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Goalkeeper Financial Valuation

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Abstract. The Goalkeeper (GK) is a central element to any football team. He is more often than not the last resort the team can depend on when facing the offense; sometimes even the only resort in circumstances like the penalty situation. For these reasons, his performance cannot be diagnosed and evaluated with the same breath as his teammates since his role is very idiosyncratic and thus deserving of an adapted model for remuneration. This is where general models like MRP fall short of giving correct estimations of the GK's performance and need to be modified to cater to exceptional cases like the penalty shootout.
In this research, we seek to derive a scheme that befits the GK. We exert considerable effort to build it on a solid financial basis by borrowing standard procedures for the financial evaluation of assets. We thus draw comparisons to interest rate behavior in financial markets in order to create a sound perspective for the GK as an asset inside a (team) portfolio.

Keywords: Financial Asset, goalkeeper, penalty

1. Introduction

In this paper, we attempt to improve some of the shortcomings of the standard method of evaluating sport athletes known as Marginal Revenue Product. Our main incentive is the difficulty in applying such methods to a particular player such as the GK whose performance relationship with respect to the rest of the team is not clearly defined. Thus the framework listed in Scully (1974) for baseball players is hard to apply in our case especially as the MRPs in the context of that sporting industry start at well below that found) found in major league football. Hence, MRP raises the question of whether the salary a player earns necessarily constitutes his (intrinsic) value to the team.

As a matter of fact, in this research, we seek to nullify this question by proposing a fair pricing of the GK as a function of his performance during penalty shootouts. We choose this spatio-temporal setup as it consists in a circumstance where he is clearly detached from the rest of the team. We can view this in a scenario where all Strikers (Ss) on both teams have scored while the GKs do not make a single save: still neither team has won. In other words, there must be a marginal impact from the GK to improve the chances of winning which accordingly sets apart the good GKs from the bad. Eventually, this should boil down to who is paid more on a national and/or league scale.

Our approach relies on fundamental financial evaluation techniques which define the basis for standard present value operations. We also incorporate risk factors that mimic cash flow uncertainty through the use of a "yield" measure that contains a certain risk spread particular to each GK. Our method has the benefit of being flexible and adaptable to any timeframe or context the user wishes to test for without the burden of missing out on key explanatory variables, a problem inherent in empirical approaches such as MRP. Since we focus solely on laying the grounds for a flexible and expandable model, we are momentarily setting aside issues such as purchasing power parity and other currency related issues by assuming a single currency.

Before laying the foundations for our evaluation in the subsequent sections, we start with an overview of the two main components which are helpful to our analysis. We then support our use of a financial rather than empirical model based on a key finding involving GKs percentage savings and its relation to conventional interest rate models. We end with an application of our method through an actual example and conclude with further research prospects.

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1.1. The Goalkeeper (GK)

The GK is one of 11 players on the field at any given time during a football match. Without him, a football match would more likely resemble a basketball game in terms of scoring as every player would be able to put a ball into a net almost 50 times its size. This is mainly the reason for the GK's unique prerogatives inside the penalty area that set him apart from his teammates though he follows the same rules as the rest of the team once he steps out of that zone.

Due to the difficult task of intercepting missile-like shots that are heading for his goal (sometimes at more than 225 km/h), he has the right to make use of his hands. Nonetheless, nothing prohibits the keeper from scoring goals and playing the role of an attacker (numerous examples have occurred particularly in the dying seconds of a game). Yet, he must remain aware never to leave his restricted area unless he is 100% sure of not putting his team at risk or he will render the net an easy target for opponents.

Although it is common for the rest of the team to take part in the defense, more or less according to their position in the team, there is, however, a situation where the GK is the only guardian of a 7.32x2.44 goal, the penalty situation.

1.2. The penalty

The penalty shootout is unquestionably the hardest challenge for a GK to handle. It was introduced in 1891 to guard against frequent usage of illegal methods (notably the hands) for stopping shots at the goal (Benjamin, 2007). Then, it became a means of distinguishing winner from loser when both teams reached a tie at the end of regulation according to the responsible body "Fédération Internationale de Football Association" (FIFA). Finally, it was used as a means of sanctioning a defender for harsh behavior against an attacker inside the GK's perimeter, thus hindering the latter's chances of success (Bar-Eli et. al, 2007). Ever since FIFA legalize it on the 27^{th} of June 1970, the penalty shootout has been part of almost all competitions (Jordet et.al 2004).

According to the FIFA Official Laws of the Game, the penalty shootout consists of having a properly identified S shoot the ball toward the goal, which is 11 meters away, in order to score for his team. This time, the GK is alone against his opponent and cannot leave his goal before the S has shot the ball ("Highlights", 2007). Initially, five players from each team take their turn at shooting a penalty. The winner is whichever team scores the higher number of penalties. If a single shootout is insufficient to proclaim a winner, an extra shot (a.k.a. "sudden death") is given to each of the teams in an iterative manner until the matter is settled.

Contrary to common perception, during this daunting task, the pressure is less on the shoulders of the GK than it is on those of the S who rarely doubts his ability to impose himself in such a seemingly easy task (reminder of the dimensions at stake) by controlling the ball's direction safely into the net.

One could regard that as mere wishful thinking. Countless accounts (Pollard and Reep (2002), etc…) have been made on Ss losing their poise as they move along the field to strike the ball (more so even before the teams have entered the shootout). And that is what the GK should capitalize on.

This is where the role of the GK comes into play, to exploit that circumstance. He has learnt to do so through arduous daily training on how to increase his chances of making a save even though the latter is relatively low probability-wise. In order to justify the idea, which remains a subject of debate nowadays, unlike the S who can undergo between 15 and 20 penalty shots in a single season (Palacios-Huerta, 2003), the experienced GK has disputed countless such duels notwithstanding the tens of interceptions per game to his account, according to Newman-Norlund (2009). He is therefore better trained at this task and holds a certain advantage thereto. This perception has been shared by other authors as well, namely Chiappori et. al (2002), who go even further to treat this experience as "natural" even in such grueling instants of uncertainty. This statement leads us towards refuting the idea that our penalty-related model could be, for instance, applicable to Ss as well, since the number of penalties encountered during a S's lifetime is much too small to make a sizeable data-source which would be used to measure his/her performance.

Such a shift in perspective has its roots in the words of an English star of the 80s who portrays the penalty shootout as a "war of nerves between S and GK". This idea is taken to a higher level in (more intense) cases where the S has no-one to stop him. It is therefore under the illusion of simplicity that the difficulty amplifies. The most common example is the free throw in basketball or the spot kick in American football.

2. Financial Valuation

2.1. Financial overview

For practical purposes, this section was inspired almost exclusively by Minquet (2005) but the knowledge base is common to any primary study on finance.

The economic value of an asset is based on return as the sum of **V** of cash flows $\mathbf{a_i}$ that it will procure. These flows are subject to temporal fluctuations related to market movement. Nevertheless, they must take into account the effect of time (a.k.a. actualization) over the lifetime **n** of the asset.

Given the compounding effect of time, interest rates multiply among themselves at each period to render a gain to the investor over the initial capital plus the sum of gains over the preceding periods. We note, however, that the asset often still holds value ($\mathbf{A_n}$) even at maturity, which could even exceed its current value (**V**) and constitute an added value in the eyes of the investor.

The following formula illustrates the association between the terms presented so far:

$$V = \frac{a_1}{(1+i_1)} + \frac{a_2}{(1+i_1)(1+i_2)} + ... + \frac{a_n}{(1+i_1).....(1+i_n)} + \frac{A_n}{(1+i_1).....(1+i_n)} \tag{2.1}$$

To simplify the equation, the multiplicative terms in the denominator can be simplified under one return value (θ) to yield the equation below:

$$V = \frac{a_1}{(1+\theta)} + \frac{a_2}{(1+\theta)^2} + ... + \frac{a_n}{(1+\theta)^n} + \frac{A_n}{(1+\theta)^n} \tag{2.2}$$

In order to find out whether her investment (**I**) is fruitful, the owner simply compares the generated cash flows (**CF**) to those that an alternative deposit of money at a bank would have landed under a given interest rate. Therefore, if the return (**r**) is greater than the financing cost (**θ**), the investment is wise from a comparative point of view. For instance, the prominent French-Algerian player Zinedine Zidane gave his Real Madrid team a player-specific overall return on his wage (all income sources combined) of 41% based on IP calculations (see below). This was almost 4 times higher than the WACC of 10.8% according to Berg and Rousseau (2006).

Eq. (2.2) simplifies if we abstract from the resale value and assume a constant return amount (coupon) **a**, then when **n** goes to infinity:

$$V = a\frac{1-(1+i)^{-n}}{i} = \frac{a}{i} \tag{2.3}$$

Furthermore, if the Net Present Value (NPV), which is the difference between the discounted **I**-generated flows and **I** itself is positive, the investment can be deemed profitable:

$$NPV = -I + \sum_{t=1}^{n} \frac{CF_t}{(1+\theta)^t} + \frac{I_n}{(1+\theta)^n} \tag{2.4}$$

There are other indicators capable of measuring the return on an investment. The most renowned is the index of profitability (**IP**) which solves the shortcoming of NPV in its ability to compare unequal investments. Otherwise, both variables give the same result even for non-equal asset lifetimes.

$$IP = \frac{\sum_{t=1}^{n} \frac{CF_t}{(1+\theta)^t} + \frac{I_n}{(1+\theta)^n}}{I} \times 100 \tag{2.5}$$

These formulae predict the future of an investment in absolute certainty conditions but this is hardly the case in reality. This is why matters translate towards an *expected* return in a risky environment. Its value is given by the sum of returns ($\mathbf{r_i}$) weighted by their respective probability of occurrence ($\mathbf{p_i}$). This is nothing but the weighted mean of returns:

$$E[r] = \sum_{i=1}^{n} r_i p_i = r^* \tag{2.6}$$

Moreover, the square root of the second moment (or variance **V[r]**) represents the variability (volatility) with respect to the mean (a.k.a. standard deviation **σ(r)**). In more mathematical terms, it takes into consideration the dispersion of returns around the mean. This measure quantifies the risk at hand:

$$\sigma[r] = \sqrt{V(r)}\ ,\ V[r] = \sum_{i=1}^{n} p_i (r_i - r^*)^2; \tag{2.7}$$

2.2. Theory

In this paper, our modus operandi is based on equations of the same type as those used in the determination of the present value of an asset. Examples can be found in Minquet (2005) who describes the return on asset (ROA) as the "constant discount rate for which the sum of future values it generates is equal to its present value". This is applicable to any asset, including players.

Yet, it is usually difficult to estimate the exact value of **r** or **i** for each maturity under consideration. This is why financial institutions relax the constraint on the identity of interest rates at each pillar and call on interest rate curves to factor the time fluctuations and calibrate their models. We will show how to apply this in the context of a penalty shootout. The intuition for our technique derives from the fact that we can modulate each penalty by a factor taken from a certain curve behaving just like a price-fixing model that adjusts to the LIBOR or even the adjusting of index issuers' risk by the use of a skew.

We now proceed to the construction of this curve and examine it via an overview of previous penalty shootout studies.

3. Key Model Factors

Our method aims at being as generic as possible. In this section, we describe the main factors affecting a shootout.

3.1. Spatial factor

According to Scully (1974), "the absolute quality of play is generally regarded as being higher in the National League". Moreover, up until the time of writing of Jordet et. al (2004), only one player had ever missed the first shot in a penalty shootout at the Copa America, i.e. 96% scoring. This percentage changes with tournament "quality" as shown at the UEFA Champions League Cup (86.3%) and World Cup (78.1%). This is important in that it motivates the existence of a spatial factor governing players' performance; one that is probably linked to an incremental burden carried by the Ss which increases with tournament "quality".

3.2. Temporal factor

Palacios-Huerta (2003) demonstrated that in order to ensure continuity in players' performance in a mixed strategy *game*, it is essential that a hypothetical remuneration for each game be constant or totally uncorrelated in time. In other words, we could simulate players' choices using a random selection process that would guarantee independence between performance and pay. Since we are concerned by the evaluation of the GK and not the S, this would allow us to replace the 5 or more Ss in a penalty session by just one, through providing such a random process.

3.3. Striker factor

Although most players are ambidextrous on the field, during penalty shootouts, players tend to use their preferred or stronger foot and do not change that on inter or intra session basis (a player could shoot more than once in a single session provided the whole team has already participated and the score is still unsettled). According to McMorris and Colenso (1996), it seems that GK anticipation of right-footed players is more refined than their left-footed peers. This is undoubtedly related to their experience, as most GKs have faced more of the former than the latter, which somewhat biases their performance to the detriment of right-footed Ss. For that matter, Chiappori et. al (2002) have found that 85% of the S population is right-footed.

3.4. Emotional factor

3.4.1. The Vasicek process

Morris (1977) quantified the importance of a point in tennis according to the hypothesis that it derives from the difference between winning the game, given that the point was scored, and that of doing so when it was not. This intuitive definition also made its appearance on the football field through the works of McGarry and Franks (2000) who drew by analogy the importance of a penalty shot from its influence on winning the match. That is why successive penalties are deemed to increase in importance due to the accentuation of the cost of losing as players progress through each stage.

Palacios-Huerta (2003) reveals that the scoring percentage deteriorates as we move toward the end of the session, independently from the fluctuations along the way (pointing to a decreasing mean). It is for that reason, firmly grounded in players' emotions, that Jordet et.al (2004) have recommended that trainers work on reducing the players' perceived importance of each shot during the shootout. Scully (1974) pointed to a "demoralization" component but failed to derive its workings in other than a simple empirical representation which involved *number of years in practice* in addition to (the questionable assessment of) the *players' bargaining power*. In this section we analyze this component further.

By quantifying Emotion in relation to the mean performance at every stage (shots 1-5) which is also a function of the preceding kicks and their results, we find ourselves with two data series of scoring percentages. We draw the first from Palacios-Huerta (2003) which is constituted by kicks from World Cups (1982-1998) in addition to the 1996 Euro. The second, borrowed from Jordet et.al (2004) is based on a selection of World Cups, European Championships and Copa America, making a total of 409 kicks. We have recorded both series in Fig. 1 below where the third series, dubbed 'Jabbour-Minquet', represents an average of both, to which we have applied the curve fitting through regression.

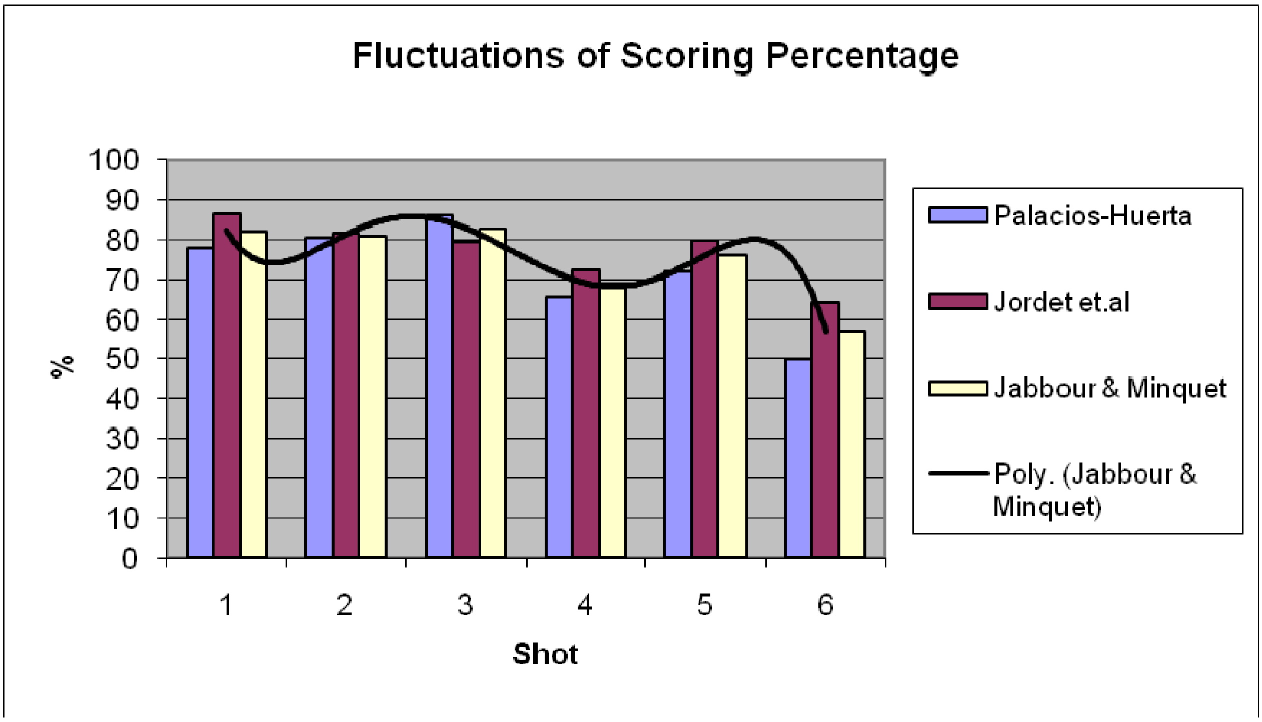

Fig. 1 : Illustration of the Penalty Process: Vasicek ($\mu = 76$, $\sigma = 8$)

As seen from the figure, although performance does not degrade linearly after each step, the largest deviation from the mean happens during the extra shots (64.3%, 50.0%, etc…) which go on through kicks 6-9 and decrease exponentially in frequency (20, 4, 2, 2). We decide to disregard these shots as a "*maturity that occurs less frequently than others*" does not apply in the context of cash flow discounting. This association will become clearer as we proceed. Nonetheless, we attribute this sharp performance decrease at the end of the shootout to a surge in stress; not to mention that the best players (usually distributed over the first *standard* five shots) have already had their go at the penalty strike.

This indicates that other factors are in play which explain, for instance, that in Palacios-Huerta (2003), the third pillar/penalty is the easiest of all while the fourth is the most difficult (both authors combined); while one could have thought initially that those were the first and last, respectively. A further explanation provided by Jordet at. al (2004) states that, usually, the finishing touch, (a.k.a. "*coup de grâce*") is the merit of the most talented Striker, every so often the team captain, which translates in a higher scoring probability towards the end.

The values we found corroborate the mean reversion at a *long-term* average of 76 (extra shots discarded). Though 5 data points might be an insufficient framework to study such a process, we nevertheless notice important aspects. A study of the correlation among shots gives a Pearson coefficient close to 21% which, in this context, is evidence of pseudo-independence. Moreover, the average we obtain is only half a standard deviation away from the suggested mean in the literature (80%). These facts give solid grounding to our results.

On the other hand, if we still take into consideration the extra shots, our mean only falls by 4 percentage points, which hints at the presence of a martingale (success rate for shot #5 = 76%) if we establish the end of the process at the fifth penalty. Otherwise we can think of a generic Vasicek process. This background will have allowed us to enhance McGarry and Franks's (2000) views on an inverted U-shape that would describe the "importance" of each kick; and replace it with a Vasicek process which accounts for mean reversion and describes the "penalty process".

A culminating point in our analysis is based on one of Palacios-Huerta's (2003) datasets. It gives the scoring percentage on penalties during the regular course of a game as a function of the match situation the teams find themselves in. This is summarized in the goal difference with regard to the opposing team: for instance, a value of '-4' indicates that the team under consideration is trailing by 4 goals during the course of the match. We expect that the more a team is comfortable with its situation the less it will focus its attention on scoring. This is also the case for the GK who will feel the pressure of being at his best level much less. This comfort, however, is not a linear function of scoring differential as shown in Fig. 2 below. Indeed, a surprising detail revealed by this figure is that it is only when a team is at its maximum comfort (ahead by 4 goals) that its players do not miss out on any penalty. On the other hand, when they are less ahead (2 goals), they find themselves in their worst shape. This is even worse than when they find themselves trailing by 2 goals, probably a time when they start to lose hope and concentration.

Conversely, two paradoxical situations - when the winner of the game is known ahead of time (-4 goals) and when everything is still at stake (0 goals) - distinguish themselves by showcasing the best performance (above 80% accuracy). The first can be explained intuitively by the fact that when a team has completely lost the essence of the game, it will focus its attention on saving face and scoring at least the "dignity goal" as it is called in Asian competitions. In contrast, when both teams are even, the two try their best to score the winner goal (sufficient) and are therefore at their strongest level.

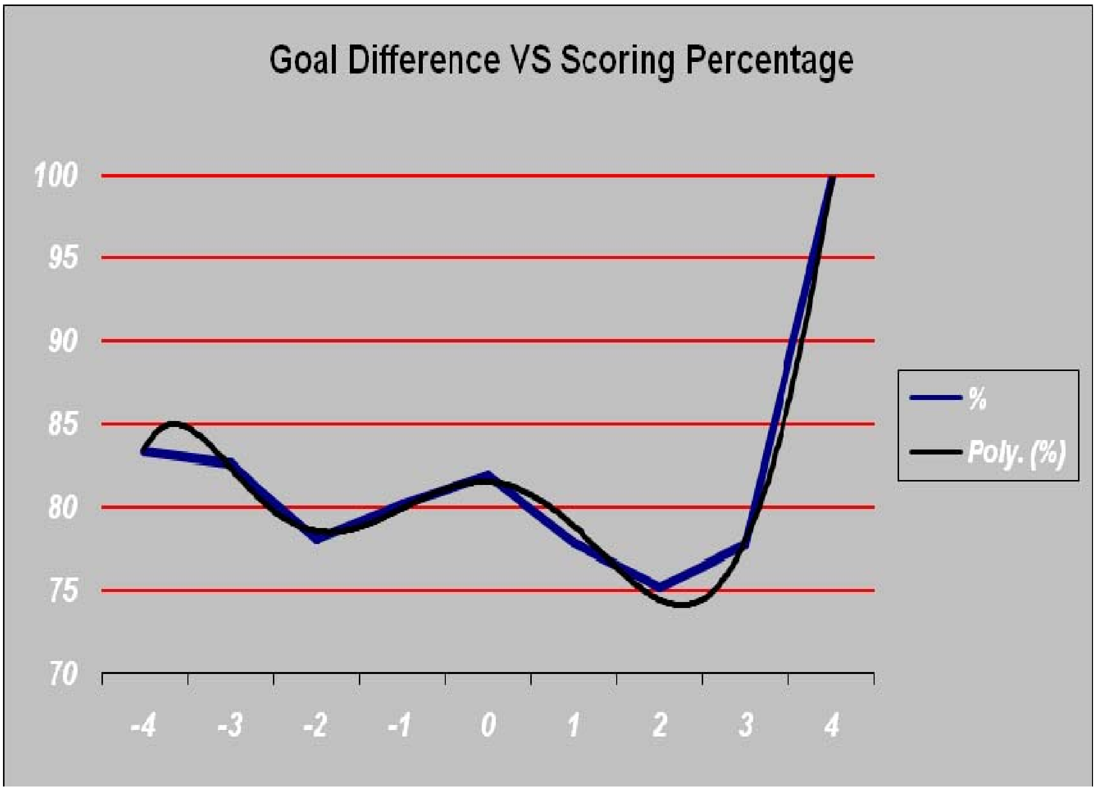

Fig. 2: Goal Difference Effect on Penalty Scoring during Regular Time

What we care about more here than the simple performance differential is its evolution with respect to the match situation. The polynomial regression in Fig. 2 points to the resemblance to the fluctuations during the regular penalty session. Once more, the behavior seems to be linked to a decreasing trend with mean reversion (Vasicek). Could it be that the same laws govern the scoring percentage during penalties in regular time and those in a shootout? We will try to elucidate this matter in the forthcoming section.

3.4.2. Bridging the Gap

The dissimilarity between the two situations where a penalty is taken during the game or the extra time shootout is that in the latter case, there is no doubt about the equality of the two teams, whereas in the model of Fig. 2, any case of goal differential can present itself. Even so, as in extra time, both teams had already started off with an equal score before reaching their current state (0-0). This inspires us to think that the same guiding principles could be governing the two situations. Yet, in order to scrutinize the shootout situation, we encounter one barrier: the fact that there are only 5 penalties during the shootout whereas there are 9 differential states according to Fig. 2.

To solve this problem, we make use of the sliding window technique, well known in network engineering for reading in data packets. This consists of *sliding* the matrix of 5 scoring percentages above those of the situations presented in Fig. 2 and calculating the correlation at each step of the slide. We apply this method to each of the series in Fig. 1 to ensure consistency in our results.

Each cone in Fig. 3 corresponds to a more or less high level of correlation pointed to by the sharpness of the cones. For instance the first two cones of the fourth column, where the goal differential is ‘1’, are the most acute of all; whereas the one at the last level below is a bit more flattened than the others. In contrast, the totally squashed areas represent the most minimal degree of correlation. Each situation (-2, -1, 0, 1, 2) denotes the center (median) of the matrix that extends over ±2 goals. For instance the matrix for the first case (-2) is [-4, -3, -2, -1, 0].

By means of our ‘Jabbour-Minquet’ generated series, we notice that the correlation accentuates above the level of 54% in a tie condition to reach 77% after a 1 goal lead (81% for the Jordet et.al curve, which brings with it the most radical change with respect to the tie case). In fact, what matters to us, be it for any of the two cases, is that the convergence with the penalty situation is most visible. In fact, we could the single goal lead case as it is totally offset by the opposing case of ‘-1’.

This allows us to certify that the law is generic even in regular time which means that the odds that depend on the circumstances and time passage are identical to those in a penalty shootout. In other words, our Emotional factor (stress, etc…) also applies to players with respect to their status towards the opposing team during regular time. This does not imply, however, that Emotion is not accentuated during penalty strikes. The average difference of 3% with the play in regular time (for all shots) is strong proof of that.

In sum, this analysis is important to respond to one sensible critique which could be addressed to our technique based on penalty sessions: are these a good proxy for evaluating GKs given that the vast majority of them is likely to have taken part in maybe only a few shootouts? Indeed, since the same laws govern the extra and game-time penalties, the context of when the penalty is taken does not really matter. We could, for instance, aggregate multiple game penalties to constitute a single session and apply our method since, to start with, we established a single (virtual) S facing our GK.

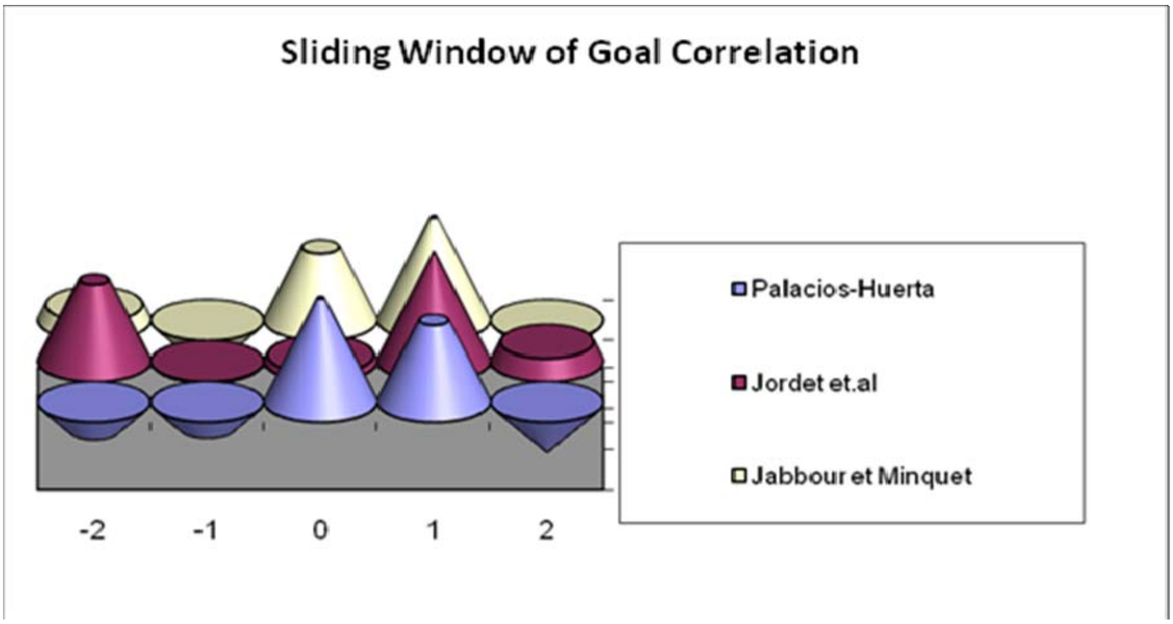

Fig. 3: Discovery of the Maximum Correlation through Sliding Window

We end this section by clarifying one important matter. The rules stated so far point to the fact that due to the difference in importance of each shot, the remuneration of the player who shoots the most pivotal one should, a priori, be the highest. Despite this, the amounts are constant in order to guarantee inter-shot independence, paving the way for de-correlation between the temporal and emotional factors. In financial terms, this is referred to as a diversification effect between different risk factors:

$$\text{Corr}(T, E) = 0 \tag{3.1}$$

4. Survey of Available Evaluation Methods

Many attempts have been made to model player characteristics. At the macro scale, methods such as MRP, or at the micro scale, by focusing on single aspect factors of the latter methods which merit their own representation as well. We will illustrate the latter with an example of stress.

4.1. Modeling stress

McGarry and Franks (2000) have attempted to model a seemingly intangible human trait, stress, through two thresholds: ‘High’ (stress = 1.0), when a player has the chance to take the lead with his team or if the losing team must score in order not to fall behind, and ‘Low’ (stress = 0.0) otherwise.

The stress-dependent scoring probability is given in formula (4.1) where **j** is the penalty number (1-5), $\mathbf{p_x}$ the scoring probabilities given in the above section and $\mathbf{p_s}$ a discrete value between 0.01 and 0.05 for **i** between 1 and 5 to reflect the level of the player taking the shot. For instance, $p_{1,1} = 0.85 - (0.01 * 1 * 0.5) = 0.845$ or $0.85 - (0.01 * 1 * 1) = 0.84$ if the score is set to 0-0 or 1-0, respectively.

$$p_{i,j} = p_x - (p_s * j * \text{stress}) \tag{4.1}$$

4.2. MRP

This section presents a method often used in the valuation of an asset called the MRP (Marginal Revenue Product) adapted to players and taken exclusively from the research of Kedar-Levy and Bar-Eli (2008).

The model posits that each player (asset) **i** should generate an expected **MRP** or **E(MRP)** resulting from the act of playing itself or other peripheral income sources. The expected net profit $\mathbf{E(d_i)}$ and return rate $\mathbf{E(r_i)}$ of the team owner are given by:

$$E(d_i) = E(MRP) - c_i \text{ where } c_i \text{ is the player's cost} \tag{4.2}$$

$$E(r_i) = E(d_i) / c_i \tag{4.3}$$

We expect $\mathbf{c_i}$ to be strictly positive; and there are no restrictions on $\mathbf{E(d_i)}$.

In order to allocate team funds in an optimal manner, the authors’ theory is to compare (regress) the performance of the players with that of the team. Using the authors’ own words, we distinguish three types of synchronization of these assets with the whole portfolio team: neutral, dragging and compensating. For instance, the performance of the GK can be *neutral* with respect to the majority of players in the team, yet he *drags* behind (as opposed to *compensates* for) the defender. This means that whenever the performance of the defender is not up to standard, it is highly probable that the GK’s own performance will suffer. In statistical terms, compensating/neutral/dragging players have a negative/null/positive correlation between them.

In essence, the authors come up with a closed-form solution to set the price (cost) of a player based on his MRP and other terms related to his behavior within the team. Note the appearance of a Sharpe and covariance terms from basic CAPM. For more details on how to derive this formula we refer the readers to the authors’ study.

$$c_i = \frac{E(MRP_i) - \left(\frac{E(R_p) - C}{\sigma_p}\right)\left(\frac{Cov(MRP_i, R_p)}{\sigma_p}\right)}{1 + C} \tag{4.4}$$

5. Adapted Valuation Model

5.1. Model derivation

Because the GK finds himself alone during the penalty shootout, it becomes impossible to define his MRP as a function of his synchronization with the rest of the team’s performance. That is why we elaborate our adapted valuation method for the specific case of the GK, which dissociates from the MRP empirical framework used in Scully (1974), who acknowledged this weakness early on. Notwithstanding the fact that MRP needs to be supplemented by other methods to recover the actual salary amounts since it has been recognized to be in excess of the amounts actually found in practice [quoting]: "players of rare talent […] command salaries in excess of their relative contribution to the team".

On the other hand, GKs cannot be valued with standard statistics as with basketball players (Field Goal Percentage, Rebounds, Assists, etc…) or adapted ratios as in Scully (1974) for baseball players (Slugging Average or Strikeout-to-Walk Ratios). This suggests the need for an adapted model for GKs.

A widely accepted process governing interest rate behavior is that of Vasicek. As showcased previously, we now know that the penalty performance fluctuation also follows the same process. This means we can substitute interest rate for performance as a measure of risk in the GK's valuation. Fittingly, we dub this process Vas-y-Ceck (VYC). This is French for "*Go Cech*", in respect to one of the best GKs at the time of writing, the Czech GK of Chelsea, Petr Cech, who went to great lengths to defend his goal.

The analogy we draw in our model is at the level of the equation pillars, which, instead of being in years (1Y, 2Y, etc…a.k.a. "tenors" in financial terms), are now pointing to the successive stages of the penalty shootout: 1P, 2P...5P. We present this by reformulating equations Eq. (2.1) and Eq. (2.2) and dropping the resale value (as is commonly done with dividend stock valuation):

$$V = \sum_{i=1}^{5} \frac{a_i}{(1+p_i)} \tag{5.1}$$

More concretely, the values of $\mathbf{p_i}$ at every tenor are taken from the distribution of scoring percentage. The person concerned with valuing the GK has the option of choosing the "marking (scoring) curve" that seems to be the most adapted, just as risk managers need to make a choice on which rate or default distribution to use in their models. By varying the values of $\mathbf{p_i}$, our practice takes into account an aspect of Emotion that impacts the GK's situation. Of course, our depiction is generic enough to incorporate any additional dosage of the factor listed previously in the form of a spread inside the equations.

The next step is to choose the values in the numerator of equation Eq. (5.1); in other words the values $\mathbf{a_i}$ (compensation) attributed to each save made by the GK. As noted earlier, we discard divergences on the importance of each shot by assuming them to be all equal to a base value $\mathbf{a_0}$. If by an exogenous procedure, we judge that a penalty shootout is equivalent to a lump sum of **A** (one simple way to get this value would be to start from a known value of the reward the team will get by winning and divide it by a number between 11 and 22, the number of players having *participated* in the win. We can of course think of more elaborate models to quantify the impact of the GK), and then divide it amongst the 5 penalties, to obtain a starting value for $\mathbf{a_0}$. Finally, by weighing (multiplying) this value by a representative factor $\boldsymbol{\beta_i}$ to quantify the impact of various influential factors (as in the previous sections) at a given time, the new value of the numerator is more representative of the monetary value of a given penalty shot. Finally, by eliminating the simplification over $\mathbf{a_i}$, we can now think of a matrix $\boldsymbol{\beta} = [\boldsymbol{\beta_i}, \mathbf{i = 1,\dots,5}]$, which would lead us to an equation such as:

$$V = a_0 \sum_{i=1}^{5} \frac{\beta_i}{(1+p_i)} \tag{5.2}$$

There remain two variables we need to add in order to render our procedure more robust. The first, in the form of a function **I(.)**, represents the possibility of default in financial terms. Indeed, since the situation of the GK comprises a certain level of risk, it is likely that he may not perform to the maximum. He should not, in this case, be eligible for the compensation he would receive had he performed at his best. Therefore, the function returns a null value in case of default to cancel the payment on a given penalty, or in the opposite case, a unitary value insuring the GK obtains what he deserves. The equation then becomes:

$$V = a_0 \sum_{i=1}^{5} \frac{I(.)\beta_i}{(1+p_i)} \tag{5.3}$$

Having determined the model's factors, we now move to its user guide. The approach we follow is adapted from Scully (1974). It consists in that any GK is equivalent, financially speaking, to a total maximum salary (**S**) that his manager could pay him for a "clean sheet" (all shots saved) over his lifetime stay with the club. By calculating the total revenue of the club (**R**) in such a scenario which would translate into a full stream of no losses, we use this method over the penalty shootout in order to match the equation **S** = **γR** where **γ** is the fraction attributed to the GK. The logic of our procedure entails that if the GK saves all possible penalties, he would be worthy of **S**. Otherwise he should be content with only a fraction γ of **S**, that symbolizes his real course. Seen from a credit instrument perspective, assuming the GK was not at his best on a given day, γ represents the fraction of his "best" which he was able to render to his team; in other words, his recovery rate (RR).

The value of γ, which therefore becomes a corrective factor, is given by equation Eq. (5.4). We signal that the value of γ is independent from $\mathbf{a_0}$.

$$\gamma = \frac{a_0 \sum_{i=1}^{5} \frac{I(.)\beta_i}{(1+p_i)}}{a_0 \sum_{i=1}^{5} \frac{\beta_i}{(1+p_i)}} = \frac{\sum_{i=1}^{5} \frac{I(.)\beta_i}{(1+p_i)}}{\sum_{i=1}^{5} \frac{\beta_i}{(1+p_i)}} \tag{5.4}$$

In sum, at each session, the caliber of the GK is reevaluated with respect to his true performance. If he meets expectations, he will be rewarded; otherwise, he will lose value, as any market asset would.

5.2. Variability and remuneration

As Blackwell once mentioned (and recited in Kast (2002)), "a distribution **l'** is more risky than another one **l''** of equal mean, if the first can be obtained through some transformation of the second. Hence, a decision-maker who is risk-averse would prefer any random variable **X** to another one **X+Y**, where **Y** has 0 mean. In other words, he would consider undesirable, any addition of variability (**Y**) that would not result in an increase of his gains".

Hence, we can determine which GK is more worthy of remuneration by analyzing the variability of his performance, in such a way that any increase of dispersion around a given mean is a loss on investment. Put differently, we could hire a GK with the same average of saves and with lesser instability in performance for a better price than an "on/off" GK. Since volatility is synonymous with risk in a financial setting, no investor would be prepared to undergo such risk for an equal down payment to the GK.

By applying this method, we can establish a calibration between different GKs which are processed through our illustration above. The link between them should be strong enough to eliminate any arbitrage.

5.3. Application to the GK

We will now demonstrate the functioning of our technique through a concrete example. We base ourselves on the UEFA Champions League Final of May 2008 which opposed Manchester United (GK: Edwin Van der Sar) to Chelsea (GK: Petr Cech).

The reasons for choosing this example is that it brings together two teams which customarily play in the same domestic (English) league but this time on an international plateau. The reason we do this is to abstract from the Spatial Factor referred to above to simplify our illustration. Otherwise, the perspectives could be affected by the fact that it is the domestic rather than international leagues which largely determine player remuneration levels. In principle, we should incorporate the Striker Factor since 2 shots (i.e. 14%) were taken with the left foot. As we will see shortly, doing so will only slightly improve the better GK's valuation and therefore does not change much to our analysis. To simplify the study, we decide to do without; bearing in mind we could always modify the related single variable later on. Finally, the Temporal Factor is considered independently from the Emotional Factor motivated by relationship (3.1). We account for the former through establishing a constant remuneration for each strike while using the penalty performance data to measure the latter.

We culled our salary data from the Paywizard website and cross-compared it with various other sources while converting the amounts to euros at the at-the- time-of-writing exchange rates. The yearly salaries (in M€) we got are Van Der Saar (5.8) and Cech (5.2), which implicitly declares the former to be the better GK of the two. We will try to prove this while at the same time showcasing that these amounts, from the perspective of this unique competition only, are well above the "efficient price".

Faithful to our process, and based on the Jabbour-Minquet curve established above, we start by determining the value of $\mathbf{a_0}$. As mentioned earlier, this variable disappears in the computation for γ; however, for illustration purposes we use a maximum salary capped at M€6 which seems a logical estimate judging by the ex ante amounts. Our reasoning remains unaffected by this estimate. Hence, the per penalty maximum reward becomes €857,142.85 for a 2 shot extended penalty session. This yields the following equation:

$$\mathrm{V} = 857{,}142.85 \times \left(\begin{array}{l} \frac{I(.)\beta_1}{(1+82.2\%)} + \frac{I(.)\beta_2}{(1+81.15\%)} + \frac{I(.)\beta_3}{(1+82.7\%)} + \frac{I(.)\beta_4}{(1+81.15\%)} + \\ \frac{I(.)\beta_5}{(1+69.1\%)} + \frac{I(.)\beta_6}{(1+76.0\%)} + \frac{I(.)\beta_7}{(1+57.15\%)} \end{array} \right) \quad (5.5)$$

The next step consists of determining the values of **β** and **I(.)**. To do so, we refer to the statistics in the table below which were taken from the Daily Mail (2009). The entries in this table depict what took place at each kick, with '0' meaning the shot resulted in a goal and '1' in a save. From this data, one would say that the two GKs seem alike. Yet, our procedure will do well at selecting the best. '0+' is used to distinguish between these two cases and those where the GK makes the correct choice but the goal was scored nevertheless, and the opposite. We have used the mean of ½ to quantify the importance of these two states (i.e. give credit to the GK for making the right decision - jumped in the same direction - and removing any if he had no impact on the save- hit the post or far shot).

Table 1: Data on the Penalty Session (UEFA Champions League Final 2008)

Shot	Score	GK	Save	Description
1	1-0	Cech	0	Opposite Direction
1	1-1	VDS	0+	Same Direction
2	2-1	Cech	0	Opposite Direction
2	2-2	VDS	0	Opposite Direction
3	2-2	Cech	1	Save
3	2-3	VDS	0+	Same Direction
4	3-3	Cech	0+	Same Direction
4	3-4	VDS	0+	Same Direction
5	4-4	Cech	0+	Same Direction
5	4-4	VDS	0+	Hit the post
6	5-4	Cech	0	Opposite Direction
6	5-5	VDS	0	Opposite Direction
7	6-5	Cech	0	Opposite Direction
7	6-5	VDS	1	Save

The mean $\hat{\beta}$ is given for when all the shots are saved (i.e. **I(.)** = 1) and thus gives the solution to the following equation. The value we obtain is around 1.715.

$$6\mathrm{M} = 857{,}142.8571 \times \hat{\beta} \left(\begin{array}{l} \frac{1}{(1+82.2\%)} + \frac{1}{(1+81.15\%)} + \frac{1}{(1+82.7\%)} + \frac{1}{(1+81.15\%)} + \\ \frac{1}{(1+69.1\%)} + \frac{1}{(1+76.0\%)} + \frac{1}{(1+57.15\%)} \end{array} \right) \quad (5.6)$$

We now construct the matrix of values β_i for each GK: Cech **[0, 0, 1.715, 1.715/2, 1,715/2, 0, 0]** and Van der Sar [**1.715/2, 0, 1.715/2, 1.715/2, 1.715/2, 0, 1.715/2**]. The equation is defined by multiplying $\hat{\beta}$ by **I(.)** (0 or 1).

The value of each of the players thus becomes:

$$\text{Cech: V= } 857{,}142.8571 \left(\begin{array}{l} \frac{0}{(1+82.2\%)} + \frac{0}{(1+81.15\%)} + \frac{1.715}{(1+82.7\%)} + \frac{0.8575}{(1+81.15\%)} + \\ \frac{0.8575}{(1+69.1\%)} + \frac{0}{(1+76.0\%)} + \frac{0}{(1+57.15\%)} \end{array} \right) = 1{,}6\mathrm{M}$$

$$\text{VDS: V}= 857{,}142.8571\left(\begin{array}{l}\frac{0.8575}{(1+82.2\%)}+\frac{0}{(1+81.15\%)}+\frac{0.8575}{(1+82.7\%)}+\frac{0.8575}{(1+81.15\%)}+\\ \frac{0.8575}{(1+69.1\%)}+\frac{0}{(1+76.0\%)}+\frac{1.715}{(1+57.15\%)}\end{array}\right)= 2{,}6\text{M}$$

We notice immediately that these values are well below the real amounts, which alludes to an excess of funds which could have been attributed to the GKs by their managers without proper financial founding. Of course, one could reproach us for being overly strict towards the GKs, a matter we will return to in our recommendations to enhance the model. Yet, one particularity of our method which justifies the choice of our example in terms of exhaustiveness, is that it is not only in line with the given values (the best GK wins more) and at the same time coherent with the initial differential between amounts, but it also credits one GK more than the other although both have saved an equal number of penalties! This strongly suggests that our procedure has conveniently taken into consideration the differential of correct decision making by the two GKs and thereby diminished the role of pure luck. From a mathematical perspective, we draw attention to the ratio of salaries = 0.64 which is nowhere equal to the simple ratio of saves 1/1 which would have logically seen the same compensation given to both GKs. This serves notice to the well-known financial statement that no two investments are alike.

Finally, a simple trick we can apply in this case to make our scheme more robust comes from equation Eq. (2.3) which gives the value of an asset at infinity. In retrospect, we admit we have too strongly diminished the importance of the last kick (from the curve we selected $\boldsymbol{\beta_7}$ = 57.15%) although, paradoxically, it was the shot that gave Manchester the victory and, markedly, set Van der Sar apart from his homolog. To counter this effect, we draw an analogy to the fact that a single save that gives a team victory is comparable, from a result standpoint, to a clean sheet on the part of the GK from start to infinity (i.e. no performance could beat it). Consequently, by Eq. (2.3), if we take the mean of the scoring percentages (74.72), and multiply it by the value quantifying each shot, we obtain the true value of $\boldsymbol{\beta_7}$:

$$\beta_7 = a_0 \text{ x } \hat{i} = 857142.9 \text{ x } 74.72 / 100 = 5.23 \qquad (5.7)$$

To end with, by replacing this value in equation Eq. (5.6), we then obtain a salary amount of 4.5M for Edwin Van Der Sar which is must closer to the original value but still caps it downward. Similarly, we could also reduce the punishment of Petr Cech (augment $\boldsymbol{\beta_3}$), by presuming that he did, emphatically, stop the penalty of the (at the time) best player in the world…Cristiano Ronaldo.

6. Conclusion

Petr Cech (€5.2), Edwin Van Der Saar (€5.8), Heurelho Gomes (€7.8), Oliver Kahn (€8.1)…These numbers, in millions, distinguish the best GKs of our time…And yet, we raise the question of being able to dissociate these GKs from a financial perspective in line with these numbers to prove their adequacy, or, more strongly prove that the higher payment to one of the GKs is tantamount to a positive performance differential.

This is how we landed on our innovation to appreciate the GK's game and compensate for the shortcomings of the standard MRP model (without discarding the possibility of a likely merger between both). We distance ourselves from the empirical approach used in Scully (1974) which is prone to have missing explanatory variables as with any empirical assessment. The author himself acknowledges the many assumptions that exist in any MRP application especially in how individual performance affects the team's own performance. Hence, a key difference in our approach is it can accompany the GK in real time to reflect his value at any moment during his lifetime profession as a player (i.e. detaching him from the current team constraints of MRP). It has also allowed us to discern the "right/efficient price" of a GK and reduce extraneous expenses based on the GK's optimal revenue. Moreover, it can adjust according to competitions with a factor that can be easily deduced with no need for complicated approximation models.

Yet, we acknowledge that our model does pose, under certain extreme circumstances, an inconvenience that can still be easily remedied. Indeed if the mathematics are applied with no reserve, then, because of the $\boldsymbol{\gamma}$ factor, the value of the GK can shrink very rapidly, and even in the case where he misses all saves, it can become null. This would defeat our purpose; therefore, to cater to this, instead of taking the value of the Maximal salary as starting salary, we can take a fraction of it, decided ad hoc based on the competition's importance to the club, and this becomes the maximal gain of a single session of penalties. Another way would consist of taking a moving average after each competition until contract maturity to factor out extremes.

On the other hand, numerous factors can be incorporated into the model to make it more flexible and less harsh towards the GK. The main one which we have cited in this paper are the Stress model and other user-defined factors such as a "superstar" factor which Scully (1974) tested as insignificant in baseball. We leave it to further research to confirm or dispute such a factor in football. In addition, the expertise, accumulated over a certain span of time, could serve the GK well during difficult times. For instance a study by Helsin and Starke (1999) discovered a correlation equal to 0.83 between the ability of the GK to recall certain action-specific criteria and his correct anticipation of the shot direction (predictive ability). This entailed a 10% higher percentage for expert GKs. Finally, to break the inter-shot independence, by assuming, for instance, that a save that comes after a miss is harder to accomplish than one that comes after another save, we can compensate the GK differently by making use of the Meyer and Erev (1998) reinforcement models such as LRP, ERP, etc…in conjunction with the aforementioned Morris (1977) theory for deducing shot importance.

7. Acknowledgements

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