr}_1\{\theta_1\} \wedge \mathrm{Cr}_2\{\theta_2\} \wedge \cdots \wedge \mathrm{Cr}_n\{\theta_n\} \tag{3}$$

for each $(\theta_1, \theta_2, \cdots, \theta_n) \in \Theta$.

Theorem 2 *(Product Credibility Theorem) Let* Θ_k *be nonempty sets on which* Cr_k *are the credibility measures,* $k = 1, 2, \cdots, n$*, respectively, and* $\Theta = \Theta_1 \times \Theta_2 \times \cdots \times \Theta_n$*. Then* $\mathrm{Cr} = \mathrm{Cr}_1 \wedge \mathrm{Cr}_2 \wedge \cdots \wedge \mathrm{Cr}_n$ *defined by Axiom 5 has a unique extension to a credibility measure on* Θ *as follows,*

$$\mathrm{Cr}\{A\} = \begin{cases} \sup\limits_{(\theta_1,\theta_2\cdots,\theta_n)\in A} \min\limits_{1\le k\le n} \mathrm{Cr}_k\{\theta_k\}, & \textit{if } \sup\limits_{(\theta_1,\theta_2,\cdots,\theta_n)\in A} \min\limits_{1\le k\le n} \mathrm{Cr}_k\{\theta_k\} < 0.5 \\ 1 - \sup\limits_{(\theta_1,\theta_2,\cdots,\theta_n)\in A^c} \min\limits_{1\le k\le n} \mathrm{Cr}_k\{\theta_k\}, & \textit{if } \sup\limits_{(\theta_1,\theta_2,\cdots,\theta_n)\in A} \min\limits_{1\le k\le n} \mathrm{Cr}_k\{\theta_k\} \ge 0.5. \end{cases} \tag{4}$$

Traditionally, a fuzzy variable is defined by a membership function (Zadeh [25]). Now we define it as a function on a credibility space just as a random variable is defined as a measurable function on a probability space.

Definition 3 *A fuzzy variable is a function from a credibility space* $(\Theta, \mathcal{P}, \mathrm{Cr})$ *to the set of real numbers.*

Note that any function on a credibility space is measurable because $\mathcal{P}$ is the power set of Θ.

A fuzzy variable ξ is said to be continuous if $\mathrm{Cr}\{\xi = x\}$ is a continuous function of x, and discrete if there exists a countable sequence $\{x_1, x_2, \cdots\}$ such that $\mathrm{Cr}\{\xi \ne x_1, \xi \ne x_2, \cdots\} = 0$.

Let $\Theta = [0, 1]$ and $\mathrm{Cr}\{\theta\} = \theta/2$ for each $\theta \in \Theta$. By using the credibility extension theorem, we may produce a credibility measure on $\mathcal{P}$. Then $(\Theta, \mathcal{P}, \mathrm{Cr})$ is a credibility space, and the identity function $\xi(\theta) = \theta$ is a fuzzy variable.

A crisp number c may be regarded as a special fuzzy variable. In fact, it is the constant function $\xi(\theta) \equiv c$ on the credibility space $(\Theta, \mathcal{P}, \mathrm{Cr})$.

Definition 4 *Let* ξ *be a fuzzy variable defined on the credibility space* $(\Theta, \mathcal{P}, \mathrm{Cr})$*. Then its membership function is derived from the credibility measure by*

$$\mu(x) = (2\mathrm{Cr}\{\xi = x\}) \wedge 1, \quad x \in \Re. \tag{5}$$

In practice, a fuzzy variable may be specified by a membership function. In this case, we need a formula to calculate the credibility value of some fuzzy event. The credibility inversion theorem provides this.

Theorem 3 *(Credibility Inversion Theorem) Let ξ be a fuzzy variable with membership function μ. Then for any set B of real numbers, we have*

$$\mathrm{Cr}\{\xi \in B\} = \frac{1}{2}\left(\sup_{x\in B}\mu(x) + 1 - \sup_{x\in B^c}\mu(x)\right). \tag{6}$$

If a fuzzy variable is defined as a function on a credibility space, then we may get its membership function via (5). Conversely, if a fuzzy variable is given by a membership function, then we may get the credibility value via (6).

For fuzzy variables, there are many ways to define an expected value operator. See, for example, Dubois and Prade [4], Heilpern [6], Campos and González [2], González [5] and Yager [24]. The most general definition of expected value operator of fuzzy variable was given by Liu and Liu [15]. This definition is applicable to both continuous and discrete fuzzy variables.

Definition 5 *(Liu and Liu [15]) Let ξ be a fuzzy variable. Then the expected value of ξ is defined by*

$$E[\xi] = \int_0^{+\infty} \mathrm{Cr}\{\xi \ge r\}\mathrm{d}r - \int_{-\infty}^0 \mathrm{Cr}\{\xi \le r\}\mathrm{d}r \tag{7}$$

provided that at least one of the two integrals is finite.

The equipossible fuzzy variable on $[a, b]$ has an expected value $(a + b)/2$. The triangular fuzzy variable (a, b, c) has an expected value $(a + 2b + c)/4$. The trapezoidal fuzzy variable (a, b, c, d) has an expected value $(a + b + c + d)/4$.

Let ξ be a continuous nonnegative fuzzy variable with membership function μ. If μ is decreasing on $[0, +\infty)$, then $\mathrm{Cr}\{\xi \ge x\} = \mu(x)/2$ for any $x > 0$, and

$$E[\xi] = \frac{1}{2}\int_0^{+\infty} \mu(x)\mathrm{d}x.$$

Let ξ be a continuous fuzzy variable with membership function μ. If its expected value exists, and there is a point x_0 such that $\mu(x)$ is increasing on $(-\infty, x_0)$ and decreasing on $(x_0, +\infty)$, then

$$E[\xi] = x_0 + \frac{1}{2}\int_{x_0}^{+\infty} \mu(x)\mathrm{d}x - \frac{1}{2}\int_{-\infty}^{x_0} \mu(x)\mathrm{d}x.$$

The definition of expected value operator is also applicable to discrete case. Assume that ξ is a simple fuzzy variable whose membership function is given by

$$\mu(x) = \begin{cases} \mu_1, & \text{if } x = a_1 \\ \mu_2, & \text{if } x = a_2 \\ \cdots & \\ \mu_m, & \text{if } x = a_m \end{cases} \tag{8}$$

where $a_1, a_2, \cdots, a_m$ are distinct numbers. Note that $\mu_1 \vee \mu_2 \vee \cdots \vee \mu_m = 1$. Then the expected value of ξ is

$$E[\xi] = \sum_{i=1}^m w_i a_i \tag{9}$$

where the weights are given by

$$w_i = \frac{1}{2}\left(\max_{1\le j\le m}\{\mu_j | a_j \le a_i\} - \max_{1\le j\le m}\{\mu_j | a_j < a_i\} + \max_{1\le j\le m}\{\mu_j | a_j \ge a_i\} - \max_{1\le j\le m}\{\mu_j | a_j > a_i\}\right)$$

for $i = 1, 2, \cdots, m$. It is easy to verify that all $w_i \geq 0$ and the sum of all weights is just 1.

The variance of a fuzzy variable provides a measure of the spread of the distribution around its expected value. A small value of variance indicates that the fuzzy variable is tightly concentrated around its expected value; and a large value of variance indicates that the fuzzy variable has a wide spread around its expected value.

Definition 6 *(Liu and Liu [15]) Let ξ be a fuzzy variable with finite expected value e. Then the variance of ξ is defined by $V[\xi] = E[(\xi - e)^2]$.*

A fuzzy variable ξ is called normally distributed (Li and Liu [12]) if it has a normal membership function

$$\mu(x) = 2\left(1 + \exp\left(\frac{\pi|x-e|}{\sqrt{6}\sigma}\right)\right)^{-1}, \quad x \in \Re, \sigma > 0. \tag{10}$$

The expected value is e and variance is σ^2.

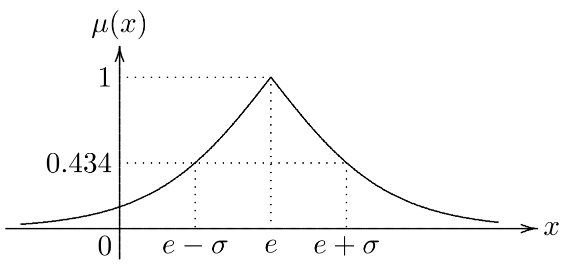

Figure 1: Normal Membership Function

Definition 7 *(Liu [16]) Let ξ be a fuzzy variable, and k a positive number. Then the expected value $E[\xi^k]$ is called the kth moment.*

A fuzzy variable ξ is called exponentially distributed (Li and Liu [12]) if it has an exponential membership function

$$\mu(x) = 2\left(1 + \exp\left(\frac{\pi x}{\sqrt{6}m}\right)\right)^{-1}, \quad x \geq 0, \, m > 0. \tag{11}$$

The second moment is m^2.

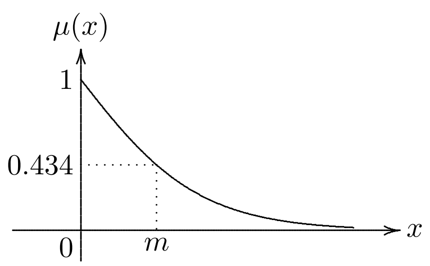

Figure 2: Exponential Membership Function

3 Entropy of Discrete Fuzzy Variables

Li and Liu [10] provided the following definition of entropy to characterize the fuzziness resulting from information deficiency which is caused by the impossibility to predict the specified value that a fuzzy variable takes.

Definition 8 *(Li and Liu [10]) Let ξ be a discrete fuzzy variable taking values in $\{x_1, x_2, \cdots\}$. Then its entropy is defined by*

$$H[\xi] = \sum_{i=1}^{\infty} S(\text{Cr}\{\xi = x_i\}) \tag{12}$$

where $S(t) = -t \ln t - (1-t)\ln(1-t)$.

It is easy to verify that $S(t)$ is a symmetric function about $t = 0.5$, strictly increases on the interval $[0, 0.5]$, strictly decreases on the interval $[0.5, 1]$, and reaches its unique maximum $\ln 2$ at $t = 0.5$.

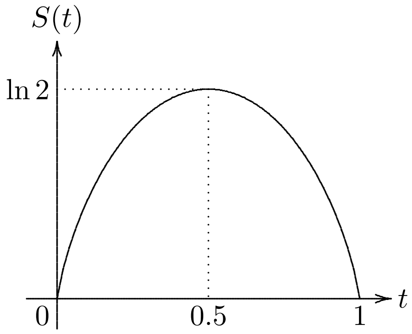

Figure 3: Function $S(t) = -t \ln t - (1-t)\ln(1-t)$

It is clear that the entropy depends only on the number of values and their credibilities and does not depend on the actual values that the fuzzy variable takes.

Suppose that ξ is a discrete fuzzy variable taking values in $\{x_1, x_2, \cdots\}$. If there exists some index k such that the membership function $\mu(x) = 1$ if $x = x_k$, and 0 otherwise, then its entropy $H[\xi] = 0$. If its membership function $\mu(x) \equiv 1$, then its entropy $H[\xi] = n \ln 2$.

Theorem 4 *(Li and Liu [10]) Let ξ be a discrete fuzzy variable taking values in $\{x_1, x_2, \cdots\}$. Then*

$$H[\xi] \geq 0 \tag{13}$$

and equality holds if and only if ξ is essentially a crisp number.

This theorem states that the entropy of a fuzzy variable reaches its minimum 0 when the fuzzy variable degenerates to a crisp number. In this case, there is no uncertainty.

Theorem 5 *(Li and Liu [10]) Let ξ be a simple fuzzy variable taking values in $\{x_1, x_2, \cdots, x_n\}$. Then*

$$H[\xi] \leq n \ln 2 \tag{14}$$

and equality holds if and only if ξ is an equipossible fuzzy variable.

This theorem states that the entropy of a fuzzy variable reaches its maximum when the fuzzy variable is an equipossible one. In this case, there is no preference among all the values that the fuzzy variable will take.

4 Entropy of Continuous Fuzzy Variables

Definition 9 *(Li and Liu [10]) Let ξ be a continuous fuzzy variable. Then its entropy is defined by*

$$H[\xi] = \int_{-\infty}^{+\infty} S(\mathrm{Cr}\{\xi = x\})\mathrm{d}x \tag{15}$$

where $S(t) = -t\ln t - (1-t)\ln(1-t)$.

For any continuous fuzzy variable ξ with membership function μ, we have $\mathrm{Cr}\{\xi = x\} = \mu(x)/2$ for each $x \in \Re$. Thus

$$H[\xi] = -\int_{-\infty}^{+\infty} \left(\frac{\mu(x)}{2}\ln\frac{\mu(x)}{2} + \left(1 - \frac{\mu(x)}{2}\right)\ln\left(1 - \frac{\mu(x)}{2}\right)\right)\mathrm{d}x. \tag{16}$$

The equipossible fuzzy variable on $[a,b]$ has entropy $(b-a)\ln 2$. The triangular fuzzy variable (a,b,c) has entropy $(c-a)/2$. The trapezoidal fuzzy variable (a,b,c,d) has entropy $(d-a)/2 + (\ln 2 - 0.5)(c-b)$.

Let ξ be an exponentially distributed fuzzy variable with second moment m^2. Then its entropy is $H[\xi] = \pi m/\sqrt{6}$.

Let ξ be a normally distributed fuzzy variable with expected value e and variance σ^2. Then its entropy is $H[\xi] = \sqrt{6}\pi\sigma/3$.

Theorem 6 *(Li and Liu [10]) Let ξ be a continuous fuzzy variable. Then $H[\xi] > 0$.*

When a continuous fuzzy variable tends to a crisp number, its entropy tends to the minimum 0. However, a crisp number is not a continuous fuzzy variable.

Theorem 7 *(Li and Liu [10]) Let ξ be a continuous fuzzy variable taking values on the interval $[a,b]$. Then*

$$H[\xi] \le (b-a)\ln 2 \tag{17}$$

and equality holds if and only if ξ is an equipossible fuzzy variable on $[a,b]$.

5 Maximum Entropy Principle

Given some constraints, for example, expected value and variance, there are usually multiple compatible membership functions. Which membership function shall we take? The *maximum entropy principle* attempts to select the membership function that maximizes the value of entropy and satisfies the prescribed constraints.

Theorem 10 *(Li and Liu [12]) Let ξ be a continuous nonnegative fuzzy variable with finite second moment m^2. Then*

$$H[\xi] \le \frac{\pi m}{\sqrt{6}} \tag{18}$$

and the equality holds if ξ is an exponentially distributed fuzzy variable with second moment m^2.

This theorem states that the exponentially distributed fuzzy variable has maximum entropy when the nonnegativity is assumed and the second moment is given in advance.

Theorem 11 *(Li and Liu [12]) Let ξ be a continuous fuzzy variable with finite expected value e and variance σ^2. Then*

$$H[\xi] \le \frac{\sqrt{6}\pi\sigma}{3} \tag{19}$$

and the equality holds if ξ is a normally distributed fuzzy variable with expected value e and variance σ^2.

This theorem states that the normally distributed fuzzy variable has maximum entropy when the variance is prescribed.

6 Cross-Entropy

Bhandari and Pal [1] defined the concept of cross-entropy for fuzzy sets. Li and Liu [13] defined the cross-entropy of fuzzy variables by using credibility measure.

Theorem 12 *(Li and Liu [13]) Let ξ and η be two discrete fuzzy variables taking values in $\{x_1, x_2, \cdots\}$. Then the cross-entropy of ξ with respect to η is defined by*

$$H^c[\xi;\eta] = \sum_{i=1}^{\infty} S^c\left(\mathrm{Cr}\{\xi = x_i\}, \mathrm{Cr}\{\eta = x_i\}\right) \tag{20}$$

where $S^c(s,t) = s\ln\left(\dfrac{s}{t}\right) + (1-s)\ln\left(\dfrac{1-s}{1-t}\right)$.

Theorem 13 *(Li and Liu [13]) Let ξ and η be two continuous fuzzy variables. Then the cross-entropy of ξ with respect to η is defined as*

$$H^c[\xi;\eta] = \int_{-\infty}^{+\infty} S^c\left(\mathrm{Cr}\{\xi = x\}, \mathrm{Cr}\{\eta = x\}\right) \mathrm{d}x. \tag{21}$$

If ξ and η have membership functions μ and ν, respectively, then $\mathrm{Cr}\{\xi = x\} = \mu(x)/2$, $\mathrm{Cr}\{\eta = x\} = \nu(x)/2$, and

$$H^c[\xi;\eta] = \int_{-\infty}^{+\infty} \left(\frac{\mu(x)}{2}\ln\left(\frac{\mu(x)}{\nu(x)}\right) + \left(1 - \frac{\mu(x)}{2}\right)\ln\left(\frac{2-\mu(x)}{2-\nu(x)}\right)\right) \mathrm{d}x. \tag{22}$$

Li and Liu [13] proved that $H^c[\xi;\eta] \ge 0$ and the equality holds if and only if ξ and η have the same membership function.

7 Quadratic Entropy

A function with properties similar to the function $S(t)$ may serve as a substitute and provide a new definition for measuring the uncertainty of fuzzy variables. Li and Liu [10] proposed that the quadratic function $Q(t) = t(1-t),\ t \in [0,1]$ can be used to define the quadratic entropy of fuzzy variables. If ξ is a discrete fuzzy variable taking values in $\{x_1, x_2, \cdots\}$, then its quadratic entropy is

$$H^q(\xi) = \sum_{i=1}^{\infty} Q(\mathrm{Cr}\{\xi = x_i\}). \tag{23}$$

If ξ is a continuous fuzzy variable, then its quadratic entropy is defined by

$$H^q(\xi) = \int_{-\infty}^{+\infty} Q(\mathrm{Cr}\{\xi = x\})\mathrm{d}x. \tag{24}$$

It is easy to prove that the quadratic entropy reaches its minimum 0 when the fuzzy variable degenerates to a crisp number, and achieves its maximum when the fuzzy variable is an equipossible one.

8 Conclusion

This paper gave an introduction to credibility theory, and reviewed the concepts of entropy of fuzzy variables. Fuzzy maximum entropy principle was also introduced via two theorems: (a) the exponentially distributed fuzzy variable has maximum entropy when the nonnegativity is assumed and the second moment is given in advance; and (b) the normally distributed fuzzy variable has maximum entropy when the variance is prescribed. Finally, the cross-entropy and quadratic entropy of fuzzy variables were documented.

Shannon entropy has shown great success in management and engineering problems. It is expected to apply fuzzy entropy to real decision problems with fuzziness. It is also believed that the fuzzy entropy may play an important role in general information theory.

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