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# Was Vardon Right ?

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**Abstract.** Following a personal experience of the beneficial effects of using the "Vardon Swing" in golf, the author has provided an analytical verification of the superiority of such an approach. The method used is essentially that due to M. Aicardi [3] however, the implementation using a more classical derivation of the equations of motion and the use of a spreadsheet with a built in optimization routine makes golf swing analysis available to a wider public.

**Keywords:** Golf Swing, Vardon, Clubhead speed.

## 1. Introduction

For many years I have been a collector of golf books; especially golf instruction books. Usually after purchasing a new acquisition I take myself off to the local driving range in an attempt to reproduce whatever "secret" the author has been expounding. Usually, the hoped for improvement is short lived at best.This failure to make a major breakthrough in my search for the best way to swing the club persisted throughout my examinations of the literature over the course of many years.In addition to perusing the instructional books I have also read widely in the theoretical literature regarding the physics of the game and I shall have more to say in this regard later.

Now, just last year, I bought a volume with the unlikely title " The Ultimate Golf Instruction Guide: Key Techniques for Becoming a Zero Handicap Golfer or Better"[1]. The author of this book is a Dr. Patrick Leonardi who is apparently a chiropractor – not a golf professional. I must be honest and admit that after reading the book I was extremely skeptical as to its value. It seemed to me that the method advocated was little more than a rerun of the "Vardon" swing using a bent left elbow – a method long since dismissed by most professional golfers. Nevertheless, I proceeded to the driving range with a large bucket of balls and started to practice what the good doctor advocates.

An hour later I returned home completely converted. Not only was I hitting the ball greater distances but also much straighter and it seemed to me with somewhat less effort. Moreover this turned out not to be a transitory experience.

As a retired scientist I decided that some kind of proof was required for the effectiveness of this approach (which I shall refer to as the Vardon swing). Most theoretical models of the golf swing have made use of a "double pendulum" model which lends itself to relatively simple analysis but which restricts consideration to a straight arm swing. Jorgensen[2] made claims regarding the superiority of the Vardon swing but did not publish details of his analysis. Here a spreadsheet application is developed utilizing a triple pendulum model which will allow discussion of the Vardon swing and indeed many other facets of the golf swing.

## 2. The Swing Model.

### 2.1. Nomenclature and Data

$R_u$ = Length of upper arm = 0.3 m $R_L$ = Length of lower arm = 0.315 m

L = Length of club below wrist hinge = 1.105 m

$M_u$ = Mass of upper arm = 5.168 kg $M_L$ = Mass of lower arm = 4.1 kg M = Mass of club = 0.394 kg

$S_u$ = First moment of mass for upper arm about shoulder joint = 0.775 kg-m

$S_L$ = first moment of mass for lower arm about elbow joint = 0.615 kg-m

S = first moment of mass of the club about the wrist joint. =0.316 kg-m

$I_u$ = Moment of inertia of upper arm about shoulder joint = 0.155 kg-$m^2$

$I_L$ = Moment of inertia of lower arm about elbow joint = 0.123 kg-$m^2$

I = Moment of inertia of club about wrist joint = 0.3 kg-$m^2$

θ, φ, β angular coordinates

x,y Cartesian coordinates with origin at final position of shoulder joint at end of downsing

t = time from start of downswing s, T = total downswing time s.

a(t) = acceleration due to shift of shoulder joint m/$s^2$

$_i$ and $\gamma_i$ parameters in the Aicardi equations, i = 1,2,3

$Q_\theta$, $Q_\varphi$, $Q_\beta$ torques applied at shoulder, elbow and wrist joints respectively, Nm.

Vc = clubhead speed at t = T, m/s.

## 2.2. Overview of the Method.

(1) The first step is to obtain the Aicardi generalized functions for Θ,φ and β.

(2) Then obtain the first and second time derivatives of the above.

(3) Now one must determine the total kinetic energy (KE) of the system . This procedure is elementary but quite lengthy and requires a fair amount of geometrical manipulation – using Pythagoras and the Cosine rule.

(4) The potential energy (PE) term in the Lagrangian must now be evaluated. Here I have followed the work of Jorgensen [2] and included terms due to both the gravitational acceleration and the acceleration due to the shoulder shift during the downswing. The treatment of the shift is elaborated later.

(5) The Lagrangian can now be written; L = KE – PE

(6) The equations of motion can now be used to determine the applied torques, e.g., $Q_\Theta = d/dt\{\frac{\partial L}{\partial \dot{\theta}}\} - \frac{\partial L}{\partial \theta}$ and similar expressions for the φ and β coordinates. Again this is an elementary procedure but the expressions are quite lengthy.

(7) Finally the spreadsheet is set up using the data characterizing the system together with the input of initial values of θ,φ, and β. Then the SOLVER routine is used to perform the optimization with constraints.

The constraints are imposed on the allowable torques. The values of the gamma parameters are used as the variables for the optimization (see Aicardi[3]).

## 2.3. The Aicardi Functions.

The analysis used here is based on a " triple pendulum" model where the three links are the upper arm, lower arm and the club (see Fig. 1). In addition a shift of the shoulder joint during the downswing is allowed. I have followed the method presented by Aicardi[3] in which a set of generalized equations containing parameters to be determined ensures that the boundary conditions of the swing will be met.

The equations for the angular coordinates are as follows:

$$\boldsymbol{\alpha} = (\gamma + 2)\,\frac{\gamma e^{\gamma}}{(1-e^{\gamma})} / \{1 - \frac{\gamma e^{\gamma}}{(1-e^{\gamma})}\}^2$$

where $\boldsymbol{\alpha}_i = \boldsymbol{\alpha}_i(\gamma_i)$ , i= 1,2,3

$$\theta(t,\alpha,\gamma) = \theta(0,\alpha,\gamma)\,\{(e^{\frac{\alpha t/T(1-e^{\gamma t/T})}{(1-e^{\gamma})}} - e^{\alpha})/(1-e^{\alpha})\}$$ where $\boldsymbol{\alpha} = \boldsymbol{\alpha}_1$ , $\gamma = \gamma_1$

$$\varphi(t,\alpha,\gamma) = \varphi(0,\alpha,\gamma)\,\{(e^{\frac{\alpha t/T(1-e^{\gamma t/T})}{(1-e^{\gamma})}} - e^{\alpha})/(1-e^{\alpha})\} \text{ where } \boldsymbol{\alpha} = \boldsymbol{\alpha}_2,\ \gamma = \gamma_2$$

$$\beta(t,\alpha,\gamma) = \beta(0,\alpha,\gamma)\,\{(e^{\frac{\alpha t/T(1-e^{\gamma t/T})}{(1-e^{\gamma})}} - e^{\alpha})/(1-e^{\alpha})\} \text{ where } \boldsymbol{\alpha} = \boldsymbol{\alpha}_3,\ \gamma = \gamma_3$$

In addition to the above it is necessary to determine the first and second time derivatives $\dot{\theta},\ddot{\theta}, \dot{\varphi},\ddot{\varphi}$ and $\dot{\beta}$, $\ddot{\beta}$ .

The computation of these is somewhat tedious but elementary and is not presented here.

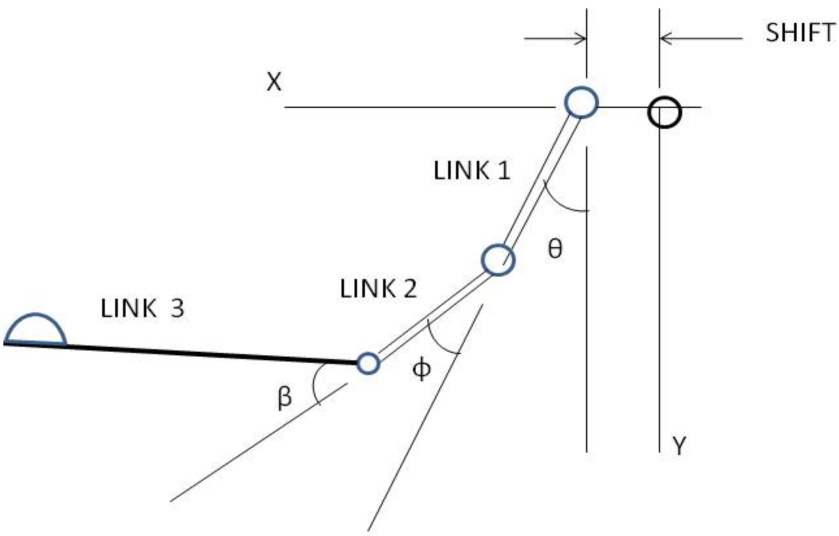


FIG. 1 The Coordinates.

## 2.4. Treatment of the Shift.

As stated above, in the development of the Lagrange equations, the potential energy terms include the effect of acceleration of the shoulder joint during the downswing.

Here I have assumed that the shoulder acceleration is given by

$a = A\sin(2\pi t/T)$ where A is the amplitude to be determined.

Integrating once gives the speed $V = AT/2\pi[1- \cos(2\pi t/T)]$ and then again the total shift in time T is

$d = AT\ t/2\pi + A(T/2\pi)^2\ \mathrm{SIN}(2\pi t/T)$ .

Choosing a suitable value for the total shift d at t=T (I used .381 m) then allows A to be determined.

This assumption is almost certainly not correct as to the manner in which the shoulder actually moves however the effect of this term on the overall argument is not critical.

## 2.5. Equations of Motion.

Professor Aicardi [3] made use of MATLAB and SIMULINK computer programs in deriving the equations of motion however these are expensive and not readily available to the average person. I have therefore chosen to use the classical Lagrange technique which (although more complex for the triple link arrangement) is adequately described in [2] and therefore only the final equations will be presented here. Note: With the Cartesian coordinates used here the shift acceleration is actually in the negative x-direction and this sign change is taken into account in the development of the potential energy term P.

## The Torque Equations.

$$Q_\theta = C_1\ddot{\theta} + C_2\ddot{\varphi} + C_3\ddot{\beta} - C_4\dot{\theta}\dot{\varphi} - C_5\dot{\theta}\dot{\beta} - C_6\dot{\varphi}\dot{\beta} - C_7\dot{\varphi}^2 - C_8\dot{\beta}^2 + C_9 - C_{18}$$

$$Q_\varphi = C_{10}\ddot{\theta} + C_{11}\ddot{\varphi} + C_{12}\ddot{\beta} - C_{13}\dot{\theta}\dot{\beta} - C_{14}\dot{\varphi}\dot{\beta} + C_7\dot{\theta}^2 - \frac{1}{2}C_{14}\dot{\beta}^2 + C_{15} - C_{19}$$

$$Q_\beta = C_3\ddot{\theta} + C_{12}\ddot{\varphi} + I\ddot{\beta} + C_{16}\dot{\theta}\dot{\varphi} + \frac{1}{2}C_5\dot{\theta}^2 + \frac{1}{2}C_{16}\dot{\varphi}^2 + C_{17} - C_{20}$$

The coefficients for the above equations of motion are listed below.

$$C_1 = I + I_u + I_L + 2R_uS_L\cos\varphi + 2R_uS\cos(\varphi+\beta) + 2R_LS\cos\beta + M_LR_u^2 + M(R_u^2+R_L^2)$$
$$+2MR_uR_L\cos\varphi$$

$$C_2 = I + I_L + R_uS_L\cos\varphi + R_uS\cos(\varphi+\beta) + 2R_LS\cos\beta + M(R_u^2 + R_uR_L\cos\varphi)$$

$$C_3 = I + S(R_u\cos(\varphi+\beta) + R_L\cos\beta)$$

$$C_4 = 2\{R_u[S_L\sin\varphi + S\sin(\varphi+\beta)] + MR_uR_L\sin\varphi\}$$

$$C_5 = 2\{S[R_u\sin(\varphi+\beta) + R_L\sin\beta]\}$$

$$C_6 = 2\{R_LS\sin\beta + R_uS\sin(\varphi+\beta)\}$$

$$C_7 = \{R_uS_L\sin\varphi + R_uS\sin(\varphi+\beta) + MR_uR_L\sin\varphi\}$$

$$C_8 = S[R_u\sin(\varphi+\beta) + R_L\sin\beta]$$

$$C_9 = g\{S_u\sin\theta + S_L\sin(\theta+\varphi) + M_LR_u\sin\theta + S\sin(\theta+\varphi+\beta) + MR_L\sin(\theta+\varphi) + MR_u\sin\theta\}$$

$$C_{10} = \{I + I_L + R_uS_L\cos\varphi + 2R_LS\cos\beta + R_uS\cos(\varphi+\beta) + M(R_L^2 + R_uR_L\cos\varphi)\}$$

$$C_{11} = I + I_L + 2R_LS\cos\beta + MR_L^2$$

$$C_{12} = I + R_LS\cos\beta$$

$$C_{13} = 2SR_L\sin\beta + R_uS\sin(\varphi+\beta)$$

$$C_{14} = 2R_LS\sin\beta$$

$$C_{15} = g\{S_L\sin(\theta+\varphi) + S\sin(\theta+\varphi+\beta) + MR_L\sin(\theta+\varphi)\}$$

$$C_{16} = C_{14}$$

$C_{17}=g\{S\sin(\theta+\varphi+\beta)\}$

$C_{18}=a\{S_u\cos\theta+S_L\cos(\theta+\varphi)+S\cos(\theta+\varphi+\beta)+M_LR_u\cos\theta+M[R_u\cos\theta+R_L\cos(\theta+\varphi)]\}$

$C_{19}=a\{S_L\cos(\theta+\varphi)+S\cos(\theta+\varphi+\beta)+M[R_u\cos\theta+R_L\cos(\theta+\varphi)]\}$

$C_{20}=aS\cos(\theta+\varphi+\beta)$

### 2.6. The Spreadsheet.

All of the above equations have been inserted on an Excel spreadsheet using a time step of 0.01 sec.

The problem becomes that of determining the maximum clubhead speed at t=T subject to constraints on the allowable torques. Provision must be made to input :

* The physical constants of the problem (see Nomenclature and Data )
* The initial values of θ, φ and β.
* Cells containing the greatest magnitudes of the torques.

The maximum clubhead speed at t=T is given by:

$$Vc = (R_u+R_L+L)\,\dot{\theta}(T) + (R_L+L)\,\dot{\varphi}(T) + L\,\dot{\beta}(T)$$

The optimization has been performed using the SOLVER routine incorporated with the Excel spreadsheet.

The parameters $\gamma_1$, $\gamma_2$, and $\gamma_3$ are the variables used for the optimization. The constraints on the torques are listed as follows on the SOLVER input:

$Q_\theta \le 0$ , $Q_\theta \ge -160$ Nm
$Q_\varphi \le 0$ , $Q_\varphi \ge -90$ Nm
$Q_\beta \le 0$ , $Q_\beta \ge -30$ Nm

## 3. Results and Discussion.

### 3.1. The Straight Arm Swing.

The straight arm swing can be investigated by choosing φ(0) = 0 in the initial conditions. The total time of the swing has been chosen to be 0.34 sec. The initial values of θ then give a measure of the height of the hands at the top of the backswing. By selecting a range of wrist cock angles β(0) we can determine the effect of wrist cock on the straight arm swing. These results are presented in FIG 2.

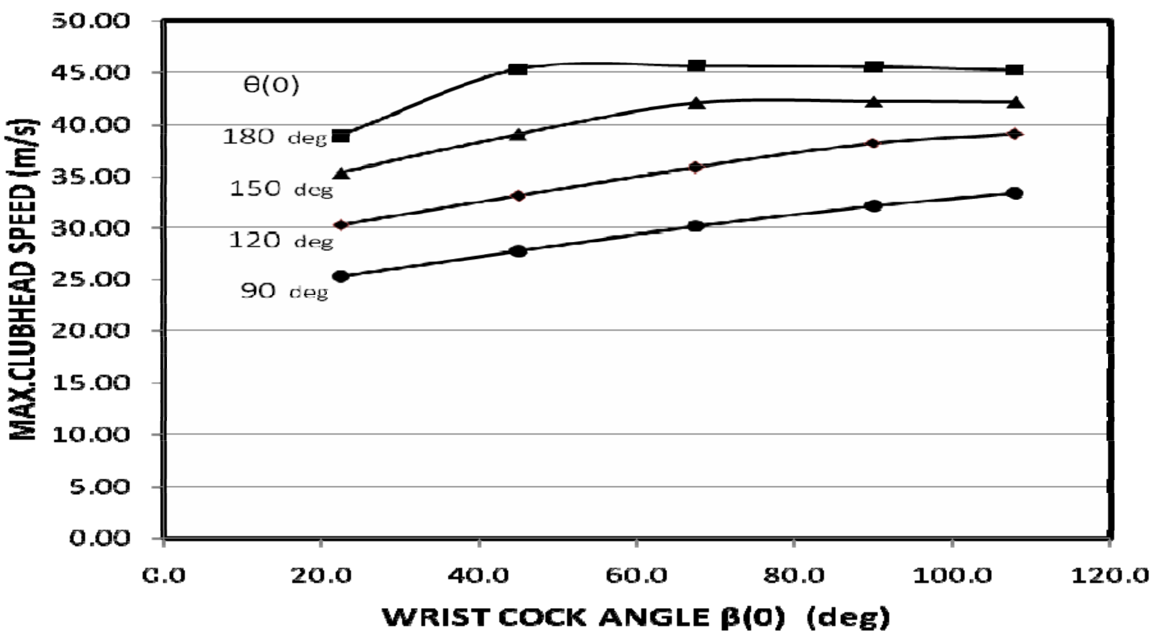

Fig. 2

The first thing that becomes apparent is the benefit of getting the hands high at the start of the downswing. For example, with a 45 deg. initial wrist cock angle, increasing the initial value of θ from 90 deg. to 180 deg. results in a 63% increase in clubhead speed at impact. Of course it should be noted that very few will be able to achieve such a high hand position. Still, even with θ(0) = 150 deg. the improvement in clubhead speed is some 40%. A further point of interest is that increasing the initial wrist cock does very little to enhance the swing speed at the higher values of θ(0).

## 3.2. The Vardon Swing.

Let us turn now to the swing in which the elbow joint has a significant bend at the beginning of the down swing. Figure 3 shows the swing speeds obtained for various initial values of θ and a fixed initial wrist cock angle of 90 deg. as the initial elbow joint angle β(0) is increased from 0 to 105 deg.

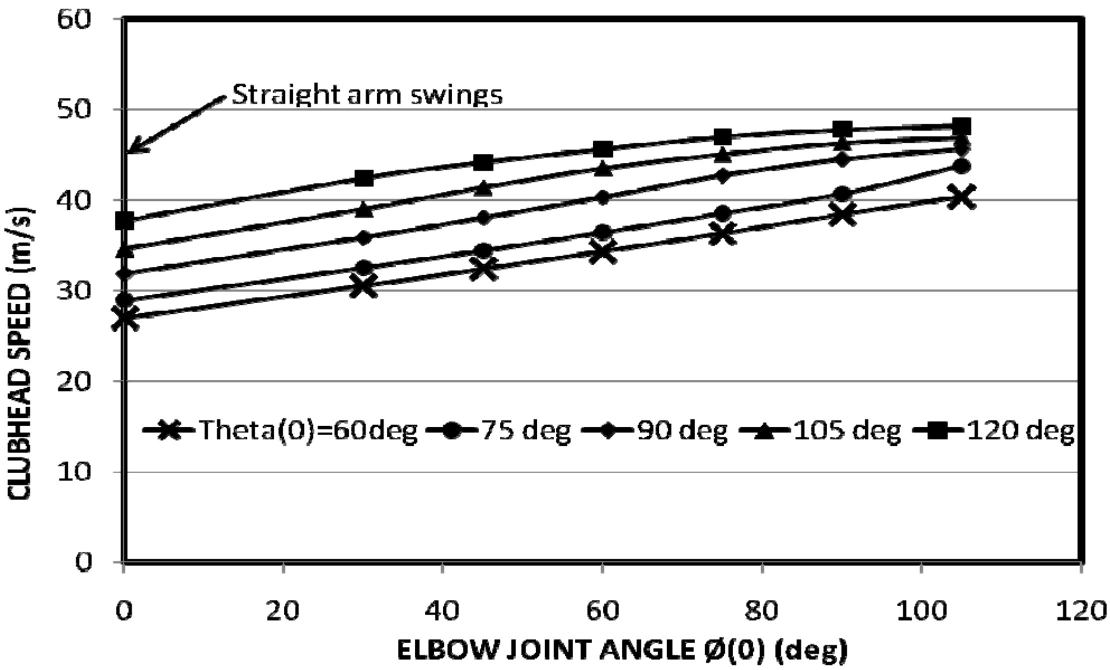


Fig. 3.

In order to get a better feel for the situation let us compare two very possible cases (Fig. 4). In the first case the golfer uses a straight left arm, raises his hands to shoulder height (θ(0) = 90 deg) and uses a 90 deg wrist cock angle (β(0) = 90 deg). This configuration leads to a clubhead speed at impact of 31.88 m/s.

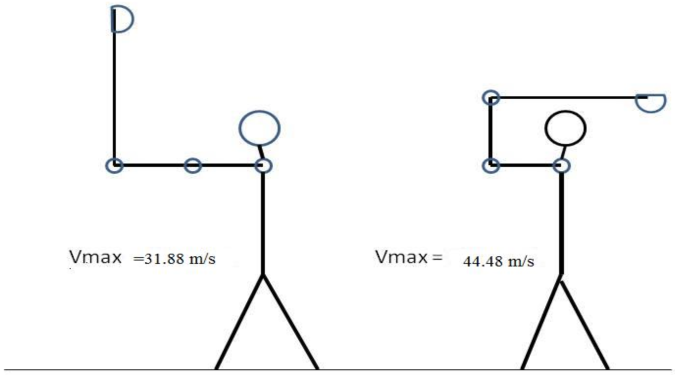


FIG. 4

Now consider the same set up except that the elbow is now bent through 90 deg. ($\varphi(0)$ = 90 deg). The clubhead speed at impact is now 44.48 m/s a whopping 39.5% increase! This translates into an increase in distance of approximately 57 m.

While it is impossible to know exactly what Vardon did there can be no doubt that a bent left arm yields superior results My own preference (following Dr. Leonardi closely) is for a lower position of the hands corresponding to $\theta(0)$ = 75 deg. This results in a clubhead speed of about 91 mph but the position is extremely easy to achieve and places very little stress on the lower back – an important factor for those of us in our later years.

As to the matter of improved accuracy I can only offer an opinion. It seems to me that this is probably due to the more upright nature of the swing resulting in a swing path that is more along the line of play.

## 4. Conclusion.

The primary motivation for this work was to provide a theoretical justification for the superiority of the “Vardon Swing”, in particular as it is propounded in Reference [2]. I believe that aim has been satisfied. It remains a mystery as to why this approach to the golf swing has received so little attention from the golf profession.

However, in the process a spreadsheet has been developed, which provides a useful tool for investigations of the golf swing, and which can be replicated by anyone with the necessary mathematical basics. All manner of “what if” exercises can be performed (changing the club, altering the total time of the swing, changing the allowable torques, examining the effect of gravity and so on).

## 5. References.

[1] Leonardi,P. *The Ultimate Golf Instruction Guide: Key Techniques for Becoming a Zero Handicap Golfer or Better.* Arizona: Silver Educational Publishing, 2004.

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[3] Aicardi,M. A Triple Pendulum Robotic Model and a Set of Simple Parametric Functions for the Analysis of the Golf Swing. *Int. J. of Sports Science and Engineering.* 2007, **1**(2): 75-86.