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Simpson's Paradox and Other Reversals in Basketball: Examples from 2011 NBA Playoffs

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Abstract. Simpson's paradox has been discussed in many scientific and social studies, including analysis of sports' data. Yet it is not well known in basketball, except in some limited cases. One reason is that the NBA official box scores of basketball games do not list the 2-point shooting statistics. In this article, we show that by decomposing the 2-point statistics from the combined field goals, Simpson's paradox appear quite frequently. Using this method, we have found many instances of Simpson's paradox from the 2011 NBA playoffs, which seem to be one of the best NBA playoffs. Other reversal phenomena are also discussed to highlight excitements in these playoffs. Moreover, we propose a method of scoring normalization to more accurately combine the three scoring types in basketball.

Keywords: Sports statistics, Data mining, Basketball, Simpson's reversal, Scoring normalization.

1. Introduction

Simpson's paradox refers to the reversal of statistical relationships when two or more groups of data are combined. For this reason, it is also called Simpson's reversal. This reversal paradox has been discussed in several sports, including baseball [1] and cricket [2]. In basketball, however, it has been discussed only for some limited cases, such as the controversy regarding the hot-hand phenomenon [3-5]. In this article, we show many instances of Simpson's paradox in the 2011 NBA playoffs, which many consider one of the best NBA playoffs. One of the reasons for the excitement in these playoffs is the reversal phenomena, not only from the point of view of the lead changes or outcome changes from the experts' predictions of the series, but also from the standpoint of statistical analysis of the games. The prognostics of many experts for numerous series in these playoffs were wrong as the outcomes were the opposite of their predictions. In many games, the team pulled ahead in the first 3 quarters, then lost the game because the opposing team turned the game around by reversing the game flow and the lead in the 4th quarter or in the final minutes of the game. The most striking reversal, however, is Simpson's paradox that appears in many of these games or series. While Simpson's paradox is often considered to be rare, which perhaps is why it is not commonly discussed in basketball, we argue that it occurs quite frequently in sports, in particular, NBA basketball. The examples in the 2011 NBA playoffs demonstrate that you do not have to be on the prowl for it.

One of the most important statistics in NBA game reports is the shooting percentage. As we know, three types of shooting can convert a play into a scoring point in basketball, the 2-point shot, 3-point shot and free throw. Yet, the NBA official box score does not separately show 2-point shooting statistics, rather the field goal (FG) and its percentage [6-7]. The latter, in fact, is the combined statistics including both 2-point and 3-point shots. Figure 1 shows a simplified version of the box score of Game 5 of the 2011 NBA Playoffs Western Conference first-round series between the top-seeded San Antonio Spurs and the last-seeded Memphis Grizzlies. To focus our attention to the scoring comparison between the two teams, we summarize the field goals and field goal attempts (FGA) along with the 3-point data extracted from Fig. 1 into Table 1a. From either Fig. 1 or Table 1a, fans see that the Grizzlies had a higher field-goal percentage (including both 2-point and 3-point shots), but a lower 3-point (3P) shooting percentage. They think that the Grizzlies must have had a higher 2-point shooting percentage in order to compensate the lower 3-point shooting, and thus a higher overall shooting. However, once the 2-point shooting statistics are summarized, fans are surprised that

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the Grizzlies had a lower 2-point shooting as well. How is it possible that a team had worse shooting in each category of the 2-point and 3-point shootings, and yet it had a better overall field goal percentage?

Visitor team: Memphis Grizzlies

Player	MIN	FG	FGA	3P	3PA	FT	FTA	......	PTS
S. Young	32	7	12	1	2	3	3	......	18
Z. Randolph	44	10	17	0	0	6	7	......	26
M. Gasol	42	5	14	0	0	1	1	......	11
T. Allen	24	2	6	0	1	3	4	......	7
M. Conley	43	9	20	0	2	2	2	......	20
D. Arthur	16	2	7	0	0	0	0	......	4
S. Battier	34	2	5	1	4	1	2	......	6
O. Mayo	20	3	4	1	1	2	2	......	9
G. Vasquez	10	1	2	0	2	0	0	......	2
Totals:		47	87	3	10	18	23		103
		47.1%		30.0%		78.3%			

Home team: San Antonio Spurs

Player	MIN	FG	FGA	3P	3PA	FT	FTA	......	PTS
R. Jefferson	33	2	7	1	4	1	1	......	6
T. Duncan	40	5	13	0	0	3	4	......	13
A. McDyess	32	2	6	0	0	0	0	......	4
M. Ginobili	42	10	18	4	9	9	10	......	33
T. Parker	44	9	20	0	2	6	9	......	24
G. Hill	30	3	7	0	1	6	6	......	12
M. Bonner	20	2	6	1	4	0	0	......	5
G. Neal	10	3	5	1	2	0	0	......	7
T. Splitter	15	3	3	0	0	0	2	......	6
Totals:		39	85	7	22	25	32		110
		45.9%		31.8%		78.1%			

Fig. 1: Simplified version of the NBA official box score of Game 5 of the 2011 NBA Playoffs Western Conference first-round series between the Spurs and Grizzlies [6]. Besides the played minutes (MIN), only the shooting and scoring statistics are included in the table.

Table 1a Shooting Statistics of Game 5 of the Spurs-Grizzlies Series (extracted from Fig. 1)

Team	FG-FGA	3P-3PA
Grizzlies	47.1% (41-87)	30.0% (3-10)
Spurs	45.9% (39-85)	31.8% (7-22)

Note: 3P stands for 3-point shots made and 3PA for 3-point attempts.

Table 1b Split table showing 3-point and 2-point shootings separately (decomposed from 1a)

Team	3P-3PA	2P-2PA
Grizzlies	30.0% (3-10)	49.4% (38-77)
Spurs	31.8% (7-22)	50.8% (32-63)

Table 1c Shooting Statistics of Game 5 of the Spurs-Grizzlies Series

Team	2-Point Shooting		3-Point Shooting		Combined	
Grizzlies	49.4%	38/77	30.0%	3/10	47.1%	41/87
Spurs	50.8%	32/63	31.8%	7/22	45.9%	39/85

Source: NBA (2011) and ESPN (2011).

This actually is an instance of Simpson's paradox. Generally, Simpson's paradox occurs when two or more categorical variables of data are combined [8-9]. In basketball, however, one often needs to decompose the box score to find an instance of Simpson's paradox because the NBA official box score does not report 2-point shooting statistics. This is likely one of the main reasons for which Simpson's paradox in basketball is not well known despite occurring commonly. Some researchers call the finding of Simpson's paradox by separating individual categories' data from the combined data the inverse Simpson's paradox [10]. Except the difference in combining or decomposing data, however, there is no other difference between Simpson's paradox and inverse Simpson's paradox. Hence, Simpson's paradox in Tables 1a and 1b can be simply presented as one condensed table (Table 1c).

In the following, we first discuss examples of Simpson's paradox that appear in the 2011 NBA playoffs. We show Simpson's paradox by decomposing the scoring columns from the NBA official box scores to separate 2-point shots. Although we have found many examples of Simpson's paradox in many games and series, we only discuss Simpson's paradox and other reversals in selected games or series from each round of the 2011 NBA playoffs. Subsequently, we present a method of scoring normalization to improve the accuracy of combining three types of shooting statistics. The normalization method is a variation of the effective field goal percentage and free throws made per field goal attempted. The latter two variables are generally considered to be among the most important factors for basketball success [11-12]. We end the discussions by drawing several conclusions.

2. Simpson's paradox in the NBA 2011 Playoffs

2.1. First Round

How often do you see winning a playoff series by the last seeded team against the top seeded team in a 7-game series of a major league, such as the NBA? In the first round of the 2011 NBA Western Conference playoffs between the top-seeded San Antonio Spurs and the last-seeded Memphis Grizzlies, 12 ESPN experts had predicted Spurs winning the series, and not a single ESPN expert had predicted the Grizzlies' winning the series [7]. As we now know, the outcomes were the opposite, and caused the reversal of fortune in what was a humiliating debacle of playoff run for the Spurs with several superstar players and 4 NBA championships. The Grizzlies, on the other hand, advanced to the NBA Western Conference semifinals for the first time in their franchise history. They had never won a single playoff game despite having been in the playoffs three times in the past. In the 2011 playoffs, however, they won the first-round series without one of their best players, Rudy Gay (due to an injury), against one of the best teams in more than a decade in the NBA.

The Grizzlies actually won the series without need to play Game 7. In fact, had the Spurs not made 2 extraordinary shots toward the end of Game 5, the Grizzlies would have won it in 5 games. While the Grizzlies were leading 3 to 1 after the first 4 games, it appeared that they were winning Game 5 as well, as they were 3 points ahead the Spurs (95-92) with only 2 seconds left in regulation [7]. Manu Ginobili of the Spurs hit a long-distance shot from a corner while falling out of bounds. The shot, however, was later ruled as a 2-point shot after a video replay, which left the Spurs trailing by 1 point at 95-94. After two free throws by the Grizzlies, the Spurs again needed a 3-point shot to tie the game, which was obviously more urgent while less than 2 seconds remained in regulation. Catching the inbound pass, a Spurs' rookie, Gary Neal, made a spectacular 3-point shot in the face of the Grizzlies' defender as time expired. These two incredible shots in the last 2 seconds of regulation reversed the trend and flow of the game as the Spurs easily outplayed the Grizzlies in overtime to take the game (110-103). Interestingly, this also created a reversal of statistical relationships that we have discussed in the introduction (table 1). As it is seen from table 1c, the Spurs did both 2-point and 3-point shootings better than the Grizzlies, but the Grizzlies shot better in the overall field goal percentage, including the 2-point and 3-point shots.

Games with the Grizzlies produce Simpson's paradox quite often, especially when they play a team that attempts many 3-point shots. This is because the Grizzlies are not a 3-point shooting team, rather a team of the paint. The Grizzlies only occasionally attempt 3-point shots in these playoffs despite the acquisition of Shane Battier (who is a 3-pointer specialist) before the trade deadline. In Game 6, for example, the Spurs' shooting guard, Manu Ginobili, again made 2 difficult 3-point shots, including a half-court shot that beat the clock. The latter 50-foot shot pulled the Spurs to a 70-66 lead going to the 4th quarter [7]. At the time, the 2 dazzling 3-point shots by Ginobili looked like the game-saving thrills that happened in Game 5. But the Grizzlies reversed the trend and changed the lead in the 4th quarter to win the game, and thus the series. An example of Simpson's reversal occurred in comparing the statistics of Ginobili and statistics of his Grizzlies' counterpart, Tony Allen (Table 2).

Table 2 Comparing 2 Players' Shooting Statistics in Game 6 of Spurs-Grizzlies Series

Team	2-Point Shooting		3-Point Shooting		Combined	
Manu Ginobili	66.7%	4/6	25%	2/8	42.9%	6/14
Tony Allen	50.0%	4/8	0%	0/1	44.4%	4/9

Source: NBA (2011) and ESPN (2011).

Both players had a so-so overall shooting game, but their 2-point shooting actually was very good: nearly 67% for Ginobili, and 50% for Allen. But their 3-point shooting was not good. While Ginobili made two very challenging 3-point shots as mentioned above, he missed six easier 3-point shots out of eight attempts. Allen missed his only 3-point attempt. For each category of 2-point and 3-point shots, Ginobili had much better shooting percentages than Allen, but Ginobili's combined shooting percentage in the game is lower than that of Allen. In other words, Simpson's reversal occurred in comparing Ginobili and Allen's shooting statistics when 2-point and 3-point shootings were combined.

2.2. Conference Semifinals (Second Round)

Simpson's paradox also occurs in comparing statistics of two players from the same team. This, for instance, happens quite frequently in comparing the statistics between Kevin Durant and Russell Westbrook of the Thunder. While Durant generally makes many shots, for both 2-point and 3-point shootings, as he has won the scoring champion of the NBA for the last two seasons, Westbrook also attempts many shots in some games. In Game 4 of the Thunders-Grizzlies series, for example, Westbrook had 33 attempts while Durant had 20. Several examples of Simpson's paradox occurred even before the game was finished with three overtimes; here we show only one example based on the statistics from the final score box (Table 3). Because the official box score does not shows 2-point shooting statistics separately, some people have quickly interpreted the outperformance of Westbrook in shooting over Durant after seeing the combined field-goal statistics. When the statistics of the 2-point and 3-point shootings are separately shown, Durant had better shooting in each category though neither player did well in 3-point shootings. The Thunders were fortunate to win this game with three overtimes. According to a statistical analysis [13], in most of their games, when Westbrook shoots more than Durant, they tend to lose; and when Durant shoots more, they tend to win. Therefore, in analyzing Simpson's paradox in basketball, not only are the shooting percentages important, but the number of shooting attempts is also important, for example, for the coaches to make the game plans, and for the teams to better delegate the shots between different players. This problem will be further discussed in conjunction with discussions of the scoring normalization in the next section.

Table 3 Comparing 2 Players' Shooting Statistics of Game 4 of Thunders-Grizzlies Series

Player	2-Point Shooting		3-Point Shooting		Combined	
Kevin Durant	57.1%	8/14	16.7%	1/6	45.0%	9/20
Russ Westbrook	48.4%	15/31	0.0%	0/2	45.5%	15/33

Note: Thunders won the game with 3 overtimes (133 to 123). Source: ESPN (2011).

In the Heat-Celtics series of the Eastern Conference Semifinals, the Heat had hot hands against the Celtics. Before the series started, the question was, could the Celtics withstand the Heat with 3 Heat's scoring generators (LeBron James, Dwayne Wade and Chris Bosh)? On the other hand, James' lack of closing ability in the regular season was a point of ridicule for people looking for a reason to laugh at the

Heat [7]. After the first 4 games, however, the Heat lead the series 3 to 1. In Game 5, the Celtics lead the game all the way from the start to less than 3 minutes to the end of the game; but then the Heat turned the game around, and reversed the lead and the outcome of the game. As a matter of fact, after Bosh made a dunk to tie the score at 87, James made two 3-point shots, a dunk and a layup in the final 2 minutes to close the game. James' reversal from the regular reason's lack of ability of closing a game to a personal 10-0 run helped finish off the Celtics by winning the series 4-to-1. This is also a reversal to the 2011 regular season games, in which the Celtics won the series 3-to-1 against the Heat.

An example of Simpson's reversal occurred in this game (Table 4). From the shooting columns in the box score of the game, the Celtics appear to shoot better than the Heat as they had 49.3% shooting average compared to 46.4% by the Heat. The Celtics also made 70% of their free throws, versus just over 68% by the Heat. However, once the regular shooting scores are combined with the free throws, the Heat had a better shooting percentage. While some may argue that it is unfair to combine regular shots with the free throws because of the different difficulties. This is a legitimate question, but the same problem exists in combining 2-point and 3-point shootings as NBA does. Before we discuss a scoring normalization method that mitigates this problem in the next section, notice that the Celtics actually made one more 2-point shot, but the Heat made twelve more free throw points out of eighteen more attempts, which is a critical difference for the game.

Table 4 Shooting Statistics of Game 5 of the Eastern Conference Semifinals Celtics-Heat Series

Team	Regular Shooting		Free Throw		Combined	
Celtics	49.3%	33/67	70.0%	14/20	54,0%	47/87
Heat	46.4%	32/69	68.4%	26/38	54.2%	58/107

Note: Heat won the game 97 to 87. Source: ESPN (2011).

2.3. Conference Finals

The Bulls swept the Heat 3-to-0 in the 2010-2011 NBA regular season [7]. The outcome of the 2011 playoffs, however, is nearly a total reversal as the Heat won 4 games in a row against the Bulls after losing the opening game. Presumably, the Bulls have a starting lineup as talented as any team in the league, including newly crowned MVP – Derrick Rose, and a bench that is also much deeper than the Heat's bench. After the Bulls won the opening game, many experts quickly jumped to say that the Bulls were going to crush the Heat, just before the Heat reversed the trend and won 4 games in a row to close the series. By analyzing the games in the series, it is easy to see that two of the Heat's three superstars performed very well, but two Bulls' superstars (MVP – Derrick Rose and power forward Carlos Boozer) did not play well. Luol Deng had the best shooting percentage among the players who made a meaningful number of points for the Bulls; yet he still did not perform better than the worst shooting superstar, Wade, among the three superstars of the Heat, except in 3-point shooting. Simpson's paradox that occurred in comparing the shooting statistics of Deng and Wade based on all the 5 games in the series (Table 5) provides us with an angle to see why the Bulls lost the series. A more detailed and also more accurate discussion is given later as it is better discussed with the normalized combination of shooting statistics.

Table 5 Comparing shooting statistics between Deng of the Bulls and Wade of the Heat in the Eastern Conference Finals (data compiled from all the 5 games in the series)

Player	2-Point Shooting		3-Point Shooting		Free Throw		Combined	
D. Wade	41.9%	31/74	20.0%	1/5	82.9%	29/35	53.5%	61/114
L. Deng	43.0%	21/49	40.7%	11/27	91.7%	11/12	48.9%	43/88

Note: Heat won the series 4 to 1. Source: NBA (2011).

2.4. NBA Finals

Although the experts were not as consistently wrong as predicting some other 2011 playoffs series, 15 out of 22 ESPN experts predicted the Heat winning the series against the Mavericks after the Heat easily eliminated the Bulls [7]. The two teams met once for playoffs in the 2006 NBA finals, in which the Heat won the championship. Most experts thought that Dwayne Wade of the Heat would continue to hammer the Mavericks, and perhaps even more so in this series because LeBron James would attract some more of Mavericks' defensive attentions. As we now know, the outcome is essentially a reversal to the 2006 Finals. In the 2006 Finals, the Mavericks initially lead the series 2-to-0, and almost won Game 3; but then the Heat turned it around, won that game and the next three games to earn their first franchise championships. In the last four games of the 2006 finals, the Mavericks had several heated confrontations with the referees. They could not keep their heads cool, and in the critical time they often had cold hands. In the 2011 Finals, the Heat initially seemed to have the upper hand by winning the opening game quite easily, and almost won Game 2. This time, however, Dirk Nowitzki of the Mavericks kept his head cool despite having a high fever. He did exactly what Dwayne Wade outdid him in the 2006 Finals in the highly challenging moments with a variety of road blocks - torn tendon, high fever, heated taunting, and double teams. Nothing stopped him to turn the games around and reverse the trend from the Heat's lead to the Mavericks' wins in the final minutes of the games. Nowitzki's hot hands in the 4th quarters of the games in the Finals were easily recognized and highly praised by the media and fans [14]. By contrast, the Heat often could not keep their heads cool, and in critical times they were often confused with how to finish, wondering who should take the shot for fear of having cold hands and missing the big shots, or wondering who should defend the Mavericks' shooter for fear of being called for foul and giving the Mavericks free throws.

Concerning the game strategies, the Miami Heat were hoping that Chris Bosh would match or even slightly underperform Dirk Nowitzki, and Dwayne Wade and LeBron James would dramatically outperform their counterparts of the Mavericks so that they win the series. The Dallas Mavericks, on the other hand, were hoping that Shawn Marion, Jason Terry and Shawn Stevenson would match or nearly match, or at least slow down, LeBron James and Dwayne Wade; and Dirk Nowitzki, Jason Kidd, Tyson Chandler and others would outperform their counterparts of the Heat to win the series. The final statistics of the series show that the Mavericks' strategy had worked. LeBron James did not have a good series because of the Mavericks' successful defensive strategy against him. Although Dwayne Wade initially performed extremely well, in fact, the best among the big three superstars or perhaps the best of all, he reverted to a normal performance in the last 2 pivotal games, partly due to injuries, partly due to the Mavericks' defense.

Table 6(a) Comparing shooting statistics between Marion of the Mavericks and James of the Heat in the 2011 NBA Finals (data compiled from all the 6 games in the series)

	2-Point Shooting		3-Point Shooting		Combined	
S. Marion	0.486	34/70	0.00	0/1	0.479	34/71
L James	0.548	34/62	0.321	9/28	0.478	43/90

Note: Mavericks won the series 4 to 2. Source: NBA (2011).

Table 6 (b) Summary statistics for Marion and James

	Shooting Points	Free-Throw Points	Total Points	Minutes played	Points/minute
S. Marion	68	14	82	214	0.383
L. James	95	12	107	262	0.408

Source: NBA (2011).

We illustrate the successful strategy of the Mavericks against LeBron James by comparing his shooting statistics to the statistics of his Mavericks' counterpart, Shawn Marion, based on the data from all the 6 games in the series (Table 6). Notice first that James apparently shot significantly better than Marion for both the 2-point and 3-point shootings. When 2-point and 3-point statistics are combined, however, James' shooting average is essentially the same as Marion's, in fact even slightly lower (Table 6a). We again have a Simpson's reversal of statistical relationships. Here we are not saying that James underperformed Marion, but his outperformance may be smaller than the statistics shown in the individual 2-point and 3-point categories (conditional associations). We calculated the offensive efficiency by summarizing the points made per playing minutes of the two players in Table 6b. While James has been often rated as the NBA best player in the last a few years, he was only slightly more efficient than Marion in the 2011 Finals as James' scoring points per minute is just slightly higher than Marion's (0.408 versus 0.383).

On the other hand, Dirk Nowitzki and Tyson Chandler performed significantly better than their counterparts of the Heat. The shooting statistics for Bosh and Nowitzki based on all the 6 games in the series illustrate the outperformance by Nowitzki (Table 7). Notice that Nowitzki shot better than Bosh in each category of 2-point and 3-point shootings, and also in the combined statistics. Incidentally, for the sake of illustrating the Yule-Simpson's effect [9], had Nowitzki missed one more 3-point shot (the numbers are in parenthesis in the last row of Table 7a: D. Nowitzki – 3pt), we would have had a reversal of statistical relationships. Similar to comparing James and Marion' statistics, we calculated the points made per playing minute of the two players to assess their offensive efficiencies (Table 7b). Nowitzki's points made per minute are remarkably higher than Bosh's points made per minute: 0.645 versus 0.470. In other words, Nowitzki performed 37%, i.e., (0.645-0.47)/0.47, better than Bosh in offensive efficiency.

Table 7 (a) Comparing shooting statistics between Nowitzki of the Mavericks and Bosh of the Heat in the 2011 NBA Finals (data compiled from all the 6 games in the series)

	2-Point Shooting		3-Point Shooting		Combined	
C. Bosh	0.418	38/91	0.000	0/1	0.413	38/92
D. Nowitzki	0.425	45/106	0.368	7/19	0.416	52/125
D. Nowitzki – 3pt	0.425	45/106	(0.316)	(6/19)	(0.408)	(51/125)

Source: NBA (2011).

Table 7 (b) Summary statistics for Nowitzki and Bosh

	Shooting Points	Free-Throw Points	Total Points	Minutes played	Points/minute
C. Bosh	76	35	111	236	0.470
D. Nowitzki	111	45	156	242	0.645

Source: NBA (2011).

3. Scoring Normalization for Combined Statistics

It is well known that combining different data can sometimes cause paradoxes or even fallacies [8-10]. Combination of various data, however, is often necessary to summarize otherwise unorganized ideas or an excessively large pool of data. We propose a scoring normalization method to mitigate the collapsibility (legitimacy of combining various data) concern in combining the shooting statistics in basketball. There exist three shooting types that can convert a play into a score: the 2-point shot, 3-point shot and free throw. Since the 3-point shooting is more difficult than the 2-point shooting, and 2-point shooting is more difficult that the free throw, applying appropriate coefficients can equalize the difficulties or at least can normalize the shootings into equivalent scoring points (ESP). For example, the normalization can use the following formulas:

$$ESPA = 2 \times \text{2-point attempts} + 3 \times \text{3-point attempts} + \text{free-throw attempts} \quad (1)$$

$$PM = 2 \times \text{2-points made} + 3 \times \text{3-points made} + \text{free throws made} \quad (2)$$

where ESPA stands for Equivalent Scoring-Point Attempts, PM stands for Points Made. Obviously the above equations can be simplified without using the free throws when only the 2-point and 3-point shootings are combined.

The rational of the normalization lies in factoring the effective possessions and effective scoring points in the statistics, and thus the combined statistics are directly related to these critical factors and final outcome of the game. In other words, the combined statistics with the scoring normalization show not only the shooting percentages, but also the effective shooting-possession points. According to Oliver (2004), the possessions are one of the four most important factors for basketball success.

Table 1N shows the normalized combination of the 2-point and 3-point shooting statistics for Game 5 of the Spurs-Grizzlies series discussed earlier (Table 1). While both Tables 1c and 1N show Simpson's reversal, Table 1N has more information. From the official box score [6, 7] or Table 1, the Spurs' field goal percentage was lower than the Grizzlies in the game. However, in Table 1N, we also see that the Spurs made the same number of points from the regular shooting as the Grizzlies, at 85 each. Notice also that the lower shooting percentage is due to the greater ESPA, 192 for the Spurs versus 184 for the Grizzlies. As ESPA is highly correlated to the possessions, it takes the possession factor into the consideration. Moreover, the fact that both teams had the same scoring points from the regular shootings tells us that the Spurs won the game by getting more often and/or shooting better on free throws, which is exactly what happened [7]. In fact, the free throw, also one of the four important factors for basketball success [12], can also be combined with the regular shots using the scoring normalization method, as formulated in equations (1) and (2).

Table 1N Shooting Statistics of Game 5 of Spurs-Grizzlies Series with Normalized Combination

Team	2-Point Shooting		3-Point Shooting		Normalized Combination	
Grizzlies	49.4%	38/77	30.0%	3/10	46.2%	85/184
Spurs	50.8%	32/63	31.8%	7/22	44.3%	85/192

Note: Spurs won the game in overtime 110 to 103. Source: ESPN (2011).

We previously discussed an example of Simpson's reversal in the final game of the Celtics-Heat series (Table 4). The normalized combination of the regular shots and the free throws are shown in Table 4N. First, notice the disappearance of the Simpson's paradox after using the normalized combination. Second, apparently, the Celtics shot equally well for the 2-point shots, better for the 3-point shots, better for the free throws, and better for the combined shooting. So why did they lose the game? Whereas this was not very clear in Table 4 or an official box score, it is better shown in the table with the normalized combination (Table 4N). Note that the last column shows both the final scores (87 for the Celtics and 97 for the Heat), and the comparison of ESPA (169 for the Celtics and 196 for the Heat). Whereas the field-goal statistics is correlated only weakly with some other important variables, ESPA reflects more clearly, though indirectly, the other variables that impact the game, such as rebounds, assists, steals, turn-over, and blocked shots etc. With 27 more points on ESPA (i.e., 196-169), the Heat won the game despite their worse shootings on the 3-point shots and free throws.

Table 4N Shooting Statistics of Game 5 of the Eastern Conference Semifinals Celtics-Heat Series

Team	2-Point Shooting		3-Point Shooting		Free Throw		Normalized Combination	
Celtics	50%	26/52	46.7%	7/15	70.0%	14/20	51.4%	87/169
Heat	50%	25/50	36.8%	7/19	68.4%	26/38	49.5%	97/196

Table 5N Comparing shooting statistics between Deng of the Bulls and Wade of the Heat in the Eastern Conference Finals (data compiled from all the five games in the series) – Normalized combination from Table 5.

Player	2-Point Shooting		3-Point Shooting		Free Throw		Normalized Combination	
Wade	41.9%	31/74	20.0%	1/5	82.9%	29/35	47.5%	94/198
Deng	43.0%	21/49	40.7%	11/27	91.7%	11/12	45.0%	86/191

Similarly, a more accurate comparison of the Deng and Wade's shooting statistics in Table 5 should be based on the normalized combination from the regular shots and the free throws (Table 5N). Simpson's paradox still exists after the normalization, since Deng apparently had better shooting percentages in 2-point shot, 3-point shot, and free throws, and yet Wade had the better overall shooting percentage. However, it is clearer, based on the new table, that Wade outperformed Deng in the points made (94 versus 86 as seen in the last column), in addition to the overall scoring percentage. Although Deng dramatically outperformed Wade in the 3-point shooting, Wade performed better in the 2-point shooting and the free throws, not because Wade shot better percentages, but because he made more attempts in those two categories. The number of field goal and the free-throw attempts reflect contributions of other variables that otherwise are not factored in a simple shooting percentage. Notice also that before the normalization, the combined data shows that Wade dramatically outperformed Deng (Table 5); but the normalized data shows only a small outperformance by Wade. This is a good description of the two players' performances in the series. Deng is a better jump shooter, and thus had a better shooting percentage in every scoring category, and Wade is a better penetrating player for layups and thus created more opportunities for free throws.

4. Concluding Remarks

As more and more sports' data are collected, data mining in sports becomes more important [15]. Simpson's paradox often appears in data mining [16] and causes significant difficulties for interpretations. Statistical analysis of the 2011 NBA playoffs games has led us to observe many examples of Simpson's paradox in basketball. It is noteworthy that in many scientific studies, disproportionality in the sample counts between subgroups that causes Simpson's paradox is often (but not always) related to a sampling bias [9, 17]. In basketball, however, disproportionality in the number of shots between two teams or two players that causes Simpson's paradox is simply an outcome of how the game being played and refereed, and thus it is not intrinsically due to a sampling bias.

Simpson's paradox can cause significant challenges for decision making because of the reversal of statistical relationships [17-20]. Under Simpson's reversal, questions, such as who is a better player or which team played better in the game, arise. Depending on looking at the individual categories or the combined statistics, the conclusions are often the opposite. We have proposed a scoring normalization method that mitigates the collapsibility concern of combining three categories of shooting statistics. We hope that this helps more accurately interpret Simpson's paradox, and evaluate players and teams in basketball.

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