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Reliability Estimation of Complex Technical Systems with Dependency Modelling: A Fuzzy Approach

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Abstract

Complex technical systems may fail unexpectedly leading to process disruption and may bring a safety-hazard situation. The system failure is an outcome of failure dependency among subsystems where the degradation process of subsystems/components is random and uncertain process. Hence, reliability assessment of complex systems is of paramount importance. Quantitative reliability assessment needs failure data, which is scarce and/or not well-recorded, thus statistically inconsistent in process industry. Most often, in such industries, the reliability is expressed linguistically (like Good, OK, Bad, etc.,) rather than quantitatively. This paper proposes an approach for quantitative reliability estimation considering failure dependency among subsystems using fuzzy failure possibility instead of failure probabilities. Condition monitoring data is used to estimate the failure possibilities with fuzzy sets. Fault tree analysis is applied to arrive at system failure possibility. The proposed approach is applied on large electric motor (2300 kW) by treating it as a system of subsystems. An order of 50% variation is found in the failure possibility estimates between with and without dependencies considerations. This approach not only helps the maintenance engineer to assess reliability but also assist in timely scheduling of suitable maintenance task.

Keywords: dependency, fault tree, fuzzy set, condition monitoring, possibility, reliability

1 Introduction

Process equipment falls into category of large and complex technical systems, which is an assembly of many subsystems. Process equipment (blowers, pumps, compressors and motors etc.,) operate on continuous mode. Unexpected failure of such equipment leads to process disruption and raises safety concerns (Sevenson, 1989). Survey conducted by the authors in an integrated steel plant on failures of large systems, revealed that the considerable number of failures are dependency failures, i.e., the system failure is a consequence of one of the subsystem failure which could not be detected in time with Condition Monitoring (CM) in spite of periodical preventive maintenance actions. Scheduling of preventive tasks is usually based on Reliability Centered Maintenance (RCM) Strategy (Moubray, 1997), but it does not envisage dependency during the reliability assessment. RCM focuses mainly on identification of the equipment failure modes and failure causes with reference to operating context (Knezevic, 1997, Hubert *et al.*, 2002) and schedules 'On Condition' (Condition Monitoring) tasks.

The Condition Monitoring (CM) data analysis primarily concentrates on raising an early warning about an impending failure and identification of root cause. CM generates huge amount of data on equipment degradation and many researchers like Wang *et al.* (2001), Jardine (2002), Carnero (2003), Brown *et al.* (2004). Dong *et al.* (2005) etc., have used the CM data to develop models for testing, optimization, scheduling and repair polices etc. But, authors could not find a remarkable work in modelling system reliability with subsystem failure dependency using CM data. Misra & Weber (1989), Onisawa (1993) and Soman (1993) have done some pioneering work in the area of dependency modelling using fuzzy sets and fault tree. In their propositions, they have dealt with subjective unreliability purely in the form of expert domain knowledge and engineering judgement. Vagueness and imprecision are inherently associated with engineering judgements. Instead of relying purely on qualitative domain knowledge/engineering judgement, authors propose an approach which uses effectively CM data (quantitative data) to estimate the component/subsystem failure possibilities using fuzzy sets.

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Dependency is often neglected in reliability estimation due its complexity and lack of data to substantiate dependency. In fault tree (FT) approach, while modelling the dependency of the top event on basic events, basic event uncertainties (probabilities) propagate to the top event (TE) through logic gates. Due to imprecise failure probabilities obtained using scarce/inconsistent failure data, many a times TE probability becomes unrealistic. Possibility theory based on fuzzy sets offers an alternate way to express TE occurrence in term of possibility rather than probability.

This paper proposes an approach to obtain failure possibility with CM data and use the same to model dependency of subsystems. System failure possibility is estimated in the form possibility fuzzy set F(x), $0<x<1$ and $\mu_R(x)=[0, 1]$. This approach offers relief to maintenance engineer from collecting troublesome failure data.

This paper is organized into four sections. Section 2 outlines the degradation process and uncertainty in failure of component/subsystem along with fuzzy representation of failure probabilities and fault tree analysis. Section 3 proposes dependency modelling with fault tree using fuzzy failure possibility. Section 4 deals with application of the proposed approach on large electric motor with four subsystems using CM data. Section 5 concludes.

2 Brief Review on Prerequisites

While dealing with the process equipment as complex technical systems, the operating context has tremendous effect on degradation process leading to failure. The CM indicators have ability to indicate degradation quantitatively and map degradation with failure. Therefore, the degradation is tackled with Condition Monitoring techniques either to get rid of a failure or mitigate the risk associated with it. Attributes of the CM data and its fault mapping ability in the system are outlined with Fuzzy set modelling. Some salient points, which enhance the readability of the paper, are presented in the following subsections.

2.1 Degradation Process – Uncertainty in Failures - Condition Monitoring

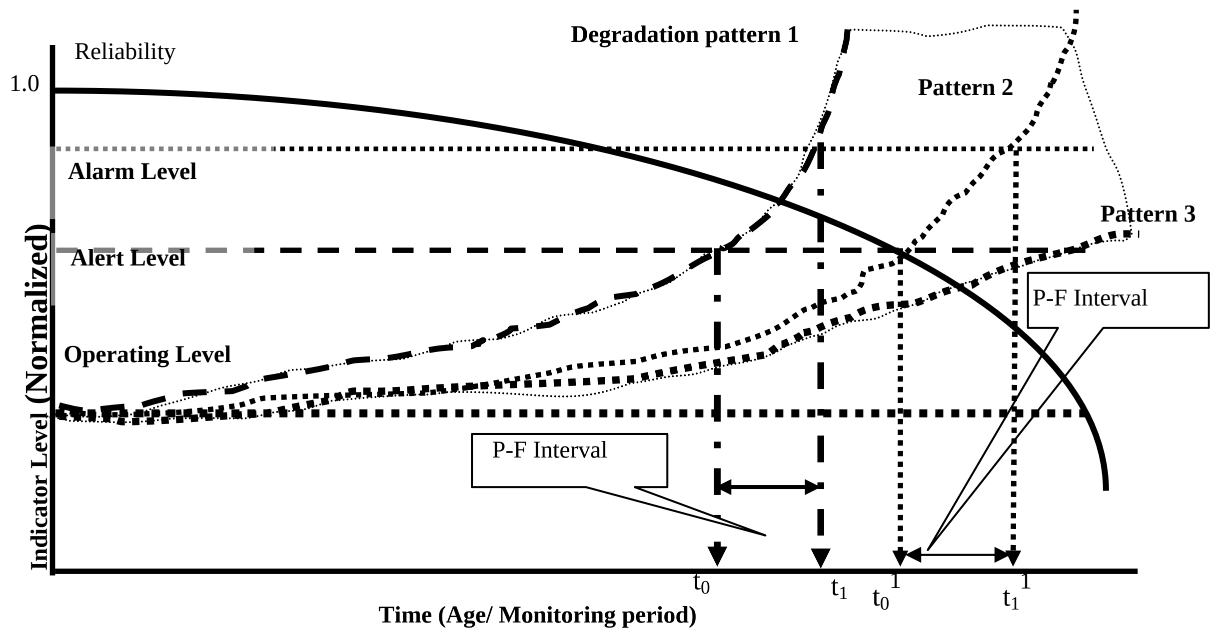

Figure 1: Degradation -condition indicator trend-reliability

Equipment degrades with use, which is a continual and irreversible process, and pulls down reliability. Degradation is measured in terms of level of condition indicators and can be considered as an indirect measure of reliability. The success of the CM program strongly relies on detection of failure progression. Normally, the failure of the equipment occurs due to interaction of many failure causes and the most predominant failure mode accelerates the degradation. Occurrence of the system failure due to interaction and dependency of various failure events can be better modelled with Fault Tree Analysis (FTA) (Misra, 1992). A comprehensive review on methods of selection and application of CM techniques can be found in (Jardine *et al.*, 2005).

Condition monitoring strategy raises an early warning about impending failure and accordingly a preventive maintenance task gets scheduled. With the CM, the equipment is withdrawn from service when degradation level is unacceptable in a planned way for a brief period for renewal, and will be put back into service. The CM continues till the next warning arises. The cyclic actions of this nature do not allow equipments to fail functionally, hence number of failure are less. Therefore, the reliability estimation becomes imprecise and far from reality with limited failure data. Randomness exists with the time for reaching the unexpected level of the condition indicator. The Potential-Functional (P-F) failure interval also is uncertain and varies widely with operating contexts as depicted in Fig.1. Evidently, the location of P-F Interval depends on the degradation pattern, which may follow pattern 1, 2 or 3 as bounded by shaded area shown in Fig. 1.

From Fig.1, it can be seen that the time to reach functional failure (t_1 or t_1^1) is not certain and precise. Therefore, the failure probability can be represented by a fuzzy set with lower and upper bound probabilities. The simple and more appropriate membership functions would be a triangular membership functions as shown in Fig.2. The support for the fuzzy set is defined by the lower and upper bound probabilities and core would be a single point with a membership grade (μ) equals to 1. The triangular membership functions can be used to approximate a gradual change of membership grade on either side of the mean probability, and the same can be decided by the engineering reasoning or domain expert knowledge as shown in Fig.2 (a), (b) and (c), respectively.

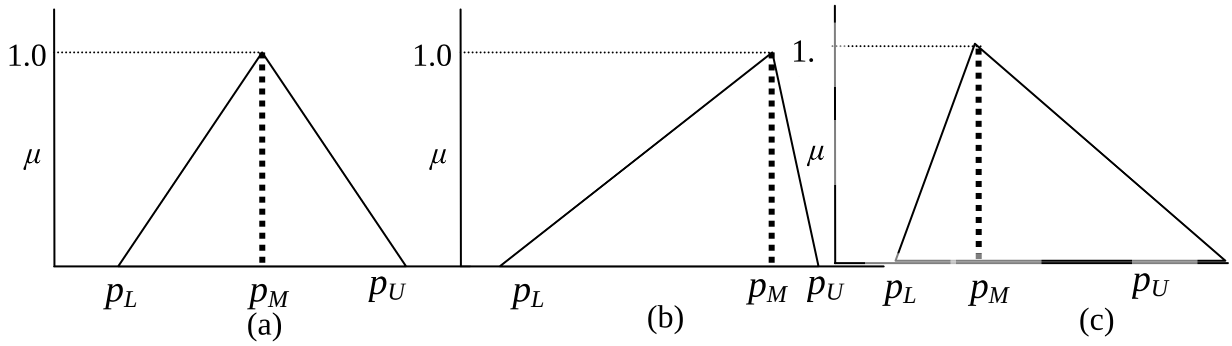

Figure 2: Triangular membership functions of failure probabilities

Fuzzy Fault Tree Analysis with Failure Possibility

There are two approaches used with Fault Tree Analysis (FTA). One is failure probability and other one is failure possibility. The Possibility theory proposed by Zadeh (1976) is a mathematical formulation complementary to probability theory, i.e., when the information is either inadequate to qualify the uncertainty of the event in terms of probability or does not satisfy the characteristics of probability; the better expression can be done with event possibility. Like probability, possibility is also defined the interval [0, 1], but does not hold all the characteristics of probability. In the case of CM data, it is frequently felt that, the information is not adequate to assign probability of occurrence of an event, but can be expressed in linguistic terms like less possible, highly possible, not possible etc. This can be best understood with the ‘Consistency Principle’, which states 1) whatever is possible, may not be probable and what is improbable need not be impossible; 2) Whatever is impossible is certainly improbable. Hence the possibility of the event is greater than or equal to its degree of probability.

Basic event probabilities can be expressed as triplet such as, $\{p_L, p_M, p_U\}$, where, p_L, p_U are lower bound, and upper bound probabilities, respectively and p_M is the mean (expected) value of the failure probabilities (fig.2). *These probabilities can be estimated using equipment withdrawal times based on condition indicator levels monitored in CM. Since the degradation, data measurement and time of measurement are fuzzy in nature (Fig.1), the fuzzy failure probabilities can be expressed [14] by suitable membership functions.* The measure of fuzziness, ‘*k*’ can be expressed as the ratio of p_M/p_U or p_M/p_L..

In light of this, Dubois *et al.* (1988) has suggested transformation of fuzzy probability to possibility and vice versa. Therefore, fuzzy failure possibility is a better measure to deal with condition monitoring data and can be expressed with a membership grade in a generalized form as:

$$F(x) = \frac{1}{1 + 20\,\mathrm{mod}(x - xo)^m} \tag{2}$$

where, $0 < x < 1$ and x_o = subjective failure possibility which helps in normalizing. $F(0) \neq 0$ *and* $F(1) \neq 1$, therefore, it satisfies the condition that there exists a possibility that a system may fail certainly and there exists also a possibility that the system may not fail at all. Hence this representation helps in dealing with random failure of equipment in a more realistic manner (Onisawa *et al.* (1993)).

For transformation of probability to possibility, the following approximation was suggested by Misra and Weber (1989), x_o is derived as function of p_M , such as

$$x_0 = f(p_M) = \frac{1}{1+(KLog(1/p_M))^3} \tag{3}$$

where, $p_M \neq 0$ and $x_o = 0$ when $p_M = 0$, $K = 1/Log(1/p_s)$, 'p_s' is the subjective standard failure probability and parameter, m (expression of fuzziness) obtained from mapping table for various degrees of fuzziness as expressed in Table 1 in terms of 'k' [Onisawa and Misra (1993)] .

Table 1: Correspondence between parameters *k* and *m*

Range of Parameter '*k*'	Parameter '*m*'
$k \leq 3$	2.0
$3 < k \leq 5$	2.5
$5 < k \leq 10$	3.0
$10 < k$	3.5

Basic events in a fault tree are related to the top event with the combination of logic gates such as, AND and OR gates. The dependency of failure of higher level component/subsystem on lower levels can be modelled with PRIORITY AND gate and EXCLUSIVE OR gates. The top event probability is obtained as a combination of 'Union' and 'Intersection' operators used on probabilities. The same can be used with the fuzzy probabilities also, but with *extension principle of fuzzy arithmetic* (Soman *et al*., 1993). When the basic event fuzzy failure probabilities are transformed to fuzzy failure possibilities, to use with FTA, the Union and Intersection operators can be expressed as following functions (Dombi, 1982), which are approximation to AND (min) and OR (max) operations such as,

$$H(x,y) = \frac{1}{1+(((1-x)/x)^{1/3}+((1-y)/y)^{1/3})^3} \tag{4}$$

where $0 < x, y < 1$, and $H(0, y) = H(x, 0) = 0$.

$$G(x,y) = \frac{((x/(1-x))^3+(y/(1-y)^3)^3)^{1/3}}{1+((x/(1-x))^3+(y/(1-y)^3)^{1/3}} \tag{5}$$

where $0 \leq x, y < 1$, and $G(1,y) = G(x, 1) = 1$.

The functions H and G are used with extension principle as the AND and OR operations in the fault tree analysis with fuzzy failure possibilities.

3 Failure Dependency Modelling with Failure Possibility

The level of dependency among components/subsystems is not same and depends on the functional and constructional configuration of the system. It the case of process industries, it depends on the operating context and can be expressed in linguistic terms like 'Highly Dependent', 'Less Dependent' and 'Moderately Dependent' etc. Therefore, probability of dependency cannot be estimated in this regard, but possibility can be expressed in terms of failure possibility. Once expert opines, the level of dependency is of particular level and its fuzziness is also approximated, the fuzzy dependency possibility can be expressed as,

$$R = \frac{1}{1+20\,\mathrm{mod}(r-r_o)^m} \tag{6}$$

where 'r_o' is subjective dependency at normalized possibility, and 'm' is the degree of dependency with fuzziness. A suitable value of 'm' can be selected from the range $1< m < 3$. Higher the value of 'm', dependency is strong. The fuzzy number so obtained is used to define the dependency relation between the components/sub-systems.

Dependency exists in the following forms (Misra, 1993):

♦ Inter Dependency of component failures, i.e., if a component/sub-system 'A' fails, Component/sub-system 'B' also fails, but failure of component/sub-system 'B' may not lead to failure of component/sub-system 'A'. Dependency of this type is called Type-1 dependency.

♦ Component/sub-system ‘A’ fails, and does not contribute to failure of system, but influences the failure of component/sub-system ‘B’ and then system fails. This is called dependency of Type- II.

Considering a system as shown in Fig.3, with two components ‘A’ and ‘B’, the system reliability can be evaluated in the following way. The failure of sub-system ‘A’ alone can not lead to failure of the system, but accelerates the failure of sub-system ‘B’ and finally leads to failure of the system. The fuzzy dependency possibility is defined in terms of fuzzy number ‘*R*’.

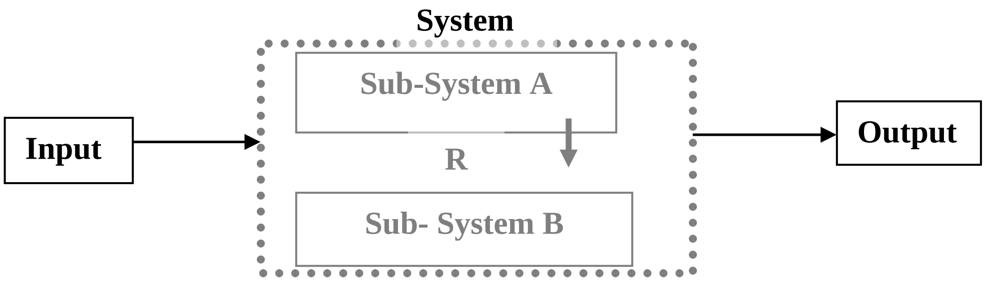

Figure 3: System configuration for type-II dependency.

Therefore, two events are to be considered for explaining the system failure, i.e.

Event #1: Failure of Sub-system A ‘AND’ then leads to failure of Sub-system B, then system fails and can be obtained as,

$$F_B' = H\,(F_A, R) \text{ (min operation)} \tag{7}$$

Event #2: Failure of Sub-System B itself, which can lead to failure of the system, i.e., F_B

Failure Possibility of System ‘F’ = $G\ (F_B', F_B)$ (max operation). (8)

Using this methodology any complex system consisting of large number of components/sub-systems can be evaluated with the help of CM data with defined dependency relations using failure possibility. The case study in the following section describes the dependency and methodology in detail.

4 Case Study: Dependency Modelling of Large Electric Motor

An electric motor of capacity, 2300 kW, 6.6 kV, 1480 RPM squirrel cage type is cooled with closed air and closed water (CACW) system. The motor is horizontally mounted and supported by three layers structure, base frame (steel), common base (steel-concrete) and reinforced concrete columns. The motor is covered under the CM program monitored periodically with the help of condition indicators (some online and some off-line), such as vibration, temperature, rotor condition etc. The motor is coupled to a compressor through a gear box, which runs at 16, 800 rpm. The motor reliability is vital, as unexpected stoppage not only hampers the production, but leads to a major safety concern of imbalance of (liquid oxygen) in the pipe line system. The motor failure possibility need to be assessed separately to integrate with other reliability estimates of gearbox, compressor and pipelines to arrive a total risk involved in operation of the compressor. The motor (system) is analyzed by considering its four major sub-systems, viz., 1) Electromagnetic system (EMS): winding and insulation etc., 2) Power Transmission System (PTS): shaft, bearings, housing etc., 3) Ventilation System (VS): cooler, coolant system, etc., 4) Support systems (SS): Base frame, foundation, anchoring etc. The fault tree of motor failure is as shown in Fig. 4.

The dependency relationship among sub-systems of large motor is as follows:

1. The ventilation system supplies cool air to motor stator and rotor windings (Electromagnetic system). The failure of ventilation system does not immediately cause failure of motor, but body temperature rises, which is monitored online. Rise of temperature of windings leads to failure of insulation, as for every 10^{o}C rise in temperature, the dielectric strength reduces to half the value. Once the ventilation system is failed, there is no heat removal from the motor and it leads to winding failure. *The time between the ventilation system failure and motor failure may vary depending on the loading and ambient temperature conditions. But the dependency is* ***very strong*** *and certainly leads to motor failure sooner or later.*

2. The Support system, which is made up of different materials and varies construction wise from point to point, offers both dynamic dampening and stiffness besides providing a base rigidity. The failure of support structure, such as development of cracks, disintegration, etc., do not offer the required dampening and stiffness to the motor bearing housings/bearings and motor as a whole. This leads to higher vibration levels (monitored offline) and higher stress at bearing housing etc., which in turn leads to failure of bearings due to high vibration or looseness. This is a gradual and continuous process of degradation and takes a long time to drive the motor power transmission system to failure zone. *This dependency can be expressed as **moderate** and failure of support structure is relatively less possible than failure of ventilation system. Motor failure may be due to either of the above two events or one of the individual failure of Electromagnetic /power transmission sub-systems*. As described earlier, ventilation system failure and subsequent failure of electromagnetic system in sequence are represented using PRIORITY AND, its failure possibility is obtained using '*H*' operator and (9),

$$P = 0.5H(F_V . F_E^{'}) \tag{9}$$

where F_v and F_E are fuzzy possibilities of ventilation and electromagnetic systems, respectively. The fuzzy failure probabilities of basic events using CM data on the motor are shown in Table 2. The lower and upper bound probability and standard failure probability are estimated from the group of six such motors working under same load and operating context.

Table 2: Fuzzy failure probabilities of sub-systems

Sub-Systems	**Lower bound Prob. (p_L)**	**Mean Prob. (p_M)**	**Upper bound Prob. (p_U)**	**Fuzziness (k) p_M/p_L**	**K 1/Log(p_s)**	**x_o**
Electromagnetic system	0.0092	0.03960	0.0700	4.30	0.715	0.498
Power Transmission System	0.0100	0.03100	0.0520	3.10	0.548	0.616
Ventilation System	0.0030	0.00415	0.0053	1.38	0.396	0.528
Support System	0.0010	0.00200	0.0030	2.00	0.376	0.492

Referring to Fig.4, the dependency of two subsystems, i.e., 1) Dependency of EMS on VS and 2) Dependency of PTS on SS are with m= 3.0 and m=2.0, respectively using (7). The dependencies fuzzy possibilities obtained using (6) are shown in Fig.5. And from the Fig.5, the plot shows the variation of the dependency with increase in degradation level of the system, which is measured in terms 'Equipment Health Index' (EHI) computed from condition indicators. EHI = 1 represents a healthy system and zero indicates a failed one (Kumar and Chaturvedi *et al.* (2005)). The subjective unreliability is a measure of equipment health indicated by EHI, which is computed periodically. The failure possibility of Electromagnetic system is high in the event of failure of ventilation system. The dependency of the EMS on VS does not decrease with time or ageing/usage. The dependency of the PTS on SS decreases after certain level of equipment degradation. This can be explained, since the SS cannot continue to support the system beyond a certain level of degradation and functional failure becomes imminent.

The motor failure (Top Event) failure possibility is obtained using the relations mentioned from (1) to (8) for which the plot is as shown in Fig.6. Using this approach, the motor failure possibility over a period of time can be approximately assessed to draw either a production schedule or a maintenance tasks. Even though the methodology offers approximate estimates, it reduces computational complexity and providing a natural language representation of failure scenario.

From the results shown in Fig.6, the membership grade variation in failure possibility with and without dependency are compared and found to be 0.77 and 0.51, respectively. The failure possibility estimated with dependency among EMS &VS and PTS &SS show that the motor failure possibility (TE) is increased by 50% than the estimate, which does not consider the dependencies. At health index 0.78 itself, there is a large variation in the failure possibility estimates. As the approaches to 0.50, the unreliability increases and system fails unexpectedly. Therefore, the approach of considering the dependency while estimating the system reliability is vital and cannot be ignored.

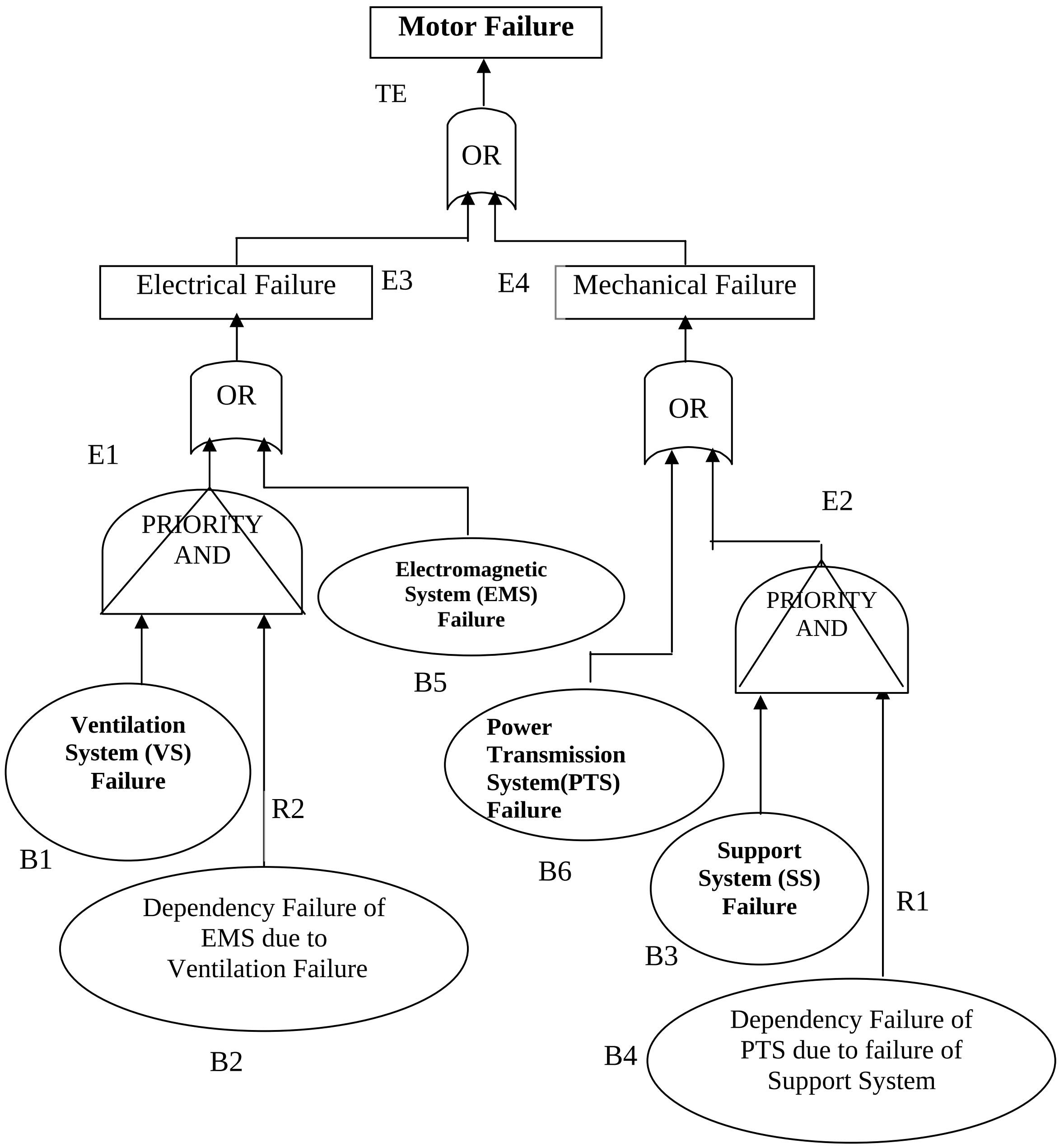

Figure 4: Fault tree of motor failure with ‘dependency’ consideration

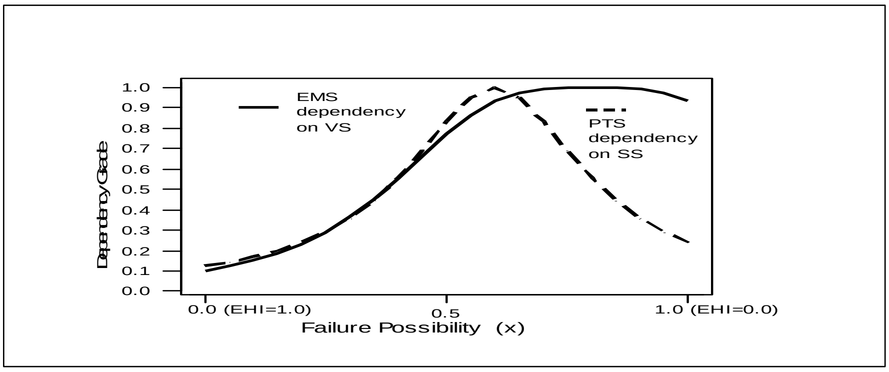

Figure 5: Dependency possibility of EMS and PTS.

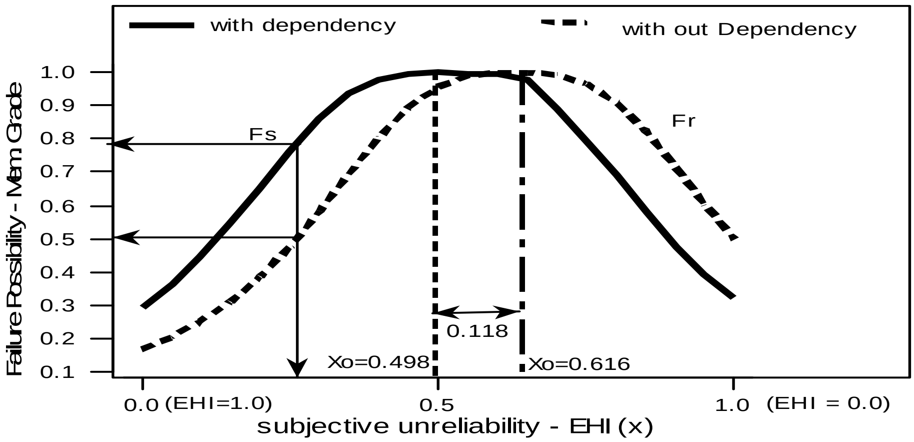

Figure 6: Motor failure possibility and subjective unreliability

5 Conclusions

The dependency modelling with fuzzy failure possibility approach offers flexibility in dealing with the CM data in terms of equipment health index. It is more advantageous to use the CM data rather than purely relying on subjective expert/domain knowledge. The method presented in this paper offered a technique to integrate, various conditions monitoring indicators (quantitative data) and domain knowledge/engineering judgement (qualitative information) through fault tree analysis. Otherwise FTA with individual condition indicator levels is difficult and may lead to contradictory analysis. The proposed method is more useful, where failure causes are monitored with the help of CM techniques. The application of this approach on large critical process equipment in a steel plant has shown encouraging results. Therefore, the same can be extended to other equipment for a quantitative assessment of reliability. The authors feel that the methods is a '*look forward*' approach, since the equipment present condition is used to estimate future unreliability of the equipment, instead of using equipment failure history.

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