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Modeling and Simulation on Energy Expenditure of Human Body in Walking

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Abstract: In fitness-exercises, walking is one of the most popular exercises, for which, the modeling and calculation of walking energy expenditure are of great importance. By the statistical analysis of the experimental data, a series of mathematical models of energy expenditure have been established, which are based on walking frequency. After examination, the precision is satisfying. Based on the grey system, we carried out the simulation on fat expenditure proportion of lower-medium exercises. Then, with the function of energy expenditure multiplied by the function of fat expenditure proportion, and calculated the integral with time, we got the carbohydrate and fat expenditure in a period of exercises. With the MATLAB simulation, the results fitted the experimental data well.

Keywords: Energy expenditure of exercise; Mathematical models; Fat expenditure; MATLAB simulation

1. Introduction

With the social progress and people's living standard improving, great changes have taken place in people’s life-style and work-style, but the number of patients who are suffering the chronic diseases, such as obesity, hypertension, diabetes, heart diseases, is growing more and more bigger, which covers numerous happy families with shadow. The medical science research indicates that nutrition redundancy and energy metabolism imbalance are the reasons for chronic metabolic diseases. Reasonable meal and scientific exercises is the key to prevention and cure of these diseases.

The reasonable meal means that a balanced meal according with personal energy expenditure characteristics. In a short word, the so-called scientific exercises are consistent with the law of exercises, that is, the appropriate exercises intensity which is suitable for each person's physical features and individual exercises expenditure patterns [1]. For the public, walking is one of the most common exercises. The most efficient and appropriate form of exercises and reasonable exercises intensity can account for the balance between human meal and energy expenditure, which also can maintain and improve health and exercises ability[2],[3] .All of these need the control of energy expenditure, if we want to control it effectively, we should acquaint ourselves with the law of the energy expenditure, and build energy expenditure models, where the personal energy expenditure models are particularly important.

2. Main Studies

2.1. Research Objects

The students aged 15 to 25 in China University of Mining and technology (Xuzhou) were selected as the main objects. At the same time the 11 to 14-year-old children and the 26 to 65 year-old staffs were chose randomly. Among them, there were 71 men and 42 women. The main research objects were the young people aged 15-25; there were 82 people in all, 54 men and 28 women respectively, 72.6% of the total research objects.

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Table 1 the situation about the main objects aged 20～25

Number	Gender	Age	Height	Wight
54	Male	21.2±1.2	173.2±6.4	64.3±8.9
42	Female	20.8±1.2	160.2±5.1	54.5±8.2

2.2. Research Methods

2.2.1 Experiment-Methods

The experiment was carried out using CORTEX MetaMax of the portable cardiopulmonary function detector, the objects who were fasting or postprandial for one hour walked on the treadmill at different speed to adapt to the treadmill, and then walked at the speed of 2,4,6,8 km/h respectively. Each speed lasted about 3 to 5 minutes.

2.2.2 Mathematical Statistics

Based on the experimental data about the gender , age, height, leg length, weight, energy expenditure of unit time (kcal / min), walking frequency (l / min) and heart rate (l / min), carried on statistical analysis on respiratory quotient(RQ), grouped by gender , excluded abnormal values, and so on. The unit energy expenditure of unit time divided by weight, we obtained the unit energy expenditure about unit weight and unit time (kcal / min / kg). Finally, the sample was grouped by age, and data were analyzed using Excel and Matlab. The linear regression models of the unit energy expenditure were built on walking frequency or walking frequency and heart rate.

2.2.3 System Modeling and Simulation

The mathematical models of the unit energy expenditure were first established based on the walking frequency, and then the total energy expenditure was calculated. According to the relationship between RQ and exercises intensity, the time of exercise [4], non-equidistant grey Logistics models and the differential equation models were built. And multiplied by weight, and then calculated the integral with time, the total fat expenditure for a period of time was obtained. Finally, the energy and fat expenditure were simulated by MATLAB.

3. The Results and Analysis of Exercises Energy Expenditure Models

3.1. Parameter Selection

3.1.1 Data Processing

At first the experimental data of the 15 to 25-year-old volunteers were processed. The average energy expenditure data were obtained after the RQ fell into stabile state. The abnormal values were eliminated with statistical methods. The same processing was done about the rest objects.

3.1.2 Correlation Analysis

Correlation analysis was done about physiological parameters and exercises parameters. And the correlation coefficients were small between unit energy expenditure and height, weight and leg length, about 0.20 on average. But the correlation coefficient was big between unit energy expenditure and the walking frequency or heart rate, about 0.88 ~ 0.90.

It may be inferred that, in the actual course of the exercises, the walking step is an uncertain factor, which is not easy to measure. In order to ensure the practical application of models accuracy, we abandoned walking step as a factor, because the speed is a function of walking step and walking frequency. That is, abandoned walking speed as a factor. Finally, the regression equation models of unit energy expenditure and that of walking frequency and heart rate were established.

3.2. Mathematical Models of Exercises Energy Expenditure

At the usual walking situation, i.e. the speed is 2km/h ~ 10km/h, the regression equations of unit energy expenditure EE (kcal/min/kg) and F (l/min) are

$$y = 0.001099F - 0.040822 \text{ (male)} \quad (1)$$

$$y = 0.000932F - 0.0299 \text{ (female)} \quad (2)$$

If considered the real-time heart rate in walking, we obtained the following more precise regression equations

$$y = 0.000717F + 0.000598H - 0.060994 \text{ (male)} \quad (3)$$

$$y = 0.000454F + 0.000669H - 0.055306 \text{ (female)} \quad (4)$$

Where, F and H are walking frequency and real-time heart rate.

3.2.1 Model Examination

After the examination, the formulas (1~4) had statistical significance （P<0.01） Each (partial) regression coefficient and intercept similarly had statistical significance (P). Please see their examination results in details in Table 2.

Table 2 the examination of the formulas **(1～4)**

	Formula（1）	Formula（2）	Formula（3）	Formula（4）
R	0.89	0.88	0.93	0.91
R^2	0.792	0.774	0.865	0.828
F	658.1	374.5	652.3	327.7

Forecast results and the errors analysis indicated that, the forecast average relative error was about 14% using the mathematical models of the unit energy expenditure according to walking frequency. The forecast average relative errors were about 11% using the mathematical models of the unit energy expenditure according to walking frequency and heart rate. According to the experimental data, in the situation that the walking intensity was changeless, the unit energy expenditure varied with the time, and waved surrounding some values. Therefore, calculated the integral with walking -time, and multiplied by the weight, that is, the predicted precision of the energy expenditure (kcal) in a period of time was better.

Table 3 compared real energy expenditure and the calculated energy expenditure of a 20-year-old male student

Time（min）	Real EE(kcal)	Calculated EE (kcal)
1	5.915	6.462
4	23.837	25.450
8	51.054	53.127
12	78.012	79.657
16	103.503	104.530
20	130.414	128.715
24	141.961	140.330

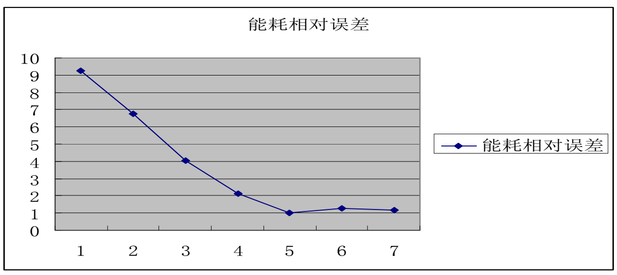


Figure 1 relative errors curve (%) of real energy expenditure and forecast energy expenditure of some 20-year-old male student

From Figure 1 we can see, computed using formula (1), the relative errors of the energy expenditure reduced gradually with the time. The relative error was 9.2% at the beginning of the walking, the relative error of the total energy expenditure was 1.2% after 24 minutes. We would get the similar results according to the data of other samples using formula (1). That is, the forecast precision would be better, and the relative errors could be controlled within 5%. Therefore, the precision of this model was satisfying.

In general, the real-time heart rate in the exercises need to obtain by the special heart rate instrument, for simple, the following calculation was done using the formula (1) or (2) which does not contain the heart rate.

$$E_j = w \times \int_0^{T_j} f(F_t) dt = \left(a \sum_{i=1}^{T_j} F_i - bT_j \right) \times w \tag{5}$$

Then, the total exercises energy expenditure was

$$E = \sum_{j=1}^{n} E_j \tag{6}$$

A and b are coefficients of formula (1) or (2) respectively, n is sum of phases parted by lasted time, T_j is the jth section of exercises, E_j is the relative energy expenditure, E is the total energy expenditure.

Moreover, according to the relationship between the age and the exercises energy expenditure, we may obtain the energy expenditure regression equations about other ages using the age-modified-coefficients method, omitted here.

3.3. Energy Materials Expenditure Modeling

3.3.1 Non-equidistance grey Logistic models

The theoretical analysis and the experimental data indicated that, we could achieve best effect of weight-losing exercises by the medium intensity exercises (45%~65%) [4]. In the medium intensity exercises course, the fat expenditure ratio was first slow and becoming more and more faster as the time went on, the fat expenditure ratio approached about 50% at 30th minutes. Then the expenditure ratio gradually changed slowly. Finally the biggest expenditure ratio reached about 85% ~ 90%[5]、[6]、[7]、[8].

Under the medium relative exercises intensity, there were experimental data of the relationship between the fat expenditure ratio and time in Table 4.

Table 4 the experimental data of the fat expenditure ratio

Time（min）	the Fat Expenditure Ratio
0	0.325
20	0.425
30	0.493
60	0.737
120	0.808
180	0.844
240	0.880

We could obtain the following formula by using non-equidistance grey Logistic models

$$Z(t) = \frac{0.8637}{1 + 1.6575e^{-0.0313t}} \tag{7}$$

Compared with the experimental data, the average relative error was 3.9%.

3.3.2 The Differential Equation Model of the Fat Expenditure

According to the knowledge about energy expenditure of aerobics [4], made the following supposition:

The velocity of fat expenditure ratio was accelerated by the carbohydrate expenditure ratio, and restrained by its expenditure ratio. And the corresponding coefficients were a and –b respectively. The fat expenditure ratio is $z(t)$.

Then, the mathematical model of fat expenditure ratio which changed with time is

$$\begin{cases} \frac{dz}{dt} = a(1 - z) - bz \\ z(t_0) = z_0 \end{cases} \tag{8}$$

This is an ordinary differential equation problem with initial-values, and its analytic solution is

$$z(t) = \frac{a}{a+b} - \left(\frac{a}{a+b} - z_0\right)e^{-(a+b)t} \tag{9}$$

To this question, made the initial value was $z_0 = 0.325$. Then carried on the fitting on the experimental data in Table 4 using the Gauss-Newton method[9], we obtained

$$z(t) = 0.9395 - 0.5708\, e^{-0.0153\, t} \tag{10}$$

Compared with the experimental data, the average relative error was 4.6%.

4. Exercises energy expenditure and relevant material energy expenditure simulation

Based on the above analysis, we could take the non-equidistance Logistic models or the differential equation models as the function of the fat expenditure ratio. Then multiplied by the total energy expenditure function, calculated the integral with time, we obtained the function of fat expenditure.

$$f(t) = \int_0^T E \times z(t)\,dt = w\int_0^T (aF_t + b) \times z(t)\,dt \tag{11}$$

A man aged 44-year-old, whose height was 183.7cm, body weight is 92.5kg. Supposed he walked at the speed of 140 l/min. The lasted time was 120 minutes (medium exercise intensity, the relaxation time during walking was ignored or the rest did not have influence on RQ).

In order to be simple, the computation was carries on with the differential equation models. Substitute formula (10) into formula (11). We obtained

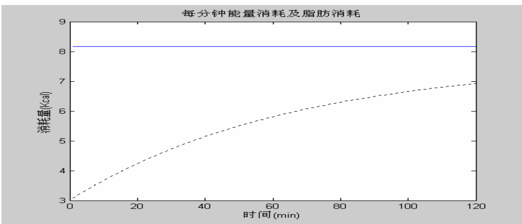
$$f(t) = 92.5 \times \int_0^{120} (a \times 140 + b) \times \left(0.9395 - 0.5708e^{-0.0153t}\right)dt \tag{12}$$

The direct integral result was 664.1kcal.

The walking lasted about 120 minutes, and the total energy expenditure was 979.2kcal, where fat expenditure was 664.1 kcal, which occupied 67.8% of the total energy supplies.

On the contrast, the total energy expenditure was 244.8kcal after 30 minutes from the beginning, the fat expenditure was 117.9 kcal, occupying 48.1% of the total energy supplies.

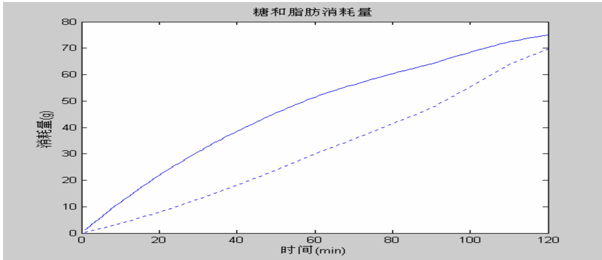
From the above, we can see that only through the long time and in the low or medium intensity exercises, can the body expend more fat effectively, see Graph 2.



Graph 2: EE [Solid] and FE [Dotted] for unit minute

This person walked for 60 minutes at frequency of 140 l/min, then walked for 30 minutes in 135 l/min, and for 20 minutes in 145 l/min. Finally he walked for 10 minutes in 130l/min as the relaxation exercises. The walking was finished (neglected the influence of relaxation time on fat expenditure, exercise intensity is 45% ~ 65% VO_2 max).

The caloric value of carbohydrate and the fat are 4.102 (kcal/g) and 9.302 (kcal/g) respectively. Based on the above formulas, the carbohydrate and fat expenditure could be calculated.



Graph 3: the carbohydrate expenditure [Solid] and FE [Dotted]

From the computation result and Graph 3, one can lose fat about 70g per day by walking for two hours, where he may lose fat about 490g (0.49kg) per week. This matches the theory in the literature [10]. Certainly, the exercise energy expenditure was considered only here, but neglected meal to the influence on weight-losing.

5. Conclusions

This article primarily researched on energy expenditure of walking exercises, as well as the corresponding carbohydrate and fat expenditure. The mathematical models of walking energy expenditure were established. Based on the experimental data, we compared the simulation of fat expenditure based on the grey system theory and the differential equation models. And carried on the simulation through MTLAB, some satisfactory results have been obtained, which has significant instruction and referential value to weight-losing. But, in order to get higher precision models and better computational methods, we also have many more work to do. For example, we should consider following two questions:

（1）For the energy expenditure models, we should do more experiments and add more information, such as body mass index and more parameters of exercises information;

（2）Since weight losing is a complex physiological and biochemical reaction process, many factors should be concerned. Exercises intensity and the time of exercise could be banded together to considered. Analyze the influencing mechanism of fat expenditure of the different exercises intensity and the different exercises time. Carry on statistical analysis and utilize optimization according to the exercise intensity and the time. If necessary classified discussion should be carried on.

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