

# Collision on Baseball Bat

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(Received February 12, 2010, accepted March 21, 2010)

**Abstract.** In this paper, we present a dynamics model based on the rigid body around fixed axis rotation. Find that the sweet spot of baseball bat is determined by radius of gyration and the center of mass, and the sweet spot is not in the end of the baseball bat. Then we design a collision model and calculated the effective elastic constant of system. We find that the ball acceleration hit by aluminum bat is larger than it by wood bat.

**Keywords:** sweet spot, rotational inertia, collision, elastic constant.

## 1. Introduction

Every hitter knows that there is a spot on the fat part of a baseball bat where maximum power is transferred to the ball when hit. The experience of hitters tells us this spot is not at the end of the bat. And H.Bordy has explained it based on theory of center-of-percussion (COP) <sup>[1]</sup>. We design a model to explain this empirical finding. The material out of which the bat is constructed also likely impact the “sweet spot”. Our model predicts different behaviours for wood or aluminium bats.

## 2. The sweet spot

If the impact force strikes the bat precisely at the COP, then the translational acceleration and the rotational acceleration in the opposite direction exactly cancel each other. The bat rotates about the pivot point but there is no net force felt by a player holding the bat in his/her hands. Because impacts at the COP result in zero net force at the pivot point, the COP location has long been identified with the sweet spot, associated with feel, of a baseball bat <sup>[1]</sup>. To find out the location of point *P*, we analyze the physical property of the bat. (See the Fig.1)

### 2.1. Model Design

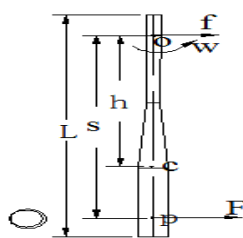


Fig.1: The physical property of the bat

Where: *O* is the pivot point, the spot a hitter holding the bat in his/her hands; *C* is the center of mass of the baseball bat; *P* is the sweet spot, the center-of-percussion.

We assume the bat is rigid body and the density is even. Now analyze the force of point *O* and *C*. At point *O*: from Angular momentum theorem <sup>[2]</sup>

$$Fs = I_o \omega \quad (1)$$

Where: *F* is the impact force on the bat from the ball;  $\omega$  is the moment angular acceleration during the

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collision;  $I_o$  is the rotational inertia to shaft. At point  $C$ : from Newton second law of motion

$$F + f = ma_c \quad (2)$$

Where:  $a_c$  the translational acceleration which the center of mass of the stick undergo.  $f$  the constraining force of the hands exerted by the bat. The distance from  $C$  to  $O$  is  $h$ , so the acceleration at  $C$  is

$$a_c = \omega \cdot h \quad (3)$$

According to formula (1), (2) and (3), we get

$$f = \left( \frac{msh}{I_o} - 1 \right) F \quad (4)$$

Where

$$I_o = mk^2 \quad (5)$$

$k$  is radius of gyration for  $O$ . If the ball hits the sweet spot,  $f=0$ . According to formula (4) and (5), we obtain:

$$s = \frac{k^2}{h} \quad (6)$$

## 2.2. Test and Solution

Assume baseball bat can be divided into three parts and the density of the wood is even. At the same time, set up Coordinate System (see the figure 2).

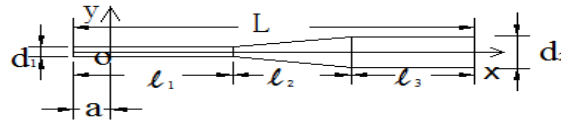


Fig.2: Coordinate System

The formula for calculating the rotational inertia:  $I = \int r_{\perp}^2 dm = \rho \int r_{\perp}^2 dV$  ( $r_{\perp}$  is the vertical distance from the mass element to the rotation shaft). The formula for calculating the center of mass:  $h = \frac{\int x dm}{m} = \frac{\rho \int x dV}{m}$  ( $x$  is the coordinates).

$$\text{Then: } s = \frac{k^2}{h} = \frac{I_o/m}{h} = \frac{\rho \int r_{\perp}^2 dV}{\rho \int x dV} = \frac{\int r_{\perp}^2 dV}{\int x dV}.$$

In the second part, we assume the variation trend is linear. Refer to the figure 2, so we have

$$s = 8 \frac{\int_{-a}^{l_1-a} \int_0^{d_1/2} (x^2 + r^2) dr dx + \int_{l_1-a}^{l_1+l_2-a} \int_0^{\frac{d_2-d_1}{2l_2}(x-l_1-a)+\frac{d_1}{2}} (x^2 + r^2) dr dx + \int_{l_1+l_2-a}^{l_1+l_2+l_3-a} \int_0^{d_2/2} (x^2 + r^2) dr dx}{\int_{-a}^{l_1-a} x d_1^2 dx + \int_{l_1-a}^{l_1+l_2-a} x \left[ \frac{d_2-d_1}{l_2} (x-l_1-a) + d_1 \right]^2 dx + \int_{l_1+l_2-a}^{l_1+l_2+l_3-a} x d_2^2 dx} \quad (7)$$

Select several real baseball bats, measure its lengths and diameters of each part (Table 1), and count the value of  $s$  by substituting into the data.

Table 1: the size of the bats

NO.	$l_1$ (cm)	$l_2$ (cm)	$l_3$ (cm)	$d_1$ (cm)	$d_2$ (cm)
1	40.50	22.50	21.00	2.40	5.50
2	32.90	26.32	29.61	2.59	6.58
3	39.90	23.10	25.20	2.94	6.72
4	35.40	23.60	30.68	2.95	5.90
5	39.60	20.16	28.80	2.88	6.48
6	38.00	28.12	22.80	3.19	6.08

Table 2: the calculation results

NO.	$L$ (cm)	$a$ (cm)	$s + a$ (cm)	$\frac{s+a}{L}$
1	84.00	14.00	72.67	0.86
2	88.83	15.50	75.10	0.84
3	88.20	15.00	75.53	0.85
4	89.68	16.00	75.52	0.84
5	88.56	16.00	74.56	0.84
6	88.92	14.00	75.65	0.85

From the table 2, the point P is not at the end of the bar. The length from the sweet spot to the end of the bat is approximately 0.16 times of the total length. That is the end of the bar is not the sweet spot.

### 3. The influence of material

The elastic constant of the material is different. Now, we analyze the collision process of the ball and the bat. The collision process is divided into two stages<sup>[3]</sup>:

1.Compression stage—from the moment of touching to the maximum deformation of them, by this time the velocities of them are equal.

2. Recovery stage—from the maximum deformation of them to the moment they separate.

#### 3.1. Compression stage

The process of their velocities changing is to see Fig.3

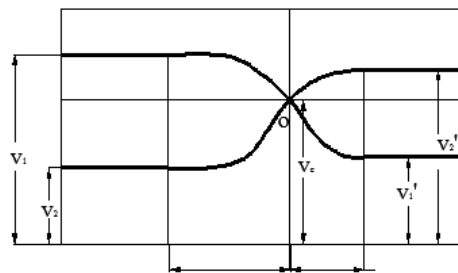


Fig.3: compression stage and recovery stage

At the moment of Compression stage just finished, the velocity of them is  $v_c$ ; both of them begin to recover because of the elastic constant; they separate until the velocity of the ball is  $v_1'$  and the velocity of the bat is  $v_2'$ . According to the law of conservation of momentum:  $m_1 v_1 + m_2 v_2 = (m_1 + m_2) v_c$ .

Where:  $k'$  is the elastic constant of the bat;  $k_0$  is the elastic constant of the ball;  $m_1$  is the mass of the ball;  $m_2$  is the mass of the bat;  $v_1$  is the initial velocity of the ball;  $v_2$  is the initial velocity of the bat.

Then we get: 
$$v_c = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

The collision process is as Fig.4.

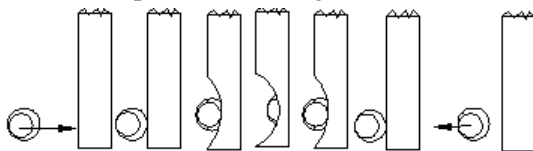


Fig.4: compression stage and recovery stage

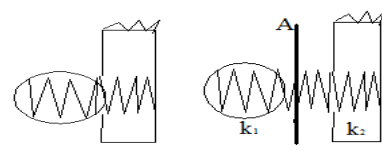


Fig.5: mechanical model about collision

We can look upon them as springs. The collision can be look as the ball and the bat hit to the wall which moves at a space  $v_c$ <sup>[4]</sup>. So the figure of mechanical model about collision is as follow: (Fig.5, 6)

From this model, the two springs are in series; the effective elastic constant of two springs is

$$k = \frac{k_0 k'}{k_0 + k'} \quad (8)$$

According to Hooker Law:

$$F = -kx \quad (9)$$

Where:  $F$  the elastic force the object undergoing.  $x$  the displacement the spring leave its equilibrium position. That is we take the relation between the collision force and the deformation for linear relationship.

### 3.2. Recovery stage

At the beginning of recovery stage, by Newton second law of motion, we calculate the acceleration of the ball at this moment:

$$a = -\frac{k}{m_1}x \quad (10)$$

Where:  $x$  is the displacement of the ball deformation. When the bat is wood, the effect elastic constant of the system:

$$k_{01} = \frac{k_0 k_1}{k_0 + k_1} \quad (11)$$

Where:  $k_1$  is the elastic constant of the wood bat;  $k_{01}$  is the effect elastic constant of the system. When the bat is aluminium, the effect elastic constant of the system:

$$k_{02} = \frac{k_0 k_2}{k_0 + k_2} \quad (12)$$

Where:  $k_2$  is the elastic constant of the aluminium bat;  $k_{02}$  is the elastic constant of the aluminium bat. On the condition of the same ball and the same deformation, using the formula (10), (11) and (12), we obtain

$$\frac{a_1}{a_2} = \frac{k_{01}}{k_{02}} = \frac{k_1 k_0 + k_1 k_2}{k_2 k_0 + k_1 k_2} \quad (13)$$

Where:  $a_1$  is the ball acceleration when the wood bat hits it;  $a_2$  is the ball acceleration when the aluminium bat hits it. Through searching for information we can know that  $k_2 > k_1$ <sup>[5]</sup>. So  $\frac{a_1}{a_2} < 1 \Rightarrow a_1 < a_2$ , that is, the ball acceleration by aluminium bat is more than it by wood bat. So aluminium bats outperforming wood bats.

## 4. Conclusion

Our model is easy to discuss and calculate and our results conform to reality. But we ignore the air resistance which can impact the collision process of the ball and the bat in reality. So our model should be improved in the future.

## 5. References

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