Plane Side Distance and Stochastic Matter

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Abstract

In this paper, we study side distance problems of the matter analysis, the definitions of side distance are extended to plane region, and some properties are obtained. We give fundamental concept for stochastic extension set according to stochastic processes methods, and spread extension set, and obtain some new results.
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Keywords: matter analysis, plane side distance, left side distance, right side distance, inside distance, outside distance, stochastic extension set, stochastic matter

1 Plane Side Distance and Its Properties

Cai [1] put forward the concept of matter element analysis and extension set, and obtained some useful results. Later, these problems have been restudied and some new results were obtained [2-5]. Cai [2] put forward the distance on line. In this paper, we study the distance on plane.

1.1 Fundamental Concept

We know concept of side distance in [2]. In this paper, we study it on plane region.

Definition 1: The distance of a point $M_0(x_0, y_0)$ to a plane region D is defined as

$$\rho(M_0, D) = \pm \min \rho(M_0, M), \ M \in \overline{D},$$

where $\overline{D}$ expresses the boundary of D. If $M_0 \in D$, then it takes negative sign, i.e., $\rho(M_0, D) < 0$. If $M_0 \notin D$, then it takes positive sign, i.e., $\rho(M_0, D) > 0$.

Obviously, we have $M_0 \in \overline{D} \Longleftrightarrow \rho(M_0, D) = 0$.

Definition 2: Suppose that D_1 and D_2 are two plane regions, and $\rho(D_1, D_2) = \min \rho(M, D_2)$, $M \in \overline{D_1}$. Then we call $\rho(D_1, D_2)$ the distance of plane regions D_1 and D_2.

Definition 3: Suppose plane region series D_a, D_0, D_{ab} and D_b satisfy the following relations: $D_a \subset D_0 \subset D_{ab} \subset D_b$, where

$$\overline{D}_{ab} = \left\{ M \mid \rho(M, D_a) = -\rho(M, D_b) \right\}.$$

Let

$$\rho_I = \rho_I(M, D_a, D_0, D_b) = \begin{cases} -\rho(M, D_a), & M \in D_a \\ \dfrac{\rho(D_b, D_0)}{\rho(D_a, D_0)} \rho(M, D_a), & M \in (D_0 - D_a) \\ \rho(M, D_b), & M \notin D_0. \end{cases}$$

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Then we call ρ_1 inside distance of M about D_a, D_0, D_b.

Definition 4: Suppose plane region series D_a, D_0, D_{ab} and D_b satisfy the following relations: $D_a \subset D_{ab} \subset D_0 \subset D_b$, where

$$\overline{D}_{ab} = \{M \mid \rho(M, D_a) = -\rho(M, D_b)\}.$$

Let

$$\rho_0 = \rho_0(M, D_a, D_0, D_b) = \begin{cases} \rho(M, D_b), & M \notin D_b \\ -\dfrac{\rho(D_a, D_0)}{\rho(D_b, D_0)}\rho(M, D_b), & M \in (D_b - D_0) \\ -\rho(M, D_a), & M \in D_0. \end{cases}$$

Then we call ρ_0 outside distance of M about D_a, D_0, D_b.

The inside distance and outside distances are called side distance, and denoted by $\rho(M, D_a, D_0, D_b)$.

1.2 Some Results

Theorem 1: If $M \notin D_b$, then we have

$$\rho(M, D_a, D_0, D_b) = \rho(M, D_b).$$

Proof: According to the above definitions, if $M \notin D_b$, then we have $M \notin D_a \cup D_b$.

Therefore, the two equations

$$\rho_I(M, D_a, D_0, D_b) = \rho(M, D_b),$$

and

$$\rho_0(M, D_a, D_0, D_b) = \rho(M, D_b)$$

hold true. The proof is complete.

Theorem 2: Suppose $D_a \subset D_0 \subset D_b$, then

(1) $M \in D_b - D_a \Longleftrightarrow \rho(M, D_a, D_0, D_b) < 0$.

(2) $M \notin D_b - D_a$ and $M \notin \overline{D}_a \cup \overline{D}_b \Longleftrightarrow \rho(M, D_a, D_0, D_b) > 0$.

(3) $M \in \overline{D}_b \cup \overline{D}_a \Longleftrightarrow \rho(M, D_a, D_0, D_b) = 0$.

Proof: (1) Let $M \in D_b - D_a$.

① To ρ_1: as $M \in D_0 - D_a$, we have $\rho(M, D_a) > 0$, then

$$\rho_I = \frac{\rho(D_b, D_0)}{\rho(D_a, D_0)}\rho(M, D_a) < 0.$$

As $M \in D_b - D_0$, then $\rho(M, D_b) < 0$. Therefore $\rho_I = \rho(M, D_b) < 0$.

② To ρ_0: as $M \in D_0 - D_a$, we have $\rho(M, D_a) > 0$, then $\rho_0 = -\rho(M, D_a) < 0$ as $M \in D_b - D_0$, and

$$\rho_0 = -\frac{\rho(D_a, D_0)}{\rho(D_b, D_0)}\rho(M, D_b) < 0.$$

Therefore

$$\rho(M, D_a, D_0, D_b) > 0.$$

Otherwise, if $\rho(M, D_a, D_0, D_b) < 0$, then we have:

① For ρ_I: if $M \in D_a$, then we have $\rho_I = -\rho(M, D_a) > 0$, a contradiction.

If $M_0 \notin D_b$, then $\rho_I = \rho(M, D_b) > 0$, a contradiction.

Therefore, we have $M \in D_b - D_a$.

② For ρ_0: if $M \in D_a$, then $\rho(M, D_a) < 0$.

Therefore, $\rho_0 = -\rho(M, D_a) > 0$, a contradiction.

If $M \notin D_b$, then $\rho_0 = \rho(M, D_b) > 0$, a contradiction.

Thus, we have $M \in D_b - D_a$

(2) Suppose $M \notin D_b \cup \overline{D}_a \cup \overline{D}_b$, if $M \notin D_b$, then we have $M \notin D_0$. Thus

① For ρ_1: $\rho_I = \rho(M, D_b) > 0$.

② For ρ_0: $\rho_0 = \rho(M, D_b) > 0$, thus $\rho(M, D_a, D_0, D_b) > 0$.

Otherwise, suppose $\rho(M, D_a, D_0, D_b) > 0$, then

① For ρ_1: if $M \in D_b - D_a$, as $M \in D_0 - D_a$, we have

$$\rho_1 = \frac{\rho(D_b, D_0)}{\rho(D_a, D_0)} \rho(M, D_a) < 0,$$

which is a contradiction . Thus, $M \notin D_0 - D_a$.

If $M \in D_b - D_0$, then $\rho_1 = \rho(M, D_b) < 0$, a contradiction.

Thus $M \notin D_b - D_0$, and $M \in D_b - D_a$.

If $M \in \overline{D}_a \cup \overline{D}_b$, then $\rho(M, D_a, D_0, D_b) = 0$, a contradiction.

Thus $M \notin \overline{D}_a \cup \overline{D}_b$.

② for ρ_0: If $M \in D_b - D_0$, then

$$\rho_0 = -\frac{\rho(D_a, D_0)}{\rho(D_b, D_0)} \rho(M, D_b) < 0,$$

which is self- contradictory.

If $M \in D_0 - D_a$, then $\rho_0 = -\rho(M, D_a) < 0$, self-contradictory.

Thus $M \notin D_b - D_a$.

(3) If $M \in \overline{D}_b \cup \overline{D}_a$, then $\rho_I = \rho(M, D_b) = 0$, $\rho_0 = \rho(M, D_b) = 0$.

Otherwise, if $\rho(M, D_a, D_0, D_b) = 0$, and $M \notin \overline{D}_a \cup \overline{D}_b$, then $\rho(M, D_a, D_0, D_b) \neq 0$, a contradiction. Thus $M \in \overline{D}_a \cup \overline{D}_b$.

If $D_{ab} = D_0$, then

$$\rho_I = \begin{cases} -\rho(M, D_a), M \in D_0 \\ \rho(M, D_b), M \notin D_0, \end{cases}$$

and

$$\rho_0 = \begin{cases} \rho(M, D_b), M \notin D_0 \\ -\rho(M, D_a), M \in D_0. \end{cases}$$

According to the above theorems, we have the following definition:

Definition 5: Suppose plane region series: $D_a \subset D_0 \subset D_b$, where

$$\overline{D}_0 = \{ M \mid \rho(M, D_a) = -\rho(M, D_b) \}.$$

Let

$$\rho = \rho(M, D_a, D_0, D_b) = \begin{cases} -\rho(M, D_a), M \in D_0 \\ \rho(M, D_b), M \notin D_0. \end{cases}$$

Then we call ρ side distance of M about D_a, D_0, D_b.

Suppose that $D_a = D_b$, then it is easily to obtain

$$\rho = \rho(M, D_a, D_0, D_b) = \begin{cases} -\rho(M, D_a), M \in D_a \\ \rho(M, D_b), M \in D_a. \end{cases}$$

Sometimes, we have

$$S(M, D_a, D_b) = \begin{cases} -1, M \in D_a \\ \rho(M, D_b) - \rho(M, D_a), M \notin D_a. \end{cases}$$

Suppose

$$K(M) = \frac{\rho(M, D_a, D_b)}{S(M, D_a, D_b)},$$

then

$$K(M) = \begin{cases} \rho(M, D_a), M \in D_a \\ \dfrac{\rho(M, D_b)}{\rho(M, D_b) - \rho(M, D_a)}, M \notin D_a. \end{cases}$$

Theorem 3: Suppose $\overline{D}_a \cap \overline{D}_b = \Phi$, then we have

(1) $M \in D_a$ or $M \notin D_b \Longleftrightarrow K(M) < 0$.

(2) $M \in D_b - D_a \Longleftrightarrow 0 < K(M) < 1$.

(3) $M \in \overline{D}_a \Longleftrightarrow K(M) = 1$.

(4) $M \in \overline{D}_b \Longleftrightarrow K(M) = 0$.

Proof: (1) If $M \in D_a$, then we have $K(M) = \rho(M, D_a) < 0$.

Otherwise, if $K(M) < 0$, and $M \in D_b - D_a$, then $\rho(M, D_a) > 0$, and $\rho(M, D_b) < 0$.
Thus

$$K(M) = \frac{\rho(M, D_b)}{\rho(M, D_b) - \rho(M, D_a)} > 0,$$

$$\rho(M, D_a) > 0,$$

and

$$\rho(M, D_a) > \rho(M, D_b).$$

Therefore, we have $K(M) < 0$.

(2) If $M \in D_b - D_a$, then $\rho(M, D_b) < 0$, $\rho(M, D_a) > 0$, thus $0 < K(M) < 1$.

Otherwise, if $0 < K(M) < 1$, then $M \in D_a, K(M) < 0$, a contradiction.

On the other hand, we have

$$0 < \frac{\rho(M, D_b)}{\rho(M, D_b) - \rho(M, D_a)} < 1,$$

which have solution

$$\begin{cases} \rho(M, D_a) > 0 \\ \rho(M, D_b) > 0 \\ \rho(M, D_b) < \rho(M, D_a) \end{cases}$$

Thus $M \in D_b - D_a$.

(3) If $M \in \overline{D}_a$, then

$$K(M) = \frac{\rho(M, D_b)}{\rho(M, D_b)} = 1.$$

Otherwise, if $K(M) = 1$, then $\rho(M, D_a) = 0$. Thus $M \in \overline{D}_a$.

(4) If $M \in \overline{D}_b$, then $K(M) = 0$.

Otherwise, if $K(M) = 0$, then $\rho(M, D_b) = 0$. Thus $M \in \overline{D}_b$.

Theorem 4: Let $\overline{D}_a \cap \overline{D}_b = \{M_z\} \neq \Phi$, and

$$K(M) = \begin{cases} \dfrac{\rho(M, D_a, D_b)}{S(M, D_a, D_b)}, S(M, D_a, D_b) \neq 0 \\ -\rho(M, D_a, D_b) - 1, S(M, D_a, D_b) = 0. \end{cases}$$

Then we have

(1) $M \in D_a$ or $M \notin D_b \Longleftrightarrow K(M) < 0$.

(2) $M \in D_b - D_a \Longleftrightarrow 0 < K(M) < 1$.

(3) $M = M_z \Rightarrow K(M) = -1$.

Proof: (1) If $M \in D_a$, then $K(M) = \rho(M, D_a) < 0$. If $M \notin D_b$, then

$$\rho(M, D_a) > \rho(M, D_b) > 0$$

Thus

$$K(M) = \frac{\rho(M, D_b)}{\rho(M, D_b) - \rho(M, D_a)} < 0.$$

Otherwise, if $K(M) < 0$, and $M \in D_b - D_a$, then we have $K(M) > 0$, a contradiction.

Thus, $M \in D_a$ or $M \notin D_b$ holds true.

(2) If $M \in D_b - D_a$, then we have $\rho(M, D_b) < 0$, and $\rho(M, D_a) > 0$.

Thus $\rho(M, D_b) - \rho(M, D_a) < 0$, i.e., $0 < K(M) < 1$.

Otherwise, if $0 < K(M) < 1$, and $M \in D_a$ or $M \notin D_b$, then we have $K(M) < 0$.

Thus we have $M \in D_b - D_a$.

(3) If $M = M_2$, then $\rho(M, D_a) = \rho(M, D_b) = 0$.

Thus

$$K(M) = -\rho(M, D_a, D_b) - 1 = -1.$$

2 Stochastic Extension Set and Stochastic Matter

2.1 Fundamental Concept

We study stochastic extension set and stochastic matter using stochastic processes theory [6-7].

Definition 1: Suppose Ω is a sample space, and (Ω, F, p) is a probability space, T is an index set, and $K(t, \omega)$ is a function from $T \times \Omega$ to real region $R = (-\infty, +\infty)$. Let

$$\tilde{A} = \{(t, \omega, y) \mid t \in T, \omega \in \Omega, y = K(t, \omega)\}.$$

Then we call $\tilde{A}$ a stochastic extension set on Ω, $y = K(t, \omega)$ (or $K_t(\omega)$) a stochastic relational function on $\tilde{A}$ (or a stochastic relational process), $K(t, \omega)$ stochastic relational degree.

Let

$$A = \{(t, \omega) \mid t \in T, \omega \in \Omega, K(t, \omega) \geq 0\}.$$

Then we call A positive region of $\tilde{A}$.

Let

$$\overline{A} = \{(t, \omega) \mid t \in T, \omega \in \Omega, K_t(\omega) \leq 0\}.$$

Then we call $\overline{A}$ negative region of $\tilde{A}$.

Let

$$J_0\tilde{A} = \{(t, \omega) \mid t \in T, \omega \in \Omega, K_t(\omega) = 0\}.$$

Then we call $J_0(\tilde{A})$ zero boundary of $\tilde{A}$.

To determine $t = t_0$, let

$$A_{t_0} = \{\omega \mid k_{t_0}(\omega) \geq 0\},$$

$$\overline{A}_{t_0} = \{\omega \mid K_{t_0}(\omega) \leq 0\},$$

and

$$J_{t_0}(\tilde{A}) = \{\omega \mid k_{t_0}(\omega) = 0\}.$$

We know, for $\tilde{A}$, when $t = t_0$, $y = k(t_0, \omega)$ is a random variable; when $\omega = \omega_0$, $y = k(t, \omega_0)$ is a relational function, and we call it a relational sample track of $\tilde{A}$.

Definition 2: Suppose $\tilde{A}$ is a stochastic extension set, T_0 is an alternation, and $T_0 U \subset U$, then we call

$$A(T_0) = \{(t, \omega) \mid t \in T, \omega \in \Omega, K(T_0(t, \omega)) \geq 0\}$$

the positive region of $\tilde{A}$ about T_0;

$$A(T_0) = \{(t, \omega) \mid t \in T, \omega \in \Omega, K(T_0(t, \omega)) \leq 0\}$$

the negative region of $\tilde{A}$ about T_0;

$$A(T) = \{(t, \omega) \mid t \in T, \omega \in \Omega, K((t, \omega)) \leq 0, K(T_0(t, \omega)) \geq 0\}$$

the extension region of $\tilde{A}$ about T_0, and

$$A_+(T_0) = \{(t, \omega) \mid t \in T, \omega \in \Omega, K(t, \omega) \geq 0, K((T_0(t, \omega)) \geq 0\}$$

the stable region of $\tilde{A}$ about T_0.

If $t = t_0$, then the sets defined in the above are random events.

2.2 Some Results

Theorem 1: Suppose $\tilde{A}$ is a stochastic extension set, T_0 is an alternation, then

(1) $T \times \Omega = A \cup \overline{A}$;

(2) $\Omega = A_{t_0} \cup \overline{A}_{t_0}$;

(3) $A \cap \overline{A} = J_0(\overline{A})$;

(4) $A_{t_0} \cap \overline{A}_{t_0} = J_{t_0}(\tilde{A})$;

(5) $A(T_0) = A(T_0) \cup A_+(T_0)$.

Proof: The results can be easily proved according to the definitions in the above.

Furthermore, we have the following result:

Theorem 2: Suppose (Ω, F, P) is a probability space, $\tilde{A}$ is a stochastic extension set, T_0 is an alternation, then

(1) $P(A_{t_0} \cup \overline{A}_{t_0}) = P(A_{t_0}) + P(\overline{A}_{t_0}) - P(J_{t_0}(\overline{A}))$.

(2) $P(A_{t_0} \cap \overline{A}_{t_0}) = P\big(J_{t_0}(\tilde{A})\big)$.

According to the probability theory, we know that $\{K(t, \omega), t \in T, \omega \in \Omega\}$ is a stochastic process. Suppose that $\{K_t(\omega), t \in T\}$ is a stochastic relational process, then for every $t \in T$, $k_t(\omega)$ is a random variable, and its distribution function is

$$P_{k(t)}(t, \xi) \triangleq P\big(\{\omega \mid k_t(\omega) < \xi\}\big).$$

We call $P_{k(t)}(t, \xi)$ the one-dimensional distribution function of $\{K_t(\omega), t \in T\}$.

In this paper, we often define an *n*-dimensional distribution function as

$$P_{k(t_1), k(t_2), \cdots, k(t_n)}(t_1, \xi_1; t_2, \xi_2; \cdots; t_n, \xi_n) \triangleq P(\bigcap_{i=1}^{n} \{\omega : K(t_i, \omega) < \xi_i\}).$$

Then we have:

Mean function

$$\mu_k(t) \triangleq E\{k_t(\omega)\} = \int_{-\infty}^{+\infty} \xi P_{K(t)}(t, \xi) d\xi .$$

Variance function

$$\text{Var}\{k(t)\} \triangleq E\Big\{\big[k(t) = \mu_k(t)\big]^2\Big\}$$

$$= \int_{-\infty}^{+\infty} \big(\xi - \mu_k(t)\big)^2 P_{K(t)}(t, \xi) d\xi.$$

Self-covariance

$$C_k(s, t) \triangleq \text{cov}\{k(s), K(t)\}$$

$$= E\big\{\big[k(s) - \mu_k(s)\big]\big[K(t) - \mu_k(t)\big]\big\}$$

$$= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \big[\xi - \mu_k(s)\big]\big[\eta - \mu_k(t)\big] P_{k(s), k(t)}(s, \xi; t, \eta) d\xi d\eta.$$

Self-relative function

$$R_k(s, t) \triangleq E\{k(s), K(t)\}$$

$$= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \xi\eta P_{K(s), k(t)}(\xi, \eta) d\xi d\eta.$$

2.3 Stochastic Matter

In the above subsections, we have given some fundamental concepts for stochastic relative functions, and with these concepts, we can obtain stochastic matter used in matter analysis.

Definition 3: Suppose *W* is a matter set defined as

$$W = \{R \mid R = (N, C, v), N \in U, v \in V\},$$

and $\tilde{A}$ is an extension set such that

$$\tilde{A} = \{(v, y) \mid v \in V, y = k(v)\}.$$

If the function $y = k(v)$ is a stochastic process, then we call

$$\widetilde{A}(R) = \left\{ (R, y) \mid R = (N, C, V) \in W, y = k(R) = k(v) \right\}$$

a stochastic matter extension set on *W*, where the matter *R* is called a stochastic matter.

It is known that all the matters are stochastic factors and can be changed to different values, so there must exists stochastic properties in them.

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