ISSN 1750-9823 (print)
International Journal of Sports Science and Engineering
Vol. 04 (2010) No. 03, pp. 165-173

Computer Simulation of "Splash Control" and Research of the Rip Entry Technique in Competitive Diving

JingGuang Qian[1] , Songning Zhang[2], Haiquan Jin[3]

[1]Nanjing Institute of Physical Education,Nanjing, China

[2]The University of Tennessee, Knoxville, USA

[3]Jiangsu Provincial Diving Team, Nanjing, China

(*Received January 8, 2010, accepted March 21, 2010*)

Abstract. The purpose of this study was to examine effects of different hand patterns used by divers on the minimization of splash at the instance of water entry for competitive diving. An impact model was developed with the human body modeled as a wedged solid object and the water as an ideal fluid. The equations of motion for the solid object were established with satisfactions of control functions and initial boundary conditions of the fluid. A finite element method was used to simulate the impact process in customized computational software. The results indicated a proportional relationship between the highest point of the unrestrained wave surface and the wedge angle during the impact. The simulation results indicated a "squared object" as the ideal shape for the water entry. In practice, instead of having palms facing each other to form a wedge at the water entry, the diver should internally rotate the arms and form a flat impact surface with the palms towards the water to effectively limit the water splash. Further mechanical analyses also suggested that, the palms should be maintained in a direction just opposite to the resultant velocity of the water in a "massaging" motion during the impact in order to effectively control the splash.

Keywords: Diving, Finite Element, Computer Simulation, Splashless Entry, Rip Entry

1. Introduction

A rip entry is a key element for water entry in competitive diving. The size of water splash may directly influence the outcome of a diving competition. Therefore, it has become a focal point of both practical and theoretical interests as to how to minimize the splash size by using various body positions and entry techniques during the entry from the initial impact to the total submerge of the entire body under water.

The splash control technique has been around for a while. At the beginning, most divers manipulate their limbs to form a sharp shaped form for the entrance. If the entry starts from the hands, both palms are usually placed facing each other with straight arms so that the body forms a wedge that has a sharp part and a blunt part. If the entry starts with the feet, the feet are fully plantarflexed with the toes entering water first. Such a body formation minimizes impact forces at the entry but usually causes a significant amount of splash. It was later accidentally discovered that a more dorsiflexed ankle position at the water entry actually caused less water splash (Rackham, 1975). Hence, divers and coaches started to experiment foot entry technique with more dorsiflexion and hand entry with hyperextended wrist (Rackham, 1975). It has gradually evolved into the current rip entry technique that requires a flat entry surface formed by overlapped hands with fully flexed and internally rotated shoulder joints.

Through trials and errors, coaches and divers have already accumulated some practical experiences and methods for the "splash control" technique. However, no breakthrough research has been performed on the theoretical basis of the technique; many aspects of the technique such as mechanisms of splash formation and optimization of splash control technique deserve further investigation (Brown, et al., 1984). Body water entry and splash formation are rather complicated phenomenon of impacts and interactions between a solid body and fluid. Impact between fluids and solids is a hot research topic; relatively effective methods analyzing the impact have been proposed over the years with some simplification process. Among them, more influential methods include the theory of similarity flow by Mackie (Mackie, 1969) and Dobrovol'skaya (Dobrovol'skaya, 1969), method of matching and gradually developing by Armand (Armand and Cointe, 1986) and Cointe (Cointe, 1989), and especially the Laplace transformation adopted by

Gavrilenko and Kubenko with applications in impact between rigid body and fluid during entry of a squared rigid body (Gavrilenko and Kubenko, 1985), an oval object (Gavrilenko, 1986), a symmetrical object (Kubenko and Gavrilenko, 1987) and, elastic spherical shell (Gavrilenko, 1989). In addition, numerical methods for the impact phenomenon have seen some further improvements in the 9-node isoparametric rectangular element (Mareal, 1978) and nonlinear boundary element method (Zhao and Faltinsen, 1993). All these research results have provided foundations for studying impact between human body and water. As to the splash formation during diving, many factors may influence the final outcome. However, the formation process can be divided into three different stages: 1) the process of initial impact between the human body and water; 2) the process from the initial body water entry till the complete submergence of the diver; 3) the process of turbulent flow formation after the complete entry. Therefore, the purpose of this study was focused on examining relationship between hand patterns and splash heights during the initial impact stage of non-rotating diving using a finite-element model.

2. Materials and Methods

In order to study the relationship between hand patterns and water splash heights, following simplifications were made during modeling and computer simulation. The water was treated as an ideal fluid (incompressible and non-viscous) and the human body as a wedge-shaped solid object (Figure 1). A physical and mathematical model of the wedged solid object and the ideal fluid during the impact was first established. The equations of motion (interaction equations between fluid and solid) for the solid object were established, which satisfied control functions and initial boundary conditions of the fluid. Computational software was developed using a finite element method to simulate the impact process of the wedged object with the fluid with the wedge angle changed from 4° to 80°.

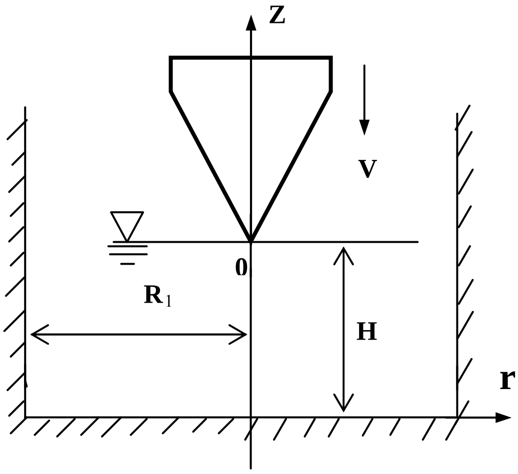

Figure 1. Initial stage prior to the impact between the wedge and fluid.

Simulations and Computation of Impact Process with Water. The human body was simplified as a wedged-shaped solid object and the water entry of the diver during the impact was treated as the impact between the wedged object and the water. According to the conservation of mass and energy, principle of momentum, and the principle of fluid viscosity, equations of continuity, equations of motion (momentum equations), energy and constitutive equations can be derived. These equations, state equations, and basic equation sets of fluid dynamics are formulated in the following forms of differential equations:

$$\begin{aligned} &\frac{\partial p}{\partial t} + div(\rho \vec{V}) = 0 \\ &\rho \frac{d\vec{V}}{dt} = \rho \vec{F} + divP \\ &\rho \frac{dU}{dt} = P \bullet S + div(kgrad(T)) + \rho q \\ &P = -pI + 2\mu(S - \frac{1}{3} I div \vec{V}) + \mu' I div \vec{V} \\ &p = f(\rho, T) \end{aligned} \tag{1}$$

Where ρ is the water density, $\vec{V}$ the velocity vector of the fluid field, $\vec{F}$ the body force applied to the water, $\boldsymbol{P}$ a stress tensor of the fluid field, p the pressure of the fluid field, T the water temperature, $\boldsymbol{I}$ the unit tensor, U internal energy of water, $\boldsymbol{S}$ the velocity tensor due to deformation in the fluid field, k the coefficient of water heat conduction, q the heat radiation coefficient, μ the first coefficient of water viscosity, μ' the second coefficient of water viscosity, and *div* divergence.

With respect to the ideal and incompressible fluid of a non-rotating motion, the fluid motion should satisfy the control equation:

$$\Delta\varphi = 0$$
$$\frac{\partial\varphi}{\partial t} + \frac{\vec{V}^2}{2} + \frac{p}{\rho} + gz = f(t) \quad (2)$$

The above equation is formed by a second-order linear partial differential equation and a potential function, and can be used to determine unknown φ (potential) and p (pressure).

Because of its symmetrical physical nature of the wedged solid, the original three-dimensional problem can be reduced to a two-dimensional planar problem by using a cylindrical coordinate system and the computation process can be therefore greatly simplified. The opposite direction of motion for the wedge represents z-axis and the left horizontal the r-axis (Figure 1). The origin of the coordinate system is set at the intersection point between the z-axis and the bottom of the pool; the equation (2) becomes the following under the cylindrical coordinate system:

$$\frac{\partial^2\varphi}{\partial r^2} + \frac{\partial^2\varphi}{\partial z^2} + \frac{1}{r}\frac{\partial\varphi}{\partial r} = 0$$
$$\frac{\partial\varphi}{\partial t} + \frac{\vec{V}^2}{2} + \frac{p}{\rho} + gz = f(t) \quad (3)$$

Where φ and p are functions of r, z and t.

The equation above has an infinite number of solutions. However, only those solutions satisfying boundary and initial conditions are meaningful. Therefore, the equation (3) must be solved under certain initial and boundary conditions.

Initial Conditions. During the water entry of the wedged object, the focus of discussions in this paper is on the first stage (t = 0) and second stage (t > 0) where the first stage is the initial condition of the entire impact process (Figure 1). More specifically, the states associated with the wedge and water in the initial stage are the initial conditions at t = 0.

$\varphi = 0 \quad (0 \leqslant r \leqslant R,\ 0 \leqslant z \leqslant H,)$

$p = \rho gz \quad (0 \leqslant r \leqslant R,\ 0 \leqslant z \leqslant H)$

where φ is the velocity potential function of the fluid field, P the function of pressure in the fluid field, ρ the water density, and g the gravitational acceleration.

Boundary Conditions. The second stage of the solid and fluid interaction, the impact stage (t > 0), is the main interest of this study (Figure 2). The free surface of the water experiences dramatic changes during the impact between the wedge and water, and the entry of the solid into water in this stage of the process. The water boundary conditions must be known in order to determine the state of the free water surface in the computation process.

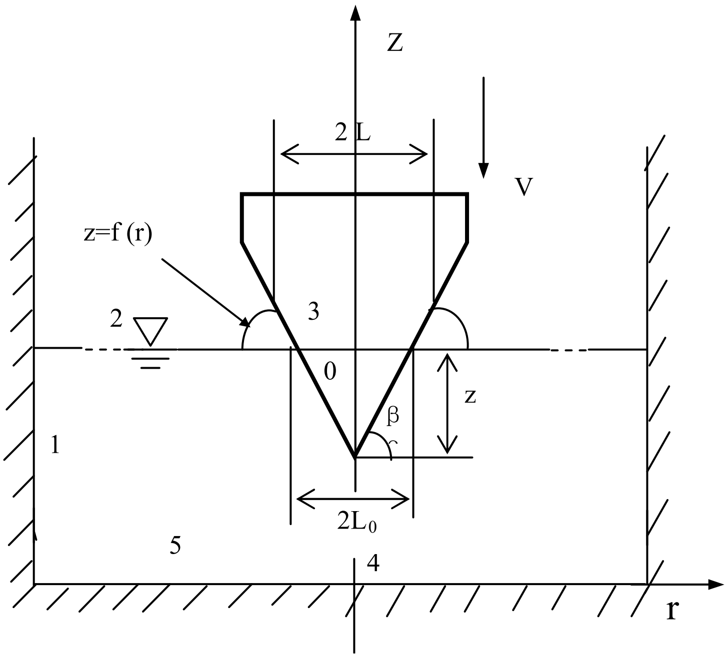

Figure 2. Impact stage between the wedge and fluid

$$v_r = \frac{\partial \varphi}{\partial r} = 0 \qquad \Gamma = \Gamma_1 \text{ for pool side}$$

$$p = P_0 \qquad \Gamma = \Gamma_2 \text{ for fluid surface}$$

$$\begin{cases} v_n = \dfrac{\partial \varphi}{\partial n} = v_l \cos(\beta) \\ 2\pi \cos\beta \int_0^L prdr - MG = Ma \end{cases} \qquad \Gamma = \Gamma_3 \text{ for interface between wedge and fluid} \tag{4}$$

$$v_r = \frac{\partial \varphi}{\partial r} = 0 \qquad \Gamma = \Gamma_4 \text{ for vertical cross-sectional area}$$

$$v_z = \frac{\partial \varphi}{\partial z} = 0 \qquad \Gamma = \Gamma_5 \text{ for pool bottom}$$

During the impact of the solid and fluid, the boundary conditions are formed by the interface surface between the solid and fluid, and the fluid surface. The boundary conditions of the fluid surface are essential boundary conditions while the natural boundary conditions include the interface of the fluid with the impact solid, and the sides and bottom of the pool. According to the equation of motion for the wedged solid and the characteristics that a non-adhesive fluid has a zero velocity along the normal direction of the wedge wall, the fluid boundary conditions can be expressed as:

where β is the oblique angle of the wedge wall (Figure 2), L the half length of the interface of the wedge and fluid, φ the velocity potential function of the fluid field, v_t the instantaneous vertical velocity of the solid, and g the gravitational acceleration.

Solution of velocity potential of ideal fluid using finite difference method. It is assumed that the approximate function (Li and Yuan, 1987; Zhang, 1986) in element e of the velocity potential function φ (r, z, t) has this form:

$$\varphi = \varphi_{.i}(t)\,\Phi_i \tag{5}$$

Where Φ_i is an interpolation function of the selected velocity potential, I = 1, 2, …, N_i^e, N_i^e the definite function of velocity in the element, $\varphi_i(t)$ the value of the velocity potential function at the time t and the knot i. By selecting the interpolating function of the velocity potential as a weighted function and establishing control functions of Galerkin integration (Wang and Shao, 1997), the strong form of integration of the velocity potential is found with consideration of the natural boundary conditions. In addition, the equation of motion (4) for the wedged object is combined into the equation (5). By setting $\delta\varphi = \Phi_j$ in the equation for the element e and through proper simplification, the following finite-element system equations for the element are derived:

$$[k_\varphi^e]_{ji}\,\varphi_i(t) = [k_F^e]_j \tag{6}$$

Where

$$\begin{aligned}[k_\varphi^e]_{ij} &= \iint_\Omega \left(\frac{\partial^2\Phi_i}{\partial r^2} + \frac{\partial^2\Phi_i}{\partial z^2} + \frac{1}{r}\frac{\partial\Phi_i}{\partial r}\right)\Phi_j \cdot d\Omega \\ &- \int_{\Gamma_1}\frac{\partial\Phi_i}{\partial r}\,\Phi_j d\Gamma - \int_{\Gamma_4}\frac{\partial\Phi_i}{\partial r}\Phi_j d\Gamma - \int_{\Gamma_5}\frac{\partial\Phi_i}{\partial z}\Phi_j d\Gamma \\ &- \int_{\Gamma_3}\left[\left(\frac{\partial\Phi_i}{\partial r}\right)^2 + \left(\frac{\partial\Phi_i}{\partial z}\right)^2\right]\Phi_j d\Gamma \\ [k_F^e]_j &= -\int_{\Gamma_3}(v_i\cos\beta)^2\Phi_j d\Gamma\end{aligned}$$

Where i = 1,2, …8 and j = 1,2, … 8.

By using a difference method, a discrete solution is sought as a function of time in equation (6) and the unsteady motion is converted into a steady motion within the unit time. Then a general matrix of the finite element functions is derived through assembly of an entire matrix of stiffness:

$$[K]_{NN}\{\varphi\}_N = \{F\}_N \quad (7)$$

This is a set of N_{th}-order linear equations. By solving this set of linear equations within the boundary conditions, the velocity potential for the entire fluid field at the time of $L+1$ [$\varphi(r, z, l+1)$] and velocities of V_r and V_z within all knots across the fluid field can be found. Therefore the pressure p can be solved for every knot.

For the unrestrained surface of the fluid during the impact process, its shape is solved using an iteration method (Zhu, 1986). Assuming the unrestrained surface as $z = f(r)$ with the flow shaped as projectile motion in general and the flow closest to the wedge as a straight line, the pressure $P^{(n)}$ can be approximated by verifying the boundary conditions of the unrestrained surface. The error pressure between the approximated pressure and the actual pressure for the unrestrained surface is given as: $\Delta P^{(n)} = P_0^{(n)} - P^{(n)}$; the water splash height can be then computed as:

$$\Delta h = \frac{P^{(n)} - P_0^{(n)}}{P_0} H_0 \quad (8)$$

When the iteration error |Δh| is within a predetermined range ($\varepsilon < 10^{-4}$), it is considered a successful solution. Otherwise, proper corrections are made to the unrestrained surface functions. Once |Δh| is solved, the shape of the unrestrained surface is determined. Through repeated iterations, the unrestrained fluid surface shape at every instant during the entire impact process is obtained. Due to the importance of splash height in judging a dive during competition, Δh was chosen as the outcome measure of the model. The water splash height was simulated during the impact at seven different oblique angles: β = 4°, 10°, 20°, 30°, 45°, 60°, and 80°. The splash height during the impact was expressed as a percent of the maximum height obtained for the oblique wedge angle at 80° during the simulation.

In addition, a 2-dimensional isoparametric 4-nodes element with an arbitrary boundary shape was used in the study. One additional node was interpolated on each side resulting an 8-nodes isoparametric element to represent the curved fluid surface. When finite element mesh is divided, a greater amount of elements was used in the areas close to the impact site with smaller distance between the adjacent nodes whereas a smaller amount of nodes was used in the areas further away towards the walls and bottom of the pool. With this approach, the splash height was effectively computed and the computation demand was greatly reduced.

3. Results

The simulated splash height during the impact for each of the seven wedge angles was presented in Table 1. The results indicated that the highest point of the unrestrained wave surface after the impact increased with an increase in the wedge angle (slope). The greatest (sharpest) wedge angle elicited the highest water splash during the impact. The difference in the splash height between the wedge angles of 80° and 4° was almost 20 times. In addition, a separate simulation was run for two wedged objects of 30 and 60 kg. The result suggested that the greater body mass and impact force increased the splash at the same oblique angle.

Table 1. Percent splash heights from different oblique angles.

	Oblique Angle						
	4º	10º	20º	30º	45º	60º	80º
Percent Splash Height	5.0%	12.5%	25.0%	31.3%	50.0%	62.5%	100%

4. Discussion

During competition, the success of a dive is partially judged by the success of the rip entry and its splash height. The lower the splash height, the higher the score for the technical component associated with the splash control. During impact between a solid and fluid, a part of impact energy is transferred to the fluid, which creates a motion of the fluid and formation of splash. However, the splash formation is also related to many other factors.

First of all the splash formation is related to fluid characteristics. Water as a special form of fluid has following unique features. (1) Incompressibility: compressibility of its volume is rather small; under the force application, it can be only displaced but not compressed. (2) Minute adhesiveness: due to this characteristic, water motion displays non-uniformity under a force application; parts of water may move in directions other than the direction of the force. (3) Under compression, water tends to move in a direction whereas least pressure is present (so called the most escapable direction). According these characteristics, motion characteristics of water during impact can be analyzed.

The splash is also related to the shape of the impact object. When the wedged object impacts the water with its sharp edge, water is compressed diagonally downward. The direction of the force is perpendicular to the inclined surface (Figure 3). The water closest to the wedge moves in the direction of the force application under the compression. Due to reaction forces from the surrounding water, its intended motion is restrained and forced to move upwards along the wall of the sharp wedge, i.e. in the most escapable direction. The first layer of water under the compression will escape in the direction first. As the wedge continues to penetrate into water, the fluid at the top has already escaped along the wall; its location becomes the most escapable direction for the water below. Therefore, the underlined water continues to escape along this direction and forms the water splash. Greater is the impact, higher the speed and thus the splash.

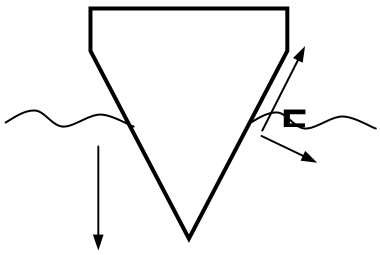

Figure 3. Impact between the wedged object and water

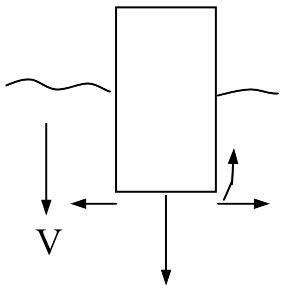

Figure 4. Impact between the squared object and water.

Even though the water escapes upwards along the slope of the wedge surfaces under pressure and forms the water splash, the splash height is related to the oblique angle of the wedge wall, which is also the projection angle of the splash. The splash height increases with increased projection angle when the escape velocity is held constant. This was also verified in our computer simulation.

When the penetrating object is not wedge-shaped but rectangle-shaped, the results of the impact are different. After the object submerges into water, the pressure is applied vertically downward. The water, under such pressure, disperses circumferentially. It does not form a most escapable direction due to the reaction forces from the surrounding water. Under such reaction forces (pressure) parts of water may move upwards along the vertical wall of the object (Figure 4). At the same time, however, the impacting object is moving downwards bringing its surrounding water with it because of its high impact velocity. When such a velocity is greater than that of the water velocity traveling upwards along the wall, no apparent water splash is formed. Therefore, instead of having palms facing each other to form a wedge at the water entry, the diver can internally rotate the arms and form the impact surface with the palms facing towards the water to effectively limit the water splash.

In practice, a competitive diver not only performs simple vertical movements but also high-speed rotations and somersaults. When an object impacts water with high-speed rotation and somersault, its velocity's direction is not purely downward and is actually determined by the translational (V_v) and rotational (ω) velocities of the object with the resultant velocity (V) directing diagonally downward at the time of impact (Figure 5). If the diver were to impact the water with the flat palms after the initial entry, the squared object would move towards one of the conners. The water would escape along the surface of the palms and the sides of the arms resulting in the similar "wedge" effect; the diver would fail to "contain" the splash. At this time, the diver should change the direction of the palms as it descends in the water, so that the palms could be always be directed 90º angle to the on-coming water flow (Figure 6a & 6b). Once the diver contacts the water, the resistance from the water can create a resistive torque that has a direction just opposite to the angular motion of the body, cause a dramatic decrease of its angular velocity quickly(ω), and produce a vertically downward resultant velocity of the body (Figure 6c). . At this time, the palms should be turned downwards from its previous diagonally downward direction to remain opposite to the resultant velocity of the water

In a forward somersault, the diver should push downward with the base of palms and the little finger's side, and keep the palm facing anteriorly downward at the water entry. In a backward somersault, the diver should push downward with the side of the thumbs and keep the palms facing posteriorly downward instead. Once the diver contacts the water, the reaction force from the water instantly creates a resistive torque that is applied in a direction just opposite to the body rotation, and dramatically reduces the speed of the body's rotation. At this time, it is critical to push with the palms in the opposite direction to keep the palms perpendicular to the direction of the water speed as discussed previously in order to rotate the palms towards a vertically downward direction in a "massaging" motion. In order to effectively control the splash, the diver not only flexes and internally rotates the shoulder joints and pushes the palms outwards to maintain the squared shape for water entry, but also "massages" the water successfully. The direction, the range of motion, and the magnitude of the "massage" motion are related to the direction of body rotation and speed at the instance of the water entry, the position of the center of gravity of the body before the dive, and the body weight of the diver.

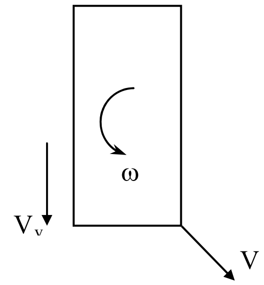

Figure 5. Impact between the squared object with high rotating speed and water.

5. Training study

After obtaining the simulated and theoretical results, twelve divers from the Jianshu professional diving team were recruited as subjects in a training study during a 2-month training period. The training emphasized "squared" rip entry concept, and implemented a technique that required the divers to have "tight body, straight body line, rigid wrist and hand, straight shoulder angle, and prompt water massaging", and a descent into water as deep as possible.

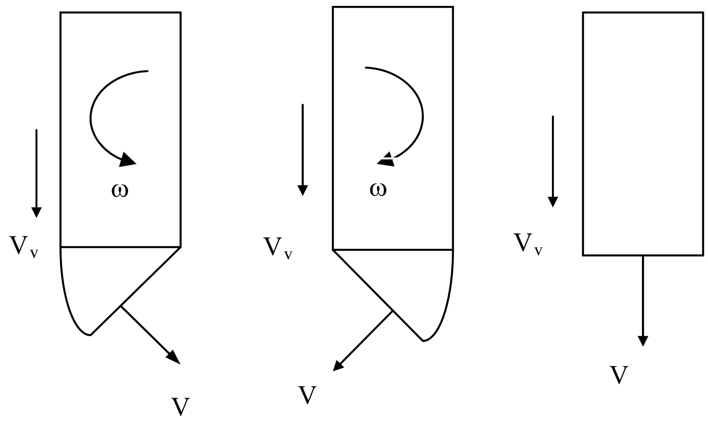

Figure 6. Wrist's motion to keep the palm perpendicular to the water's resultant velocity.

In a platform diving events that have high descending velocity, it is easier to contain the splash than a springboard event with proper control. Both effectiveness and quality of the rip entry are actually better in platform events, mainly due to its greater descending height and longer time so that the athlete has enough preparation for water entry. According to our previous theoretical analysis, the descending body with high velocity causes the water flow closest to the body to move downward. When this movement velocity is greater than the upward escaping velocity of water due to reaction to the downward compression, no apparent splash will occur. In addition, the vertical descending velocity of the palm is far greater than the horizontal velocity due to the rotation of the body at the time of entry; the range of necessary "massaging" motion is smaller. Thus the rip entry technique for a platform event is simpler. With sufficient strength in the wrist, elbow and shoulder joints and capable to maintaining a rigid and straight body posture, a platform diver can obtain smaller splash and crisper entry sound to provide good impression to judges and audience. If the diver does not have enough strength and cannot maintain a "squared" rigid body posture, any giving in a body joint would result a significant amount of water splash. Furthermore, high impact forces are usually experienced using a flat hand pattern and may cause an injury with any such giving (le Viet, et al., 1993). Therefore, we devoted a large amount of training on techniques with emphases placed on the rip entry technique so that the participating divers could master the skill. Specific strength training was also introduced with aims to prevent injuries.

The training of "water massaging" technique is paid specially attention for the entry action with high velocity of forward or backward somersault so that to maintain the hand is vertical to the direction of sum of velocity. Different standards are proposed in the practice of training for the deferent level of diving athletes. The training for those experimental athletes is emphasized on explaining the mechanism and principal of the rip entry techniques and the movement practice to improve their perceiving and capability on the rip entry techniques. These experimental athletes usually have been trained for a long time and have some of experiments on the entry techniques, however they are not so clear in the details of the movement so that they often failed to complete the movement since their perceiving on the rip entry techniques is passive.

The training for those young divers is emphasized on some assistant practice to let them perceive "water massaging" technique on the basis of understanding the technique of "flat hand" pattern. The training period for "the rip entry" techniques can be shorted in this way.

Under the guidance of the theoretical and practical experimental results, the participating divers all mastered the rip entry techniques with a combination of the "squared object" entry and "massaging". In past training practice, it used to take long-term and repeated practice for athletes to passively perceive and master the technique. With the understanding of the mechanisms of splash control, the divers were more actively involved in the training process and we were able to improve the training efficiency and levels. The success rate of the rip entry of two of the best divers of the group reached above 80% in practice. Xu Hao received gold medals in the double event in the 2002 Asian, and Yang Lan earned the gold medals in the double event of the 2002 Chinese National Diving Championship.

6. Conclusion

The simulation results indicated that the slope of the wedge was inversely proportional to the impact magnitude and the decline of the body velocity, but proportional to the splash height. The splash height changes by a factor of nearly 20 times between 80° and 4° wedge angles. (2) The splash height is also closely related to the hand pattern used in diving at the time of impact. Using a technique with an internally rotated and fully flexed (straight) shoulder joint and both hands forming a "squared" surface, a diver can effectively reduce the splash but increase the risk of injury. (3) It is imperative to "massage" the water after the initial entry to maintain the palms opposite to the direction of the water velocity to effectively minimize the splash according to the direction and speed of the body rotation. (4) The rip entry techniques with a combination of the "squared object" entry and "massaging" is emphasized in the training for divers, the training period for "the rip entry" techniques can be shorted and the success rate can be improved in this way.

The limitations of the study are mainly associated with necessary simplifications of the impact process between the wedge and water. The human body is modeled as a wedged object, water is treated as ideal fluid, and the splash height is estimated with a finite element method. Due to these simplifications, the simulated results can certainly deviate from a realistic situation and cannot yield precise outputs. Therefore, much of discussion was based on qualitative observations. Further studies are warranted to improve the model and simulation technique to achieve more precise quantification of the outcome.

7. References

[1] Armand, J. L. and R. Cointe. Hydrodynamic impact of a cylinder. *In: Offshore Mechanics and Arctic Engineering Symposium. Tokyo, Japan.* 1986, pp. 609-634.

[2] Brown, J. G., L. D. Abraham and J. J. Bertin. Descriptive analysis of the rip entry in competitive diving. *Research Quarterly for Exercise and Sport.* 1984, **55:** 93-102.

[3] Cointe, R. Two-dimensional water-solid impact. *Journal of Offshore Mechanics and Arctic Engineering*. 1989, **111**: 109-114.

[4] Dobrovol' skaya, Z. N. On some problems of similarity flow of fluid with a free surface. *Journal of Fluid Mechanics.* 1969, **36**: 805-829.

[5] Gavrilenko, V. V.. Transient loading as an ellipsoid of revolution penetrates a fluid. *Soviet Applied Mechanics*. 1986, **22**: 797-802.

[6] Gavrilenko, V. V. Determination of the stress-strain state of thin elastic spherical shells penetrating into a compressible fluid. *Soviet Applied Mechanics.* 1989, **24**: 859-866.

[7] Gavrilenko, V. V. and V. D. Kubenko. Plane problem of rigid body penetration into a compressible fluid. *Soviet Applied Mechanics.* 1985, **21**:345-352.

[8] Kubenko, V. D. and V. V. Gavrilenko, Axisymmetric problem of the penetration of rigid bodies into a compressible fluid. *Soviet Applied Mechanics*. 1987, **23**: 152-158.

[9] Li, D. and G. Yuan. *The numerical method for two dimensional unstable fluid motion*. Beijing: Science Press, 1987.

[10] Mackie, A. G. The water entry problem. *Quarterly Journal of Mechanics and Mathematics.* 1969, **22**: 1-17.

[11] Mareal, P. V. *Hydrodynamic impact analysis. (EPRI NP-824, Research Project 812-3)*. California: Electric Power Research Institute, 1978.

[12] Rackham, G. Entry Techniques. In: G. Rackham (Eds.). *Diving complete.* London: Faber and Faber LTD. 1975, pp. 203-217.

[13] Wang, C. and M. Shao. *Fundamental principles and numerical solutions in finite element methods*. Beijing: TsingHua University Press, 1997.

[14] Zhang, B. *Finite element methods in fluid mechanics.* Beijing: Mechanics and Industry Press, 1986.

[15] Zhao, R. and O. Faltinsen. Water entry of two-dimensional bodies. *Journal of Fluid Mechanics*. 1993, **246**: 593-612.

[16] Zhu, J. *Computational fluid mechanics.* Beijing: Science Press, 1986.