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A Multi-criteria product mix problem considering multi-period and several uncertainty conditions

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Abstract. This paper considers a multi-period and multi-criteria product mix problem minimizing the total cost, maximizing the total profit and minimizing the inventory levels under various random and ambiguous conditions. The proposal model is an extended model including some standard product mix problems and the model integrating manufacturing and distribution planning decisions in supply chains, and it represents more complex situations in real world. Since this problem is not well-defined problem due to including random and fuzzy variables, it is hard to solve it directly. Therefore, introducing chance constraints, main problem is transformed into a nonlinear programming problem. Furthermore, for solving it more efficiently, the solution method using the mean absolute deviation is constructed and so it is easier to obtain an optimal solution than previous analytical solution procedures.

Keywords: multi-criteria product mix problem, multi-period model, stochastic programming, fuzzy programming

1 Introduction

In many corporations and industries, there are many decision making problems such as scheduling problem, logistics, data mining and resource allocation problem. Product mix problem is also one of resource allocation problems and the most important industrial problems. It is important for decision makers in corporations and industries to predict future returns, to grasp the ability of machines and human resources, to receive efficient information of market demand and to decide an optimal asset allocation maximizing the total profit or minimizing the total cost under some constraints. Product mix problem also plays an important role in the prediction of future return and economic strength of the firm. Until now, many researchers have considered minimizing the total costs derived from production processes of firms, and recently many mathematical models have been proposed (In recent researches, Letmathe and Balakrishnan[9], Li and Tirupati[10], Morgan and Daniels[14], Mula, et al.[15]). However, minimizing the total cost does not mean maximizing the total future profit of the firm; i.e., decision makers need consider not only minimizing the total cost but also maximizing the total profits. To accomplish both objects, they also consider what products, how many and when they produce. In addition to this, there are many probabilistic and possibilistic factors in the production processes; breakdown of machines, ability of employee, lack of efficient information, and so on. Recently, some researchers have studied production planning problems including ambiguous situations (Mula, et al.[16, 17], Vasant[21], Wan[22]). In Hasuike [2], we have considered product mix problems under both randomness and fuzziness. Furthermore, In Hasuike and Ishii[4], we have considered the multi-criteria product mix problem, particularly minimizing the total emission of environmental pollutants.

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In most of previous available solution methods used to solve product mix problems called Manufacturing and Distribution Planning Decisions (MDPD) in supply chains, the objectives and model inputs are often assumed to be deterministic and crisp values. However, in practical production processes, the environmental coefficients and related parameters including available labor levels, machine capacities, market demand and cost coefficients, are normally randomness or fuzziness because of the lack of the reliable information, decision maker's subjectivity or unobtainable over the planning horizon. Therefore, there exist recent studies considering these uncertainty situations in supply chains (Petrovic et al.[18], Sabri and Beamon[19], Gen and Syarif[1], Liang[11]). Furthermore, Liang et al.[12] has developed a fuzzy multi-objective linear programming (FMOLP) model to solve multi-product and multi-time period MDPD problems with multiple fuzzy objectives in a supply chain. However, in study[12], only the fuzzy situation has been considered without including the random situations such as objective situations derived from the statistical analysis. In order to include the more complicate and wider productive situations in the supply chain, we need to consider both randomness and fuzziness, simultaneously. Therefore, in this paper, we develop the multi-product and multi-time period MDPD problems with randomness and fuzziness extending the model[12].

In mathematical programming problems, problems with several uncertainty situations are formulated as stochastic and fuzzy programming problems. Until now, there are some researches with respect to stochastic and fuzzy programming problems (Katagiri, et al.[6, 7], Inuiguchi and Ramik[5], Liu[13], Vajda[20]). Our proposed problem in this paper includes randomness and fuzziness, and is not a well-defined problem. Therefore, using chance constraints, we transform main problem into the deterministic equivalent problem. Furthermore, since this problem is a nonlinear programming problem, in order to solve it more efficiently and rapidly, we construct the efficient solution method introducing the mean-absolute deviation with respect to the total variance.

This paper is organized as follows. In Section 2, we consider a standard product mix problem based on the SCM model and introduce the multi-period and multi-criteria product mix problem. In Section 3, we extend this problem to the problem considering random and fuzzy environments. Our proposal problem is not a well-defined problem due to including random variables and fuzzy numbers. Therefore, we do the equivalent transformation of main problem and show that our proposal model is equivalent to a linear programming problem. Furthermore, we consider the relation between our model and previous models. In section 4, in order to compare our proposal model and previous some basic models, a numerical example is given. Finally in Section 5, we conclude this paper.

2 Formulation of standard multi-period product mix problem

First of all, we describe a multi-product and multi-time periods MDPD problem. In this paper, we assume that the logistics center in a supply chain attempts to determine the MDPD plan for N types of homogeneous commodities from one factory to J destinations (distribution centers) to satisfy market demand over a planning horizon.

2.1 Notation of each variable

The notation of each variable in this paper is as follows:

(Index set)

n: Index of product type, $(n = 1, 2, \cdots, N)$
j: Index of destination, $(j = 1, 2, \cdots, J)$
t: Index of planning time period, $(t = 1, 2, \cdots, T)$

(Decision variables)

x_{nt}: Production volume for nth product in period t
w_{nt}: Inventory level for nth product in period t
y_{nt}: Backordering volume for nth product in period t

u_{ntj}: Unit distributed for nth product from the factory to destination j in period t

(Parameters)

r_{nt} : Return coefficient per unit for n th product in period t
a_{nt} : Total regular cost coefficient per unit in production processes for nth product in period t
b_{nt} : Backordering cost per unit for nth product in period t
c_{nt} : Inventory carrying cost per unit for nth product in period t
p_{ntj} : Delivery cost per unit for nth product from the factory to destination j in period t
v_n : Regular size per unit for nth product
e : Escalating factor for each cost
D_{ntj} : Demand for nth product of destination j in period t
l_{nt} : Hour of labor per unit for nthproduct in period t
F_t : Maximum labor levels available for the factory in period t
m_{nt} : Hour of machine per unit for nth product in period t
M_t : Maximum machine capacity available for the factory in period t
q_{nhj} : Warehouse space per unit for nth product from the factory to destination j in period t
W_{tj} : Maximum warehouse space available for destination j in period t
B : Total budget

2.2 Formulation of multi-period product mix problem

Using these variables and parameters in Subsection 2.1, we introduce a multi-period product mix problem (MPPMP). In this paper, we consider the multiple objectives for solving the multi-product and multi-time period MDPD problems by reviewing the literature and considering practical situations. In real-world situations, most MDPD problems have been minimized the total cost in the production process such as recent studies of Gen and Syarif[10] and Liang[12]. However, minimizing the total cost does not always mean maximizing the total profit, equivalently. Furthermore, in the concept of Theory of Constraint (TOC), minimizing the total inventory level is considered much better. Therefore, by considering these assumptions, we introduce the following three-objective for our proposal MPPMP:

(Objective functions)

(1) Minimizing the total cost

$$Z_1 = \sum_{n=1}^{N}\sum_{t=1}^{T} a_{nt}x_{nt}(1+e)^t + \sum_{n=1}^{N}\sum_{t=1}^{T} b_{nt}y_{nt}(1+e)^t \tag{1}$$

$$+\sum_{n=1}^{N}\sum_{t=1}^{T} c_{nt}w_{nt}(1+e)^t + \sum_{n=1}^{N}\sum_{t=1}^{T}\sum_{j=1}^{J} p_{ntj}u_{ntj}(1+e)^t \tag{2}$$

(2) Maximizing the total profit

$$Z_2 = \sum_{n=1}^{N}\sum_{t=1}^{T} r_{nt}x_{nt}(1+e)^t \tag{3}$$

(3) Minimizing the total inventory levels in the last period T

$$Z_3 = \sum_{n=1}^{N} w_{nT} \tag{4}$$

Then, the constraints are as follows:

(Constraints)

(a) Constraints on carrying inventory

$$w_{nt-1} - y_{nt-1} + x_{nt} - w_{nt} + y_{nt} = \sum_{j=1}^{J} u_{ntj},\ \forall n, \forall t \tag{5}$$

(b) Constraints on demand

$$u_{ntj} = D_{ntj},\ \forall n, \forall t, \forall j \tag{6}$$

(c) Constraints on labor levels and machine capacity

$$\sum_{n=1}^{N} l_{nt} x_{nt} \leq F_t,\ \sum_{n=1}^{N} m_{nt} x_{nt} \leq M_t,\ \forall t \tag{7}$$

(d) Constraints on warehouse space for each destination

$$\sum_{n=1}^{N} q_{ntj} u_{nt} \leq W_{tj},\ \forall t, \forall j \tag{8}$$

(e) Constraints on initial total budget

$$Z_1 \leq B \tag{9}$$

(f) Non-negative constraints on decision variables

$$x_{nt} \geq 0,\ y_{nt} \geq 0,\ w_{nt} \geq 0,\ u_{ntj} \geq 0,\ \forall n, \forall t, \forall j \tag{10}$$

With respect to this basic problem, in the case that all parameters are fixed values, this problem is equivalent to a linear programming problem. Therefore, we obtain its optimal product mix analytically using linear programming approaches such as Simplex method and Interior point method.

3 Product mix problem under uncertainty situations

In Section 2, we have introduced the basic multi-period product mix problem based on Supply Chain Management (SCM). In this problem, many previous studies have considered that the parameters have been assumed to be fixed values. However, the available labor levels, machine capacity for each product and the market demand for each destination are often treated as random variable and fuzzy numbers due to statistical analysis based on incomplete and/or unobtainable information and decision maker's subjectivity. Therefore, in this paper, we focus on developing a random and fuzzy MPPMP to optimize the integrated MDPD plan in random and fuzzy environments.

3.1 Formulation of random and fuzzy multi-period and multi-criteria product mix problem

In previous study[12], it has assumed that decision makers have already adopted the triangular fuzzy number to represent the fuzzy market demand, available labor levels and machine capacity in the original problem. However, with respect to some coefficients such as the hours to occur in the regular production process such as l_{nt} and m_{nt}, most of companies have enormous amount of data and obtain the objective value derived from the statistical analysis. Therefore, we should deal these parameters with random variables. Furthermore, with respect to future return r_{nt} and demand D_{ntj}, these change randomly corresponding to the environment in each period, and so we also deal them with random variables. Furthermore, r_{nt} is closely related to D_{ntj}. For example, if a decision maker considers that she or he manages to earn more highly profit and plan to raise the price too much, demand D_{ntj} may decrease drastically, and vice versa. Considering these conditions, in this paper, we assume that the relation between r_{nt} and D_{ntj} represents the following linear relation.

$$r_{nt} = \alpha_0 + \sum_{j=1}^{J} \alpha_j D_{ntj} \tag{11}$$

where each α_j is assumed to be a fixed coefficient. Consequently, with respect to each parameter, we have the following assumptions:

(Parameters assumed to be fuzzy numbers)

$$a_{nt}, \quad b_{nt}, \quad c_{nt}, \quad p_{ntj}, \quad q_{nhj}, \quad F_t, \quad M_t, \quad W_{tj}$$

(Parameters assumed to be random numbers)

$$r_{nt}, \quad D_{ntj}, \quad v_n, \quad l_{nt}, \quad m_{nt}$$

In this paper, we assume that each fuzzy number is a triangle fuzzy numbers (for example, the h-cut of $\tilde{\alpha}_{nt}$ is $\left[a_{nt}^L(h), a_{nt}^U(h)\right]$ and each random variable occurs according to a normal distribution (for example, $D_{ntj} \sim N\left(\bar{d}_{ntj}, \sigma_{nt}^2\right)$), Therefore, in the main problem, Z_1 is a fuzzy number due to including the fuzzy numbers and Z_2 is a random variable due to including the random variables. Thereby, we reformulate a random/fuzzy MPPMP as follows:

$$\begin{aligned}
\text{Minimize} \quad & \tilde{Z}_1 \cong \sum_{n=1}^{N}\sum_{t=1}^{T} a_{nt}x_{nt}(1+e)^t + \sum_{n=1}^{N}\sum_{t=1}^{T} b_{nt}y_{nt}(1+e)^t \\
& \quad + \sum_{n=1}^{N}\sum_{t=1}^{T} c_{nt}w_{nt}(1+e)^t + \sum_{n=1}^{N}\sum_{t=1}^{T}\sum_{j=1}^{J} p_{ntj}u_{ntj}(1+e)^t \\
\text{Maximize} \quad & \tilde{Z}_2 \cong \sum_{n=1}^{N}\sum_{t=1}^{T} r_{nt}x_{nt}(1+e)^t = \sum_{n=1}^{N}\sum_{t=1}^{T}\left(\alpha_0 + \sum_{j=1}^{J}\alpha_j D_{ntj}\right)x_{nt}(1+e)^t \\
\text{Minimize} \quad & Z_3 \cong \sum_{n=1}^{N} w_{nT} \\
\text{subject to} \quad & w_{nt-1} - y_{nt-1} + x_{nt} - w_{nt} + y_{nt} = \sum_{j=1}^{J} u_{ntj}, \forall n, \forall t, \\
& u_{ntj} = D_{ntj}, \forall n, \forall t, \forall j, \\
& \sum_{n=1}^{N} l_{nt}x_{nt} \le F_t, \sum_{n=1}^{N} m_{nt}x_{nt} \le M_t, \forall t, \\
& \sum_{n=1}^{N} q_{ntj}u_{nt} \le W_{tj}, \forall t, \forall j, \\
& Z_1 \le B, \\
& x_{nt} \ge 0, y_{nt} \ge 0, w_{nt} \ge 0, u_{njt} \ge 0, \forall n, \forall t, \forall j
\end{aligned} \tag{12}$$

This problem is not a well-defined problem due to including the random variables and fuzzy numbers. Therefore, for the transformation into the deterministic equivalent problem, we introduce the chance constraints. In this paper, we deal all fuzzy objective function and constraints with the possibility chance constraints. Furthermore, using the stochastic programming approach and fuzzy programming approach based on the studies of Hasuike[4] and Katagiri[7], problem (12) is transformed into the deterministic equivalent problem:

$$
\begin{aligned}
&\text{Minimize} \quad f_1 \\
&\text{Maximize} \quad f_2 \\
&\text{Minimize} \quad f_3 = \sum_{n=1}^{N} w_{nT} \\
&\text{suject to} \quad \sum_{n=1}^{N}\sum_{t=1}^{T} a_{nt}^L(h)x_{nt}(1+e)^t + \sum_{n=1}^{N}\sum_{t=1}^{T} b_{nt}^L(h)y_{nt}(1+e)^t \qquad (13)\\
&\qquad + \sum_{n=1}^{N}\sum_{t=1}^{T} c_{nt}^L(h)w_{nt}(1+e)^t + \sum_{n=1}^{N}\sum_{t=1}^{T}\sum_{j=1}^{J} p_{ntj}^L(h)u_{ntj}(1+e)^t \leq f_1, \\
&\sum_{n=1}^{N}\sum_{t=1}^{T}\left(\alpha_0 + \sum_{j=1}^{J}\alpha_j \bar{d}_{ntj}\right) x_{nt}(1+e)^t - k_\beta \sqrt{Var(Z_2)} \geq f_2, \\
&\sum_{j=1}^{J} \bar{d}_{ntj} - k_{\frac{\beta}{2}} \sqrt{Var(D_{nt})} \leq w_{nt-1} - y_{nt-1} - x_{nt} - w_{nt} + y_{nt}, \forall n, \forall t, \\
&w_{nt-1} - y_{nt-1} + x_{nt} - w_{nt} + y_{nt} \leq \sum_{j=1}^{J} \bar{d}_{ntj} + k_{\frac{\beta}{2}} \sqrt{Var(D_{nt})}, \forall n, \forall t, \\
&u_{ntj} = D_{ntj}, \forall n, \forall t, \forall j, \\
&\sum_{n=1}^{N} \bar{l}_{nt}x_{nt} + k_\beta \sqrt{x^t V_t x} \leq F^L{}_t(h), \sum_{n=1}^{N} \bar{m_{nt}}x_{nt} + k_\beta \sqrt{x^t V_m x} \leq M_t{}^L(h), \forall t, \\
&\sum_{n=1}^{N} q_{ntj}^U(h)u_{nt} \leq w_{tj}{}^L(h),)\forall t, \forall j, \\
&Z_1 \leq B, \\
&x_{nt} \geq 0, y_{nt} \geq 0, w_{nt} \geq 0, u_{ntj} \geq 0, \quad \forall n, \forall t, \forall j
\end{aligned}
$$

Since this problem is equivalent to a multi-objective nonlinear programming problem, it is much harder to solve it directly and analytically. Therefore, in order to solve it more efficiently, we do the transformation of this problem.

3.2 Transformation of main problem to solve more efficiently

In the case we solve problem (13), the main difficulty is that this problem includes square root terms. In order to remove this difficulty, we introduce a mean-absolute deviation representing as $W[R_i(x)]$ based on the previous researches of Hasuike[3] and Konno[8]. Furthermore, in this paper, all random variables are assumed to be normal distributions. Subsequently, with respect to the relation between mean-absolute deviation $W[R_i(x)]$ and variance $\sigma_i^2(x)$ in the case of normal distributions, the following theorem holds based on the study of Konno[8].

Theorem 1. *[8] In the case that each random variable occurs according to the normal distribution,*

$$\sigma_i^2(x) = \frac{\pi}{2}\left(W\left[R_i(x)\right]\right)^2$$

holds.

Using this theorem and introducing the parameter η, problem (13) is equivalently transformed into the following form by extending our previous study[3]:

$$
\begin{aligned}
\text{Minimize} \quad & f_1 \\
\text{Maximize} \quad & f_2 \\
\text{Minimize} \quad & f_3 = \sum_{n=1}^{N} w_{nT} \\
\text{suject to} \quad & \sum_{n=1}^{N}\sum_{t=1}^{T} a_{nt}^{L}(h) x_{nt}(1+e)^t + \sum_{n=1}^{N}\sum_{t=1}^{T} b_{nt}^{L}(h) y_{nt}(1+e)^t \\
& \quad + \sum_{n=1}^{N}\sum_{t=1}^{T} c_{nt}^{L}(h) w_{nt}(1+e)^t + \sum_{n=1}^{N}\sum_{t=1}^{T}\sum_{j=1}^{J} p_{ntj}^{L}(h) u_{ntj}(1+e)^t \le f_1, \\
& \sum_{n=1}^{N}\sum_{t=1}^{T} \left(\alpha_0 + \sum_{j=1}^{J} \alpha_j \bar{d}_{ntj}\right) x_{nt}(1+e)^t - k_\beta \sqrt{\frac{\pi}{2}} \left(\sum_{i=1}^{I} p_{Z_2}^{(i)} \eta_{Z_2}^{(i)}\right) \ge f_2, \\
& \eta_{Z_2}^{(i)} - \sum_{n=1}^{N}\sum_{t=1}^{T} \left(r_{nt}^{(i)} - \bar{r}_{nt}\right) x_{nt}(1+e)^t \ge 0, \\
& \eta_{Z_2}^{(i)} - \sum_{n=1}^{N}\sum_{t=1}^{T} \left(r_{nt}^{(i)} - \bar{r}_{nt}\right) x_{nt}(1+e)^t \ge 0, \\
& \sum_{j=1}^{J} \bar{d}_{ntj} - K_{\frac{\beta}{2}} \sqrt{\frac{\pi}{2}} \left(\sum_{i=1}^{I} p_{d_{nt}}^{(i)} \eta_{d_{nt}}^{(i)}\right) \le w_{nt-1} - y_{nt-1} + x_{nt} - w_{nt} + y_{nt}, \forall n, \forall t, \\
& w_{nt-1} - y_{nt-1} + x_{nt} - w_{nt} + y_{nt} \le \sum_{j=1}^{J} \bar{d}_{ntj} + K_{\frac{\beta}{2}} \sqrt{\frac{\pi}{2}} \left(\sum_{i=1}^{I} p_{d_{nt}}^{(i)} \eta_{d_{nt}}^{(i)}\right), \forall n, \forall t, \\
& \eta_{d_{nt}}^{(i)} - \sum_{j=1}^{J} \left(d_{ntj}^{(i)} - \bar{d}_{ntj}\right) \ge 0, \eta_{d_{nt}}^{(i)} + \sum_{j=1}^{J} \left(d_{ntj}^{(i)} - \bar{d}_{ntj}\right) \ge 0, \\
& u_{ntj} = D_{ntj}, \forall n, \forall t, \forall j, \\
& \sum_{n=1}^{N} \bar{l}_{nt} x_{nt} + K_\beta \sqrt{x^t V_t x} \le F_t^L(h), \sum_{n=1}^{N} \bar{m}_{nt} x_{nt} + K_\beta \sqrt{x^t V_t x} \le M_t^L(h), \forall t, \\
& \eta_l^{(i)} - \sum_{n=1}^{N} \left(l_{nt}^{(i)} - \bar{l}_{nt}\right) \ge 0, \eta_l^{(i)} + \sum_{n=1}^{N} \left(l_{nt}^{(i)} - \bar{l}_{nt}\right) \ge 0, \\
& \eta_m^{(i)} - \sum_{n=1}^{N} \left(m_{nt}^{(i)} - \bar{m}_{nt}\right) \ge 0, \eta_m^{(i)} + \sum_{n=1}^{N} \left(m_{nt}^{(i)} - \bar{m}_{nt}\right) \ge 0, \\
& \sum_{n=1}^{N} q_{ntj}^{U}(h) u_{nt} \le W_{tj}^{L}(h), \forall t, \forall j, \\
& Z_1 \le B, \\
& x_{nt} \ge 0, y_{nt} \ge 0, w_{nt} \ge 0, u_{nt} \ge 0, \forall n, \forall t, \forall j
\end{aligned}
\tag{14}
$$

By introducing Theorem 1 and parameter η, we find that problem (14) is equivalently transformed into the multi-objective linear programming problem. Second, with respect to multi-objective function, decision makers often set the goal to each objective function and consider its aspiration level to the achievement of objective value more than the goal. Furthermore, taking the vagueness of human judgment and flexibility for the execution of a plan into account, each aspiration level is assumed to be a satisfaction function. Then, decision makers need to consider the condition that they satisfy the aspiration level more than a goal. In this paper, we introduce the following linear satisfaction functions:

$$\mu_1(f_1)=\begin{cases}1 & \left(f_1\le f_1^L\right)\\ \frac{f_1^U-f_1}{f_1^U-f_1^L} & \left(f_1^L<f_1\le f_1^U\right)\\ 0 & \left(f_1^U<f_1\right)\end{cases},\ \mu_2(f_2)=\begin{cases}1 & \left(f_2^U\le f_2\right)\\ \frac{f_2-f_2^L}{f_1^U-f_1^L} & \left(f_2^L\le f_2\le f_2^U\right)\\ 0 & \left(f_2<f_2^L\right)\end{cases},$$

$$\mu_3(f_3)=\begin{cases}1 & \left(f_3\le f_3^L\right)\\ \frac{f_3^U-f_3}{f_3^U-f_3^L} & \left(f_3^L<f_3\le f_3^U\right)\\ 0 & \left(f_3^U<f_3\right)\end{cases} \tag{15}$$

where all upper values f_k^U and lower value f_k^L are fixed. Using these membership functions and introducing a parameter h, problem (14) is equivalently transformed into the following problem:

$$\begin{aligned}
\text{Maximize}\quad & h\\
\text{suject to}\quad & \mu_1(f_1)\ge h,\quad \mu_2(f_2)\ge h,\quad \mu_3(f_3)\ge h, \qquad (16)\\
& \sum_{n=1}^{N}\sum_{t=1}^{T}a_{nt}^T(h)x_{nt}(1+e)^t+\sum_{n=1}^{N}\sum_{t=1}^{T}b_{nt}^T(h)y_{nt}(1+e)^t,\\
&\qquad +\sum_{n=1}^{N}\sum_{t=1}^{T}c_{nt}^L(h)w_{nt}(1+e)^t+\sum_{n=1}^{N}\sum_{t=1}^{T}\sum_{j=1}^{J}p_{ntj}^L(h)u_{ntj}(1+e)^t\le f_1,\\
& \sum_{n=1}^{N}\sum_{t=1}^{T}\left(\alpha_0+\sum_{j=1}^{J}\alpha_j\bar{d}_{ntj}\right)x_{nt}(1+e)^t-K_\beta\sqrt{\frac{\pi}{2}}\left(\sum_{i=1}^{I}p_{Z_2}^{(i)}\eta_{Z_2}^{(i)}\right)\ge 0,\\
& \eta_{Z_2}^{(i)}-\sum_{n=1}^{N}\sum_{t=1}^{T}\left(r_{nt}^{(i)}-\bar{r}_{nt}\right)x_{nt}(1+e)^t\ge 0,\\
& \eta_{Z_2}^{(i)}+\sum_{n=1}^{N}\sum_{t=1}^{T}\left(r_{nt}^{(i)}-\bar{r}_{nt}\right)x_{nt}(1+e)^t\ge 0,\\
& \sum_{j=1}^{J}\bar{d}_{ntj}-K_{\frac{\beta}{2}}\sqrt{\frac{\pi}{2}}\left(\sum_{i=1}^{I}p^{(i)}_{d_{nt}\eta^{(i)}_{d-nt}}\right)\le w_{nt-1}-y_{nt-1}+x_{nt}-w_{nt}+y_{nt},\forall n,\forall t\\
& w_{nt-1}-y_{nt-1}+x_{nt}-w_{nt}+y_{nt}\le\sum_{j=1}^{J}\bar{d}_{ntj}+K_{\frac{\beta}{2}}\sqrt{\frac{\pi}{2}}\left(\sum_{i=1}^{I}p_{d_{nt}}^{(i)}\eta_{d_{nt}}^{(i)}\right),\forall n,\forall t\\
& u_{ntj}=D_{ntj},\forall n,\forall t,\forall i,\\
& \sum_{n=1}^{N}\bar{l}_{nt}x_{nt}+K_\beta\sqrt{x^tV_tx}\le F_t^L(h),\quad \sum_{n=1}^{N}\bar{l}_{nt}x_{nt}+K_\beta\sqrt{x^tV_tx}\le M_t^L(h),\forall t,\\
& \eta_l^{(i)}-\sum_{n=1}^{N}\left(l_{nt}^{(i)}-\bar{l}_{nt}\right)\ge 0,\quad \eta_l^{(i)}+\sum_{n=1}^{N}\left(l_{nt}^{(i)}-\bar{l}_{nt}\right)\ge 0,\\
& \eta_m^{(i)}-\sum_{n=1}^{N}\left(m_{nt}^{(i)}-\bar{m}_{nt}\right)\ge 0,\quad \eta_m^{(i)}+\sum_{n=1}^{N}\left(m_{nt}^{(i)}-\bar{m}_{nt}\right)\ge 0,\\
& \sum_{n=1}^{N}q_{ntj}^U(h)u_{nt}\le W_{tj}^L(h),\forall t,\forall j,\\
& Z_1\le B,\\
& x_{nt}\ge 0,y_{nt}\ge 0,w_{nt}\ge 0,u_{nt}\ge 0,\forall n,\forall t,\forall j
\end{aligned}$$

Therefore, we consider the following problem assume to be each linear satisfaction function:

$$
\begin{aligned}
&\text{Maximize} \quad h \\
&\text{suject to} \quad \sum_{n=1}^{N}\sum_{t=1}^{T} a_{nt}^{T}(h)x_{nt}(1+e)^t + \sum_{n=1}^{N}\sum_{t=1}^{T} b_{nt}^{L}(h)y_{nt}(1+e)^t + \sum_{n=1}^{N}\sum_{t=1}^{T} c_{nt}^{L}(h)w_{nt}(1+e)^t \qquad (17) \\
&\qquad + \sum_{n=1}^{N}\sum_{t=1}^{T}\sum_{j=1}^{J} p_{ntj}^{L}(h)u_{ntj}(1+e)^t \le (1-h)f_1^U + hf_1^L, \\
&\sum_{n=1}^{N}\sum_{t=1}^{T}\left(\alpha_0 + \sum_{j=1}^{J}\alpha_j \bar{d}_{ntj}\right)x_{nt}(1+e)^t - K_\beta\sqrt{\frac{\pi}{2}}\left(\sum_{i=1}^{I} p_{Z_2}^{(i)}\eta_{Z_2}^{(i)}\right) \ge (1-h)f_2^L + hf_2^U, \\
&\eta_{Z_2}^{(i)} - \sum_{n=1}^{N}\sum_{t=1}^{T}\left(r_{nt}^{(i)} - \bar{r}_{nt}\right)x_{nt}(1+e)^t \ge 0, \\
&\eta_{Z_2}^{(i)} + \sum_{n=1}^{N}\sum_{t=1}^{T}\left(r_{nt}^{(i)} - \bar{r}_{nt}\right)x_{nt}(1+e)^t \ge 0, \\
&\sum_{n=1}^{N} w_{nT} \le (1-h)f_3^U + hf_3^U, \\
&\sum_{j=1}^{J}\bar{d}_{ntj} - K_{\frac{\beta}{2}}\sqrt{\frac{\pi}{2}}\left(\sum_{i=1}^{I} p_{d_{nt}}^{(i)}\eta_{d_{nt}}^{(i)}\right) \le w_{nt-1} - y_{nt-1} + x_{nt} - W_{nt} + y_{nt}, \forall n, \forall t, \\
&w_{nt-1} - y_{nt-1} + x_{nt} - w_{nt} + y_{nt} \le \sum_{j=1}^{J}\bar{d}_{ntj} + K_{\frac{\beta}{2}}\sqrt{\frac{\pi}{2}}\left(\sum_{i=1}^{I} p_{d_{nt}}^{(i)}\eta_{d_{nt}}^{(i)}\right) \forall n, \forall t, \\
&\eta_{d_{nt}}^{(i)} - \sum_{j=1}^{J}\left(d_{ntj}^{(i)} - \bar{d}_{ntj}\right) \ge 0, \quad \eta_{d_{nt}}^{(i)} + \sum_{j=1}^{J}\left(d_{ntj}^{(i)} - \bar{d}_{ntj}\right) \ge 0, \\
&u_{ntj} = D_{ntj}, \forall n, \forall t, \forall j, \\
&\sum_{n=1}^{N}\bar{l}_{nt}x_{nt} + K_\beta\sqrt{x^t V_t x} \le F_t^L(h), \quad \sum_{n=1}^{N}\bar{m}_{nt}x_{nt} + K_\beta\sqrt{x^t V_t x} \le M_t^L(h), \forall t, \\
&\eta_l^{(i)} - \sum_{n=1}^{N}\left(l_{nt}^{(i)} - \bar{l}_{nt}\right) \ge 0, \quad \eta_l^{(i)} + \sum_{n=1}^{N}\left(l_{nt}^{(i)} - \bar{l}_{nt}\right) \ge 0, \\
&\eta_m^{(i)} - \sum_{n=1}^{N}\left(m_{nt}^{(i)} - \bar{m}_{nt}\right) \ge 0, \quad \eta_m^{(i)} + \sum_{n=1}^{N}\left(m_{nt}^{(i)} - \bar{m}_{nt}\right) \ge 0, \\
&\sum_{n=1}^{N} q_{ntj}^{U}(h)u_{nt} \le W_{tj}^{L}(h), \forall t, \forall j, \\
&Z_1 \le B, \\
&x_{nt} \ge 0, y_{nt} \ge 0, w_{nt} \ge 0, u_{ntj} \ge 0, \forall n, \forall t, \forall j
\end{aligned}
$$

In the case that parameter h is fixed, this problem is equivalent to a linear programming problem, and so it is obvious that this problem is solved more efficiently and analytically than main problem (13) by using efficient linear programming approaches and a bisection algorithm for parameter h. Then, we consider its dual problem and obtain the optimal product mix more efficiently than problem (13). Furthermore, In the case that random variables is treated as the only expected values, our proposed model is equivalent to the previous model[12]. Consequently, our proposed model include the almost all the pervious product mix problems and SCM problems.

4 Numerical example

In order to compare our proposal model with previous models, we give the following numerical example. We assume that two products are produced and the time horizon is three periods. Tab. 1 shows data for each variable. Then, all fuzzy variables are symmetric triangle fuzzy numbers represented as $\langle a, b \rangle$, and all random variables occur according to normal distributions. Then, the value of escalating factor is -0.05. In this paper, we compare our proposal model (12) with two previous models; (a) basic product mix model minimizing the total cost satisfying the total profit is more than and equal to the target value (not including random variables and fuzzy numbers), and (b) Liang model based on study[12] (not including random variables and not considering total profit and inventories). Subsequently, each inverse function for fuzzy goal is $f_1 = 2500 - 500h$, $f_2 = 10000 + 2000h$ and $f_3 = 10 - 5h$, respectively. Consequently, we solve three models and each optimal product mix is as Tab. 2. From the result of Tab. 2, while the total cost of our proposal model is a little larger than the other models, the total profit is much larger than the other models. This means that our proposal model is focused on maximizing the total profit rather than minimizing the total cost. Then, the final inventory volumes are as the following Tab. 3.

Table 1. Data for each parameter in production processes

	Period	Production 1	Production 2
Return (considering each demands; according to normal distribution)	Period 1	N(150,10)	N(100,15)
	Period 2	N(200,10)	N(120,15)
	Period 3	N(250,20)	N(150,15)
Regular cost (according to a triangle fuzzy number)	Period 1	$< 20, 2 >$	$< 10, 1 >$
	Period 2	$< 20, 2 >$	$< 10, 1 >$
	Period 3	$< 20, 2 >$	$< 10, 1 >$
Back ordering cost (according to a triangle fuzzy number)	Period 1	$< 25, 3 >$	$< 12, 2 >$
	Period 2	$< 25, 3 >$	$< 12, 2 >$
	Period 3	$< 25, 3 >$	$< 12, 2 >$
Inventory carrying cost (according to a triangle fuzzy number)	Period 1	$< 0.3, 0.05 >$	$< 0.15, 0.05 >$
	Period 2	$< 0.3, 0.05 >$	$< 0.15, 0.05 >$
	Period 3	$< 0.3, 0.05 >$	$< 0.15, 0.05 >$
Initial Inventory		5	5

Table 2. Optimal product mix for each model

		Model (a)	Model (b)	Proposal model
Product 1	Period 1	5	7.670	0.272
	Period 2	0	9.659	12.218
	Period 3	25	16.684	23.128
Product 2	Period 1	5	5.658	6.004
	Period 2	15	9.242	32.208
	Period 3	25	24.551	24.977
Expected total cost		1259.34	1267.09	1443.55
Expected total profit		11385.7	11108.6	14474.0

Table 3. Final inventory volumes

	Model (a)	Model (b)	Proposal model
Product 1	0	0	0.051
Product 2	0	0.658	0.003

Tab. 3 shows that we largely reduce the final inventory volumes as well as the other models. Therefore, we find that our proposal model considers minimizing the total inventory level strictly.

5 Conclusion

In this paper, we have proposed the extend product mix problem model considering various uncertainty situations and multi-period including SCM model. Since our proposed model includes the previous fuzzy manufacturing and distribution planning decisions, this model is applied to more widely and complicatedly practical product mix problems. Furthermore, it has been hard to solve our proposed model due to including random variable and fuzzy numbers. Therefore, introducing chance constraints and transformation into the deterministic equivalent problem, we have obtained that our proposed model has been equivalent to a linear programming problem. By the development of information technology and computers, it is easy to solve much larger scale linear programming problem. Therefore, our proposed model is expected to apply to more various types of practical product mix problems.

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