

## An optimal grey based approach based on TOPSIS concepts for supplier selection problem

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**Abstract.** Supplier selection is a multi-attribute decision making (MADM) problem that is affected by quantitative and qualitative factors, some of which may conflict with each other. Since most of the input information is not known accurately, selecting the right suppliers becomes more difficult. Grey theory is one of the new mathematical methods used to analyze systems with uncertain and incomplete information. In this article, we firstly described two previous grey based methods and then, we proposed a new method based on TOPSIS concepts in grey theory to deal with the problem of selecting suppliers. The new method calculates the weighted connection between each of the alternatives sequence and the positive and negative referential sequence to compare the ranking of grey numbers and select the most desirable supplier. Through this article, it is demonstrated that the new method is a good means of evaluation, and it is also more optimal than the two methods.

**Keywords:** supplier selection, MADM, new grey based method, TOPSIS concepts, optimal method.

### 1 Introduction

In manufacturing industries the component parts and raw materials can be up to 70% of the product cost. In such circumstances, the purchasing department can play a key role in cost reduction, and supplier selection is one of the most important functions of purchasing management<sup>[10]</sup>.

When a relatively small amount of parts are externally procured, the total demand or requirement can be provided by only one supplier. Such a single sourcing scenario seems tenable especially in the last decade, which has seen an important shift in the sourcing strategy of many firms, moving from the traditional concept of having many suppliers to rely largely on one source with which a long term win-win partnership is established. In these situations, the decision consists of selecting one supplier for one order in order to meet the total buyer's requirement<sup>[11]</sup>.

As reported by Aissaouia et al.<sup>[1]</sup>, a wide variety of slightly different models have been suggested for single sourcing, which are described in the following literatures. Using pairwise comparison, a more accurate scoring approach that has been applied on supplier selection is the analytical hierarchy process (AHP). This technique is proposed by Narasimhan<sup>[20]</sup>, Partovi et al.<sup>[22]</sup>, Nydick and Hill<sup>[21]</sup>, Barbarosoglu and Yazgaç<sup>[2]</sup>, Yahya and Kingsman<sup>[31]</sup>, Masella and Rangone<sup>[18]</sup>, Tam and Tummala<sup>[26]</sup>, Lee et al.<sup>[15]</sup>, Handfield et al.<sup>[11]</sup> and Colombo and Francalanci<sup>[7]</sup> in order to cope especially with determining scores. Linear weighting (LW) models have been firstly endorsed by Wind and Robinson<sup>[29]</sup> in supplier selection to evaluate the supplier's performance. This method is relatively easy to implement, and produces useful and reasonably reliable data. Benton<sup>[4]</sup> also employed mathematical programming (MP) to select only one supplier to supply all needed items. And finally, for supplier selection problem, total cost approach (TCA) was used by Monczka and Trecha<sup>[19]</sup>, and Smytka and Clemens<sup>[25]</sup>.

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Although linear weighting is a very simple method, it depends heavily on human judgment and also the attributes are weighted equally, which rarely happens in practice<sup>[16]</sup>. On the other hand, AHP can not effectively consider risk and uncertainty in evaluating the supplier's performance because it presumes that the relative importance of attributes affecting supplier performance is known with certainty<sup>[9, 16]</sup>. The drawback of MP is that it requires arbitrary aspiration levels and cannot accommodate qualitative attributes<sup>[14, 16]</sup>. And finally, TCA-based models attempt to include all quantifiable costs in the supplier choice process that are incurred throughout the purchased item's life cycle<sup>[11]</sup>.

Supplier selection is a multi-attribute decision making (MADM) problem that is affected by several quantitative and qualitative factors, some of which may cause conflict to others. Depending on the purchasing strategies, the criteria or attributes have different importance and there is a need to weight them. Moreover, in a real situation problem, most of the input information is not known precisely. Thus, supplier selection problem has many uncertainties and becomes more difficult<sup>[16]</sup>. In these cases, the theory of fuzzy sets is usually used for handling uncertainty. Recently, a fuzzy based approach<sup>[28]</sup> has been proposed to deal with the supplier selection problem under uncertainty. Although fuzzy set theory is adequate to deal with uncertain and imprecise data, it can not handle incomplete data and information.

Grey theory, proposed by Deng in 1989<sup>[8]</sup>, similar to fuzzy set theory, is an effective mathematical tools to deal with systems analysis characterized by imprecise and incomplete information. The theory is based on the degree of information known. If the system information is unknown, it is called a black system; if the information is fully known, it is called a white system. And a system with information known partially is called a grey system. The advantage of grey theory over fuzzy theory<sup>[3, 32]</sup> is that grey theory takes into account the condition of the fuzziness; that is, grey theory can deal flexibly with the fuzziness situation<sup>[8, 16]</sup>.

Deng<sup>[8]</sup> had also proposed grey relational analysis (GR) in the grey theory that was proved to be an accurate method for multiple attribute decision making problems<sup>[12, 17, 27]</sup>, especially for those problems with very unique characteristic<sup>[5, 30]</sup>. The GR method is based on the minimization of maximum distance from the ideal referential alternative. Subsequently, Zhang et al.<sup>[33]</sup> presented the method of grey relational analysis (GR) as a means to reflect uncertainty in multiple attribute models through interval fuzzy numbers.

Besides, for supplier selection problem, Li et al.<sup>[16]</sup> proposed a new grey based approach (LI) under an uncertain environment, using interval fuzzy numbers. They calculate a grey possibility degree between compared suppliers alternatives set and ideal referential supplier alternative to determine the ranking order of all alternatives of supplier and to select the ideal supplier based on grey numbers.

The drawback of the two methods is that the negative ideal referential alternative is not considered to evaluate and rank the alternatives. Sometimes, the selected solution which has the minimum distance from the ideal solution may also have a short distance from the negative ideal solution as compared to other alternatives<sup>[23]</sup>.

In this paper, to eliminate the drawback, we propose a new grey based method (NG) based on the concepts of technique for order preference by similarity to ideal solution (TOPSIS) to evaluate and select the best supplier. Based on TOPSIS<sup>[13]</sup> concepts, the chosen alternative should have the shortest distance from the positive ideal solution (PIS) and the farthest from the negative ideal solution (NIS)<sup>[6]</sup>. In the NG method, the weighted connection between compared alternatives set and ideal and negative ideal referential alternatives are calculated to compare the ranking of grey numbers and select the most desirable supplier. In this paper, the three grey based methods, GR, LI and NG, are also compared with each other. Through this article, we demonstrated that consideration of both ideal and negative ideal solution to solve the multiple attribute decision making problems is very effective in choosing the most desirable alternatives. We also use Li's et al.<sup>[16]</sup> article assumption, data and information.

The remainder of this paper is arranged as follows. Section 2 describes preliminaries which include the concept of TOPSIS and the grey theory concepts. Section 3 introduces the NG method in grey theory. Then, in Section 4, an illustrative example is presented by applying the proposed method to the supplier selection problem, after which we discuss and show the optimality and effectiveness of the method. Finally, conclusions are presented in Section 5.

## 2 Preliminaries

### 2.1 TOPSIS for MCDM<sup>[6]</sup>

Multiple attribute decision making (MADM) is used to select an alternative from several alternatives according to various criteria. The technique for order preference by similarity to ideal solution (TOPSIS) was first developed by Hwang and Yoon<sup>[13]</sup>, based on the concept that the chosen alternative should have the shortest distance from the positive ideal solution (PIS) and the farthest from the negative ideal solution (NIS) for solving a multiple criteria decision making problem. In short, the ideal solution is composed of all best values attainable of criteria, whereas the negative ideal solution is made up of all worst values attainable of criteria.

### 2.2 Basic definitions in grey theory

Grey theory is one of the new mathematical theories born out of the concept of the grey set, which was proposed by Deng<sup>[8]</sup>. It is an effective method used to solve uncertainty problems with discrete data and incomplete information<sup>[16]</sup>. In this section, we briefly review some relevant definitions and the calculation process in grey theory<sup>[16, 33]</sup>. Let  $G = [\underline{G}, \overline{G}] = \{x \mid \underline{G} \leq x \leq \overline{G}, \underline{G}, \overline{G} \in R\}$ . We call  $G = [\underline{G}, \overline{G}]$  an interval number. If  $0 \leq \underline{G} \leq \overline{G}$ , we call interval number  $G = [\underline{G}, \overline{G}]$  a positive interval number. Let  $X = ([\underline{G}_1, \overline{G}_1], [\underline{G}_2, \overline{G}_2], \dots, [\underline{G}_n, \overline{G}_n])$  be an n-dimension interval number column vector.

**Definition 1.** If  $G_1 = [\underline{G}_1, \overline{G}_1]$  and  $G_2 = [\underline{G}_2, \overline{G}_2]$  are two arbitrary interval numbers, the distance from  $G_1 = [\underline{G}_1, \overline{G}_1]$  to  $G_2 = [\underline{G}_2, \overline{G}_2]$  is

$$|G_1 - G_2| = \max(|\underline{G}_1 - \underline{G}_2|, |\overline{G}_1 - \overline{G}_2|) \quad (1)$$

**Definition 2.** If  $G_1 = [\underline{G}_1, \overline{G}_1]$  and  $G_2 = [\underline{G}_2, \overline{G}_2]$  are two arbitrary interval numbers, then

$$G_1 + G_2 = [\underline{G}_1 + \underline{G}_2, \overline{G}_1 + \overline{G}_2] \quad (2)$$

**Definition 3.** If  $G_1 = [\underline{G}_1, \overline{G}_1]$  and  $G_2 = [\underline{G}_2, \overline{G}_2]$  are two arbitrary interval numbers, then

$$G_1 - G_2 = [\underline{G}_1 - \overline{G}_2, \overline{G}_1 - \underline{G}_2] \quad (3)$$

**Definition 4.** If  $G_1 = [\underline{G}_1, \overline{G}_1]$  and  $G_2 = [\underline{G}_2, \overline{G}_2]$  are two arbitrary interval numbers, then

$$G_1 \times G_2 = [\min(\underline{G}_1 \underline{G}_2, \underline{G}_1 \overline{G}_2, \overline{G}_1 \underline{G}_2, \overline{G}_1 \overline{G}_2), \max(\underline{G}_1 \underline{G}_2, \underline{G}_1 \overline{G}_2, \overline{G}_1 \underline{G}_2, \overline{G}_1 \overline{G}_2)] \quad (4)$$

**Definition 5.** If  $G_1 = [\underline{G}_1, \overline{G}_1]$  and  $G_2 = [\underline{G}_2, \overline{G}_2]$  are two arbitrary interval numbers, then

$$G_1 \div G_2 = [\underline{G}_1, \overline{G}_1] \times \left[ \frac{1}{\underline{G}_2}, \frac{1}{\overline{G}_2} \right] \quad (5)$$

**Definition 6.** If  $k$  is an arbitrary positive real number, and  $G = [\underline{G}, \overline{G}]$  is an arbitrary interval number, then the number product between  $k$  and  $G = [\underline{G}, \overline{G}]$  is

$$K \cdot [\underline{G}, \overline{G}] = [K\underline{G}, K\overline{G}] \quad (6)$$

### 2.3 Comparison of grey numbers

Li et al.<sup>[16]</sup> proposed a degree of grey possibility to compare the ranking of grey numbers.

**Definition 7.** If  $G_1 = [\underline{G}_1, \overline{G}_1]$  and  $G_2 = [\underline{G}_2, \overline{G}_2]$  are two arbitrary interval numbers, the possibility degree of  $G_1 \leq G_2$  can be expressed as follows<sup>[24]</sup>:

$$p\{G_1 \leq G_2\} = \frac{\max(0, L^* - \max(0, \overline{G}_1 - \underline{G}_2))}{L^*} \quad (7)$$

where  $L^* = L(G_1) + L(G_2)$ .

### 3 Proposed approaches

The latest grey based method (LI) based on a grey possibility degree was proposed by Li et al.<sup>[16]</sup> to evaluate and select the best supplier. This method is very suitable for solving the group decision making problem with uncertain and incomplete information<sup>[16]</sup>. By considering the method, it can be concluded that there exists a certain degree of similarity between the input and operation of the model and the GR method<sup>[16, 33]</sup>. Here, similar to the two methods, we propose a new grey based method (NG) based on TOPSIS concepts and compare it with the two methods as well. Same as the LI method, we also extend the GR and NG methods for solving the group decision making problem. In this section, we use Li's et al.<sup>[16]</sup> assumptions and information. it was assumed that  $S = \{S_1, S_2, \dots, S_m\}$  is a discrete set of m possible supplier alternatives,  $Q = \{Q_1, Q_2, \dots, Q_n\}$  is a set of n attributes of suppliers, and  $W = \{W_1, W_2, \dots, W_n\}$  is the vector of attribute weights. They also considered the ratings of suppliers and attribute weights as linguistic variables that can be expressed in grey numbers by the 1-7 scale shown in Tab. 1 and Tab. 2, respectively.

**Table 1.** The scale of attribute ratings  $G$ <sup>[16]</sup>

Scale	G
Very poor (VP)	[0, 1]
Poor (P)	[1, 3]
Medium poor (MP)	[3, 4]
Fair (F)	[4, 5]
Medium good (MG)	[5, 6]
Good (G)	[6, 9]
Very good (VG)	[9, 10]

**Table 2.** The scale of attribute weights  $w$ <sup>[16]</sup>

Scale	w
Very low (VL)	[0.0, 0.1]
Low (L)	[0.1, 0.3]
Medium low (ML)	[0.3, 0.4]
Medium (M)	[0.4, 0.5]
Medium high (MH)	[0.5, 0.6]
High (H)	[0.6, 0.9]
Very high (VH)	[0.9, 1.0]

Firstly, the LI, GR and NG methods are described. Then, a comparative analysis is used to determine the differences between the models as well as percentage of improvement of the proposed new method (NG).

#### 3.1 The grey based approaches

In this section, the LI, GR and NG methods are described as follows, respectively.

##### 3.1.1 The LI method

This method calculates the grey possibility degree between compared suppliers alternatives set and ideal referential supplier alternative to determine the ranking order of all alternatives based on grey numbers. The procedures of the method are summarized as follows<sup>[16]</sup>:

**Step 1.** Arrange a committee of DMs and identify the attribute weights of suppliers. Assume that a decision group has  $K$  persons, then the attribute weight of attribute  $Q_j$  can be calculated in Eq. (8).

$$W_j = \frac{1}{K} [W_j^1 + W_j^2 + \dots + W_j^K] \tag{8}$$

where  $W_j^K$  ( $j = 1, 2, \dots, n$ ) is the attribute weight of  $K$ th DMs and can be described by linguistic variable.

**Step 2.** Use linguistic variables for the ratings to make an attribute rating value. Then, the rating value can be calculated in Eq. (9).

$$G_{ij} = \frac{1}{K} [G_{ij}^1 + G_{ij}^2 + \dots + G_{ij}^K] \tag{9}$$

where  $G_{ij}^K$  ( $i = 1, 2, \dots, m; j = 1, 2, \dots, n$ ) is the attribute rating value of  $K$ th DMs and can be described by the grey number  $G_{ij}^K = [G_{ij}^K, \overline{G_{ij}^K}]$ .

**Step 3.** Construct the grey decision matrix D that the structure of the matrix can be expressed in Eq. (10).

$$D = \begin{bmatrix} G_{11} & G_{12} & \cdots & G_{1n} \\ G_{21} & G_{22} & \cdots & G_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ G_{m1} & G_{m2} & \cdots & G_{mn} \end{bmatrix} \quad (10)$$

where  $G_{ij}$  are linguistic variables based on the grey number.

**Step 4.** Normalize the grey decision matrix in Eq. (11): the process is to transform different scales and units among various criteria into common measurable units to allow comparisons across the criteria. Assume  $G_{ij}$  to be the element of the evaluation matrix  $D$  of alternative  $i$  under evaluation criterion  $j$  then an element  $G_{ij}^*$  of the normalized evaluation matrix  $D^*$  can be calculated in Eq. (12) and Eq. (13).

$$D^* = \begin{bmatrix} G_{11}^* & G_{12}^* & \cdots & G_{1n}^* \\ G_{21}^* & G_{22}^* & \cdots & G_{2n}^* \\ \vdots & \vdots & \ddots & \vdots \\ G_{m1}^* & G_{m2}^* & \cdots & G_{mn}^* \end{bmatrix} \quad (11)$$

Where for a benefit attribute,  $G_{ij}^*$  is expressed as:

$$G_{ij}^* = \left[ \frac{G_{ij}}{G_j^{\max}}, \frac{\overline{G_{ij}}}{G_j^{\max}} \right]$$

$$G_j^{\max} = \max_{1 \leq i \leq m} \{G_{ij}\} \quad (12)$$

Where for a cost attribute,  $G_{ij}^*$  is expressed as:

$$G_{ij}^* = \left[ \frac{G_j^{\min}}{G_{ij}}, \frac{G_j^{\min}}{\overline{G_{ij}}} \right]$$

$$G_j^{\min} = \min_{1 \leq i \leq m} \{G_{ij}\} \quad (13)$$

The normalization method mentioned above is to preserve the property that the ranges of the normalized grey number belong to  $[0, 1]$ .

**Step 5.** Establish the weighted normalized grey decision matrix in Eq. (14). Considering the different importance of each attribute, the weighted normalized grey decision matrix can be established as

$$D^* = \begin{bmatrix} V_{11} & V_{12} & \cdots & V_{1n} \\ V_{21} & V_{22} & \cdots & V_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ V_{m1} & V_{m2} & \cdots & V_{mn} \end{bmatrix} \quad (14)$$

where  $V_{ij} = G_{ij}^* \times W_j$ .

**Step 6.** Make the ideal alternatives as referential alternatives for the grey based model. For  $m$  possible supplier alternatives set  $S = \{S_1, S_2, \dots, S_m\}$ , the ideal referential supplier alternative is  $S^{\max} = \{G_1^{\max}, G_2^{\max}, \dots, G_n^{\max}\}$  which can be obtained in Eq. (15).

$$S^{\max} = \{[\max_{1 \leq i \leq m} V_{i1}, \max_{1 \leq i \leq m} \overline{V_{i1}}], \dots, [\max_{1 \leq i \leq m} V_{in}, \max_{1 \leq i \leq m} \overline{V_{in}}]\} \quad (15)$$

**Step 7.** Calculate the grey possibility degree between compared supplier alternatives set  $S = \{S_1, S_2, \dots, S_m\}$  and ideal  $S^{\max}$  referential supplier alternatives in Eq. (16).

$$P_i = P\{S_i \leq S^{\max}\} = \frac{1}{n} \sum_{j=1}^n P\{V_{ij} \leq G_j^{\max}\} \quad (16)$$

When  $P_i$  is smaller, the ranking order of  $S_i$  is better. Otherwise, the ranking order is worse.

### 3.1.2 Grey relation (GR) analysis

This method is based on the minimization of maximum distance from the ideal referential alternative. The GR method with interval fuzzy number<sup>[23]</sup> has nine steps. The first six steps of the GR method can be described as the same as the LI method. And, the last three steps for ranking the alternatives are shown as follows:

**Step 7.** Establish the connected weighted normalized decision matrix in Eq. (17) that is the distance between each of the alternatives sequence with the ideal referential alternative sequence.

$$\Delta^+ = \begin{bmatrix} \Delta_{11} & \Delta_{12} & \cdots & \Delta_{1n} \\ \Delta_{21} & \Delta_{22} & & \\ \vdots & \vdots & \ddots & \vdots \\ \Delta_{m1} & \Delta_{m2} & \cdots & \Delta_{mn} \end{bmatrix} \quad (17)$$

where  $\Delta_{ij}$  is the distance between  $V_{ij}$  and  $G_j^{\max}$ .

**Step 8.** Calculate the grey relation coefficient between each of the alternatives sequence with the ideal referential alternative sequence in Eq. (18).

$$\xi_{ij} = \frac{\min_i \min_j \Delta_{ij} + \rho \max_i \max_j \Delta_{ij}}{\Delta_{ij} + \rho \max_i \max_j \Delta_{ij}} \quad (18)$$

$\rho$  is the distinguished coefficient ( $\rho \in [0, 1]$ ). The value of  $\rho$  reflects the degree to which the minimum scores are emphasized relatively to the maximum scores. A value of 1.0 would give equal weighting. In this paper value of  $\rho$  is 1.

**Step 9.** Determine the grade of grey relation of each alternative to the ideal solution in Eq. (19).

$$C_i = \left(1 - \frac{1}{n} \cdot \sum_{j=1}^n \xi_{ij}\right) \quad i = 1, 2, \dots, m \quad (19)$$

When  $C_i$  is smaller, the ranking order of  $S_i$  is better. Otherwise, the ranking order is worse.

### 3.1.3 The proposed new grey based method

The NG method calculate the weighted connection between compared alternatives set and ideal and negative ideal referential alternatives to compare the ranking of grey numbers and select the most desirable supplier. The new method has six steps. The first four steps of the method can be described as the same as the GR and LI methods. And, the last two steps for ranking the alternatives are shown as follows:

**Step 5.** Make the ideal and negative-ideal alternatives as referential alternatives for the Grey based model. For  $m$  possible supplier alternatives set  $S = \{S_1, S_2, \dots, S_m\}$ , the ideal and negative-ideal referential supplier alternatives are  $S^{\max} = \{G_1^{\max}, G_2^{\max}, \dots, G_n^{\max}\}$  and  $S^{\min} = \{G_1^{\min}, G_2^{\min}, \dots, G_n^{\min}\}$  respectively, and can be obtained in Eq. (20) and (21) respectively.

$$S^{\max} = \left\{ \left[ \max_{1 \leq i \leq m} \underline{G}_{i1}^*, \max_{1 \leq i \leq m} \overline{G}_{i1}^* \right], \dots, \left[ \max_{1 \leq i \leq m} \underline{G}_{in}^*, \max_{1 \leq i \leq m} \overline{G}_{in}^* \right] \right\} \quad (20)$$

$$S^{\min} = \left\{ \left[ \min_{1 \leq i \leq m} \underline{G}_{i1}^*, \min_{1 \leq i \leq m} \overline{G}_{i1}^* \right], \dots, \left[ \min_{1 \leq i \leq m} \underline{G}_{in}^*, \min_{1 \leq i \leq m} \overline{G}_{in}^* \right] \right\} \quad (21)$$

**Step 6.** Here, it is proposed that the weighted connection between each of the alternatives sequence and the referential sequence is calculated in Eq. (22).

$$\Gamma_i = \frac{1}{n} \sum_{j=1}^n \left( \left( \frac{|G_j^{\max} - G_{ij}^*|}{G_j^{\max} - G_j^{\min}} \right) \times W_j \right) \quad (22)$$

where  $W_j (j = 1, 2, \dots, n)$  is the attribute weight that can be depicted by linguistic variable. It is obtained by averaging Tab. 2.

When  $\Gamma_i$  is smaller, the ranking order of  $S_i$  is better. Otherwise, the ranking order is worse.

### 3.2 Comparative analysis

Up to this stage, the LI, GR and NG methods have been explained. To compare the qualities of them, the normalized suppliers' ratings obtained in Eq. (16), Eq. (19) and Eq. (22) are used as coefficients of an objective function in linear programming, separately, while satisfying the suppliers' capacity and the buyer's demand. Subsequently, order quantities ( $X_i$ ) are assigned to the suppliers such that the Total Value of Purchasing (TVP) becomes minimum. And finally, a method with the smaller TVP is better. The procedure is shown as follows:

**Step 1.** Normalize the suppliers' ratings obtained by Eq. (16), Eq. (19) and Eq. (22), as follows:

$$\text{For the LI method, } P_i^* = \frac{P_i}{\sum_{i=1}^m P_i}, \quad i = 1, 2, \dots, m, \quad (23)$$

$$\text{For the GR method, } C_i^* = \frac{C_i}{\sum_{i=1}^m C_i}, \quad i = 1, 2, \dots, m, \quad (24)$$

$$\text{For the NG method, } \Gamma_i^* = \frac{\Gamma_i}{\sum_{i=1}^m \Gamma_i}, \quad i = 1, 2, \dots, m, \quad (25)$$

The process is to transform different rating scales for the three methods into common measurable scales to allow comparisons across the criteria.

**Step 2.** Build the linear programming<sup>[10]</sup>. The notation, objective function and constraints of the linear programming are listed as follows:

#### Notations

- $P_i^*$  Normalized final rating of  $i$ th supplier which is calculated by Eq. (23),
- $C_i^*$  Normalized final rating of  $i$ th supplier which is calculated by Eq. (24),
- $\Gamma_i^*$  Normalized final rating of  $i$ th supplier which is calculated by Eq. (25),
- $X_i^P$  Order quantity for  $i$ th supplier when  $P_i^*$  are coefficients of the objective function,
- $X_i^C$  Order quantity for  $i$ th supplier when  $C_i^*$  are coefficients of the objective function,
- $X_i^\Gamma$  Order quantity for  $i$ th supplier when  $\Gamma_i^*$  are coefficients of the objective function,
- $V_j$  Capacity of  $i$ th supplier,
- $D$  Demand for the buyer.

#### Objective function

As  $C_i$  and  $X_i$  denote the ratings and the numbers of purchased units from the  $i$ th supplier, respectively, Eq. (26) is designed to minimize the total value of purchasing (TVP):

$$\min(TVP) = \sum_{i=1}^m C_i X_i \quad (26)$$

#### Constraints

The constraints of the problem are supplier capacity and buyer's requirement which are formulated as follows:

Capacity constraint: As supplier  $i$  can provide up to  $V_i$  units of the product and its order quantity ( $X_i$ ) should be equal or less than its capacity, these constraints are shown in Eq. (27).

$$0 \leq X_i \leq V_i, \quad i = 1, 2, \dots, m. \quad (27)$$

Demand constraint: As sum of the assigned order quantities to  $n$  suppliers should meet the buyer's requirement, it can be stated in Eq. (28).

$$\sum_{i=1}^m X_i = D \quad (28)$$

**Step 3.** Rank the methods. According to step 2, the method with smaller TVP is better.

## 4 Application and analysis

As stated earlier, Li's et al.<sup>[16]</sup> article data and information is used for the numerical example. There are six suppliers  $S_i (i = 1, 2, \dots, 6)$  selected as alternatives against four attributes  $Q_j (j = 1, 2, 3, 4)$ . The four attributes are product quality, service quality, delivery time and price respectively.  $Q_1, Q_2$  and  $Q_3$  are benefit attributes, the greater values being better.  $Q_4$  is cost attributes, the smaller values are better.

Based on the information, firstly, the LI, GR and NG methods are employed to evaluate and rank the suppliers. Then, the comparative analysis is used to compare the three methods. The calculation procedures are described as follows.

### 4.1 The grey based approaches

In this section, the calculation procedures for the three methods are described as follows.

#### 4.1.1 The LI method<sup>[16]</sup>

**Step 1.** Make the weights of attributes  $Q_1, Q_2, Q_3$  and  $Q_4$ . A committee of four DMs,  $D_1, D_2, D_3$  and  $D_4$  has been formed to express their preferences and to select the best suppliers. According to Eq. (8), the evaluation values of attribute weights from four MDs can be obtained and the results are shown in Tab. 3.

**Table 3.** Attribute weights for six suppliers<sup>[16]</sup>

$Q_j$	$D_1$	$D_2$	$D_3$	$D_4$	$w_j$
$Q_1$	VH	H	H	H	[0.675, 0.925]
$Q_2$	H	VH	VH	H	[0.750, 0.950]
$Q_3$	MH	H	H	MH	[0.550, 0.750]
$Q_4$	M	M	MH	MH	[0.450, 0.550]

**Step 2.** Make attribute rating values for six supplier alternatives. According to Eq. (19), the results of attribute rating values are shown in Tab. 3

**Step 3.** Establish the grey decision matrix. According to Eq. (10), we can obtain the grey decision matrix of suppliers.

**Step 4.** Establish the grey normalized decision table. According to grey normalized decision matrix shown in Eq. (11), the grey normalized decision table is shown in Tab. 5.

**Step 5.** Establish the grey weighted normalized decision table. According to the grey weighted normalized decision matrix shown in Eq. (14), the grey weighted normalized decision table is shown in Tab. 6.

**Step 6.** Make the ideal  $S^{\max}$  supplier as referential alternative. According to Eqs. (15), the ideal  $S^{\max}$  supplier is shown as follows:

$$S^{\max} = \{[0.470, 0.925], [0.550, 0.950], [0.383, 0.750], [0.350, 0.550]\}$$

**Step 7.** Calculate the grey possibility degree between compared supplier alternatives set  $S = S_1, S_2, \dots, S_6$  and ideal referential supplier alternative. According to Eq. (16), the results of the grey possibility degree are shown as follows:

$$\begin{aligned} P_1 &= 0.539 & P_2 &= 0.575 & P_3 &= 0.789 \\ P_4 &= 0.747 & P_5 &= 0.771 & P_6 &= 0.840 \end{aligned}$$

The smaller one is better.

$$S_1 > S_2 > S_4 > S_5 > S_3 > S_6$$

**Table 4.** Attribute rating values for supplier<sup>[16]</sup>

$Q_j$	$S_i$	$D_1$	$D_2$	$D_3$	$D_4$	$G_{ij}$
$Q_1$	$S_1$	G	MG	G	G	[5.75, 8.25]
	$S_2$	MG	G	F	MG	[5.00, 6.50]
	$S_3$	F	F	MG	G	[4.75, 6.25]
	$S_4$	F	MG	MG	F	[4.50, 5.50]
	$S_5$	MG	F	F	MG	[4.50, 5.50]
	$S_6$	G	MG	MG	MG	[5.25, 6.75]
$Q_2$	$S_1$	G	G	MG	MG	[5.50, 7.50]
	$S_2$	G	MG	MG	G	[5.50, 7.50]
	$S_3$	F	F	P	F	[3.25, 4.50]
	$S_4$	P	MP	MP	P	[2.00, 3.50]
	$S_5$	MP	MP	P	MP	[2.50, 3.75]
	$S_6$	MP	P	P	MP	[2.00, 3.50]
$Q_3$	$S_1$	G	MG	MG	G	[5.50, 7.50]
	$S_2$	MG	G	G	G	[5.75, 8.25]
	$S_3$	G	G	F	MG	[5.25, 7.25]
	$S_4$	G	MG	MG	G	[5.50, 7.50]
	$S_5$	MG	F	F	MG	[4.50, 5.50]
	$S_6$	F	F	MG	F	[4.25, 5.25]
$Q_4$	$S_1$	F	G	G	G	[5.50, 8.00]
	$S_2$	G	G	F	MG	[5.75, 8.25]
	$S_3$	VG	VG	G	G	[7.50, 9.50]
	$S_4$	G	MG	G	G	[5.75, 8.25]
	$S_5$	MG	MG	G	MG	[5.25, 6.75]
	$S_6$	G	VG	VG	G	[7.50, 9.50]

**Table 5.** rey normalized decision table<sup>[16]</sup>

$S_i$	$Q_1$	$Q_2$	$Q_3$	$Q_4$
$S_1$	[0.70, 1.00]	[0.73, 1.00]	[0.67, 0.91]	[0.66, 0.96]
$S_2$	[0.61, 0.79]	[0.73, 1.00]	[0.70, 1.00]	[0.72, 1.00]
$S_3$	[0.58, 0.76]	[0.43, 0.60]	[0.64, 0.88]	[0.55, 0.70]
$S_4$	[0.55, 0.67]	[0.27, 0.48]	[0.67, 0.91]	[0.64, 0.91]
$S_5$	[0.55, 0.67]	[0.33, 0.50]	[0.55, 0.55]	[0.78, 1.00]
$S_6$	[0.64, 0.82]	[0.27, 0.48]	[0.52, 0.64]	[0.55, 0.70]

**Table 6.** Grey weighted normalized decision table<sup>[16]</sup>

$S_i$	$Q_1$	$Q_2$	$Q_3$	$Q_4$
$S_1$	[0.47, 0.93]	[0.55, 0.95]	[0.37, 0.68]	[0.30, 0.53]
$S_2$	[0.41, 0.73]	[0.55, 0.95]	[0.38, 0.75]	[0.33, 0.55]
$S_3$	[0.39, 0.70]	[0.33, 0.57]	[0.35, 0.66]	[0.25, 0.39]
$S_4$	[0.37, 0.62]	[0.20, 0.44]	[0.37, 0.68]	[0.29, 0.50]
$S_5$	[0.37, 0.62]	[0.25, 0.48]	[0.30, 0.50]	[0.35, 0.55]
$S_6$	[0.43, 0.76]	[0.20, 0.44]	[0.28, 0.48]	[0.25, 0.39]

#### 4.1.2 Grey Relation (GR) analysis<sup>[33]</sup>

**Step 7.** Establish the connected weighted normalized decision matrix with the ideal referential alternative sequence. According to Eq. (17), we can obtain the connected weighted normalized decision matrix of suppliers.

**Step 8.** Calculate the grey relation coefficient between each of the alternatives sequence with the ideal referential alternative sequence. According to Eq. (18), we can obtain the grey relation coefficients.

**Step 9.** Determine the grade of grey relation of each alternative to the ideal solutions. According to Eq. (19), the grade of grey relation of each alternative to the ideal solutions is shown as follows:

$$C_1 = 0.054 \quad C_2 = 0.098 \quad C_3 = 0.283$$

$$C_4 = 0.277 \quad C_5 = 0.298 \quad C_6 = 0.336$$

The smaller one is better.

$$S_1 > S_2 > S_4 > S_3 > S_5 > S_6$$

### 4.1.3 The new grey based method (NG)

**Step 5.** Make the ideal  $S^{\max}$  and negative ideal  $S^{\min}$  suppliers as referential alternatives. According to Eqs. (20) and (21), the ideal  $S^{\max}$  and negative ideal  $S^{\min}$  suppliers are shown as follows:

$$S^{\max} = \{[0.679, 1.000], [0.733, 1.000], [0.697, 1.000], [0.778, 1.000]\}$$

$$S^{\min} = \{[0.545, 0.667], [0.267, 0.467], [0.515, 0.636], [0.553, 0.700]\}$$

**Step 6.** Calculate the weighted connection between each of the alternatives sequence and the referential sequence. According to Eq. (25), the results of the weighted connection are shown as follows:

$$\Gamma_1 = 0.0325 \quad \Gamma_2 = 0.0665 \quad \Gamma_3 = 0.1725$$

$$\Gamma_4 = 0.1827 \quad \Gamma_5 = 0.1959 \quad \Gamma_6 = 0.2171$$

The smaller one is better.

$$S_1 > S_2 > S_3 > S_4 > S_5 > S_6$$

Here, the weighted distance between each supplier with  $S^{\max}$  and  $S^{\min}$  is calculated to demonstrate the new method is able to consider both PIS and NIS to evaluate the suppliers. According to Eq. (1), the formulas to calculate the distance are shown as follows, respectively.

$$D_i^{\max} = \sum_{j=1}^n |G_j^{\max} - G_{ij}^*| \times W_j, i = 1, \dots, m$$

$$D_i^{\min} = \sum_{j=1}^n |G_{ij}^* - G_j^{\min}| \times W_j, i = 1, \dots, m$$

According to above, the results of distance are shown in Tab. 7 and Fig. 1.

**Table 7.** Distance between each supplier with  $S^{\max}$  and  $S^{\min}$

$S_i$	$D_i^{\min}$	$D_i^{\max}$
$S_1$	0.36533	0.04269
$S_2$	0.31829	0.08591
$S_3$	0.13291	0.27172
$S_4$	0.10125	0.30295
$S_5$	0.08029	0.32433
$S_6$	0.04348	0.35108

Tab. 7 and Fig. 1 illustrate that the supplier which is close to  $S^{\max}$  and far from  $S^{\min}$  has highest priority in the ranking, and vice versa. Now it can be concluded that this method can easily consider both ideal and negative ideal solution to evaluate and rank the suppliers. The relationship between  $S_i$  with  $S^{\max}$  and  $S^{\min}$  is determined is follows:

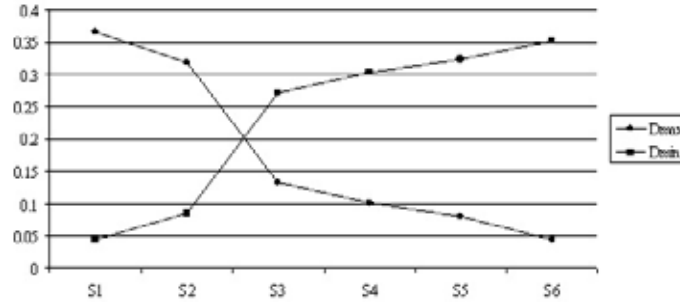


Fig. 1. Distance between each supplier with  $S^{\max}$  and  $S^{\min}$

- (1) If  $S_i = S^{\max}$ , then  $F_i = 0$
- (2) If  $S_i = S^{\min}$ , then  $F_i = 1$
- (3) If  $S^{\min} \leq S_i \leq S^{\max}$ , then  $0 \leq F_i \leq 1$

Up to this stage, we have understood that each method has its own result. And, we cannot determine which method is more optimal. The following comparative analysis will help us compare the three results and find the optimal one.

### 4.2 Comparative analysis

In this section, the LI, GR and NG methods are compared together. The essential information to establish a single objective linear programming are: the buyer’s demand is 3000 and the suppliers’ capacity are 800, 700, 900, 900, 850 and 750, respectively.

**Step 1.** Normalize the suppliers’ ratings obtained by Eqs. (16), (19), (22). According to Eqs. (23) ~ (25), the results of the normalized suppliers’ ratings are shown in Tab. 8.

Table 8. Normalized the LI, GR and NG methods

$S_i$	the LI method	The GR method	The NG method
$S_1$	0.126	0.040	0.037
$S_2$	0.135	0.073	0.077
$S_3$	0.185	0.210	0.199
$S_4$	0.175	0.206	0.211
$S_5$	0.181	0.221	0.226
$S_6$	0.197	0.250	0.250
Sum	1.000	1.000	1.000

**Step 2.** Establish the single objective linear programming to assign optimum order quantities ( $X_i$ ) to each supplier for the three methods. According to Eq. (26), the objective function, and according to Eq. (27) and Eq. (28), the constraints are shown as follows:

**For the LI method:**

**For the GR method:**

**For the NG method:**

$$\min(TVP) = \sum_{i=1}^6 P_i^* X_i^P$$

$$\min(TVP) = \sum_{i=1}^6 C_i^* X_i^C$$

$$\min(TVP) = \sum_{i=1}^6 \Gamma_i^* X_i^F$$

$$s.t X_i^P \leq V_i \quad i = 1, 2, \dots, 6$$

$$s.t X_i^C \leq V_i \quad i = 1, 2, \dots, 6$$

$$s.t X_i^F \leq V_i \quad i = 1, 2, \dots, 6$$

$$\sum_{i=1}^6 X_i^P = 3000$$

$$\sum_{i=1}^6 X_i^C = 3000$$

$$\sum_{i=1}^6 X_i^F = 3000$$

$$X_i^P \geq 0 \quad i = 1, 2, \dots, 6$$

$$X_i^C \geq 0 \quad i = 1, 2, \dots, 6$$

$$X_i^F \geq 0 \quad i = 1, 2, \dots, 6$$

This LP problem is solved using Solver from Microsoft Excel or LINDO. The results of the optimum order quantities (Xi) for these methods, TVP and the percentage of the improvement (PI) are shown in Tab. 9 In this

**Table 9.** Order quantities ( $X_i$ ) for the three methods.

$S_i$	The LI method	The GR method	The NG method
$X_1$	800	800	800
$X_2$	700	700	700
$X_3$	0	600	900
$X_4$	900	900	600
$X_5$	600	0	0
$X_6$	0	0	0
TVP	461.4	394.35	389.1
PI%		14.53	1.35

case, it is shown that consideration of both PIS and NIS to evaluate the suppliers decreases TVP from 461.4 in the LI method as well as 394.35 in the GR method to 389.1. Moreover, PI shows that the NG method is 1.35% better than the GR method and the GR method is 14.53% better than the LI method.

To demonstrate that the NG method is really more optimal, the methods are also compared via four different combinations of weights that are summarized in Tab. 10 For the given four different combinations of

**Table 10.** The solution qualities for the three methods via TVP with different weights

Different Cases	Weight				TVP			PI (%)	
	$Q_1$	$Q_2$	$Q_3$	$Q_4$	LI	GR	NG	LI to GR	GR to NG
Case1	[0.95,1]	[0.05,0.1]	[0.05,0.1]	[0.05,0.1]	469.14	410.94	377.31	14.16	8.91
Case2	[0.05,0.1]	[0.95,1]	[0.05,0.1]	[0.05,0.1]	469.62	397.23	351.92	18.22	12.88
Case3	[0.05,0.1]	[0.05,0.1]	[0.95,1]	[0.05,0.1]	458.55	343.97	276.82	33.31	24.26
Case4	[0.05,0.1]	[0.05,0.1]	[0.05,0.1]	[0.95,1]	463.97	374.15	315.95	24.01	18.42

the weights, Tab. 10 shows TVP values of the GR method are better than the LI method and the percentages are between 14.16% and 33.31%. Moreover, the NG method is more optimal than the GR method, and the percentages are between 8.91% and 24.26%. For each method, the suppliers' scoring, for the four cases are constructed in Fig. 2. In Fig. 2, it is shown that in the LI method, sensitivity of suppliers' scoring to variation of attributes' weight is very low. It may be a weakness of the LI method and based on Tab. 10, TVP does not considerably change with attributes' weight variations. For LI, maximum value for TVP happened in Case 2 (469.62) and its minimum value is in Case 3 (458.55). The difference between the two TVP is 11.07 (2.36%). While the difference (maximum TVP - minimum TVP) for the GR and NG methods are 66.97 (16.3%) and 100.49 (26.63%), respectively. It can be concluded that, besides of being more optimal than the LI and GR methods, the NG method is also more sensitive to variation of attributes' weight.

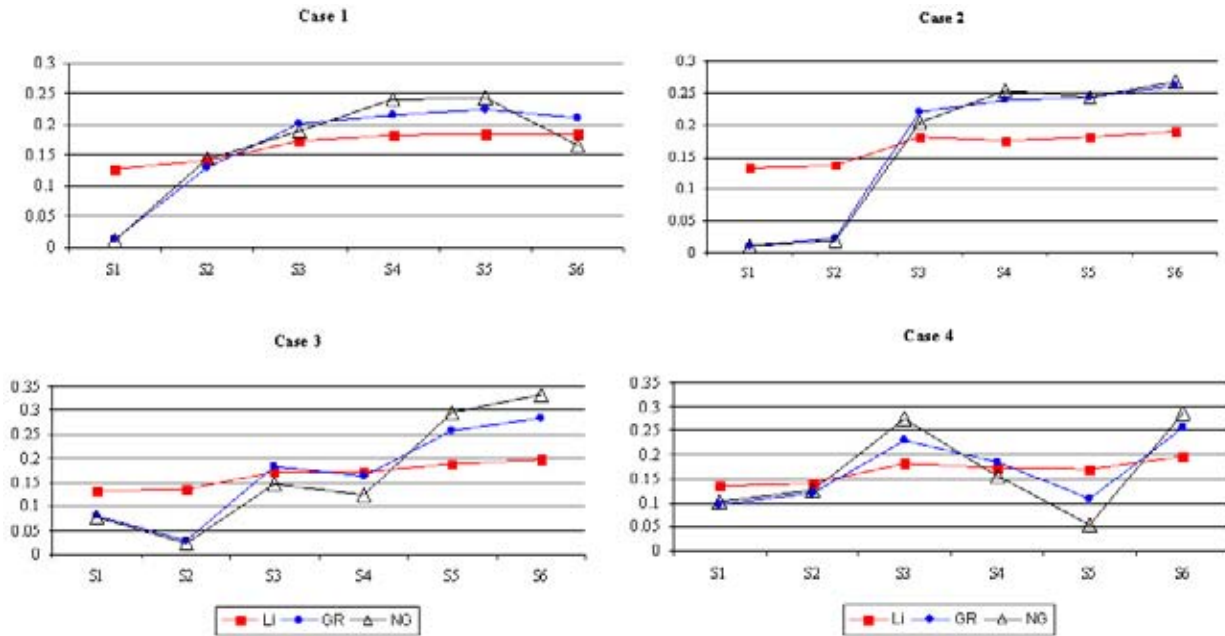


Fig. 2. The suppliers' scoring-For different Cases

## 5 Conclusion

Supplier selection is a MADM problem that in conventional MADM methods, the ratings and the weights of attributes must be known precisely<sup>[13, 16]</sup>. As Li et al.<sup>[16]</sup> declared, in many situations DMs' judgments are often uncertain and cannot be estimated by an exact numerical value. Thus, supplier selection problem has many uncertainties and becomes more difficult. Grey theory is a new mathematical field that is one of the methods used to study the uncertainty of a system. Moreover, the advantage of grey theory over fuzzy sets theory<sup>[32]</sup> is that grey theory can deal flexibly with both the fuzziness situation and incomplete information<sup>[16]</sup>.

In this paper, we proposed a new grey based approach based on the concept of TOPSIS to deal with the supplier selection problem in an uncertain environment. Moreover, we compared the new method with grey relation (GR)<sup>[8, 33]</sup> analysis and Li's et al.<sup>[16]</sup> grey method (LI). Li's et al.<sup>[16]</sup> data, information and assumptions were used in order to introduce the new grey based method (NG). Similar to Li's et al.<sup>[16]</sup> research, the ratings of attributes are described by linguistic variables that can be expressed in grey numbers. In the NG method, the weighted connection between each of the alternatives sequence and both PIS and NIS referential sequence was calculated to compare the ranking of grey numbers and select the most desirable supplier. Through this article, we demonstrated that this approach considers both the ideal and negative ideal solution to solve the multiple attributes decision making problem for selecting the best supplier. A comparative analysis showed that the NG method is more optimal than the LI and GR methods.

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