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Coordination mechanism for tow-level supply chain with one manufacturer and one buyer under credit period

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Abstract. Achieving effective coordination among members of the supply chain has become a pertinent research issue in the supply chain management. This is because coordination has the advantage of reducing total cost and of improving system performance.

In this paper, a coordination mechanism is derived under credit period for a two-level supply chain that includes a single manufacturer and a single buyer for a single product. A centralized system is designed to minimize the total variable cost per unit time. Then, theorems are developed to determine the optimal ordering policies and bounds are provided for credit period to develop an algorithm. Numerical examples reveal that our optimization procedure is effective. Cost saving due to coordination is shared as a result of agreement between the two parties involved.

Keywords: supply chain management, coordination mechanism, credit period

1 Introduction

Globalization in our modern times is giving rise to a multitude of continual changes and developments in the business environment including regulatory changes, increasing competition intensity, increasingly demanding customers, new information technologies, mergers and acquisitions, among others. The supply chain management (SCM) is the term used to describe the management of materials and information across the entire supply chain, from suppliers to component producers, to final assemblers, to distribution (warehouse and retailers), and ultimately to the consumer. In fact, it often includes after-sales service and returns or recycling as well.

SCM involves coordinating and integrating activities and processes among different business functions for the benefit of the entire supply chain. The integration of multiple functions and enterprises, particularly in a global supply chain context, is complex and requires coordination. Hence, coordination between elements of supply chain for improving its performance has received a great deal of attention from researchers. Some coordination strategies, such as the quantity discount, credit option, and payback/return policy, are often used to regulate the relationship among parties involved in the supply chain. The quantity discount policy has attracted much attention as a coordination mechanism[9, 11, 15, 19, 24, 25]. Pasternack[20], Emmons and Gilbert[7], Shiang Lau and Hing-Ling Lau[13], and Lee and Rhee[14] studied the payback/return policy in the supply chain.

However, some authors have used delay in payment as a coordination mechanism in their models. Kingsman[12] used the delay-in-payment option in the development of his mathematical model. Davis and Gaither developed optimal ordering policies for firms that are offered one-time opportunity extended payment[6]. Goyal explored a single item EOQ model under permissible delay in payments[8]. Kim et al. (1995) determined the optimum length of credit period for the product supplier sells to retailers in order to

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maximize profit. Chung presented a procedure to determine the optimal time interval of permissible delay in payment model by minimizing the total annual variable cost function[5]. Chu et al.[3], Chu and Chung[4] and Jamal et al.[10], determined the optimal payment time if delay beyond a fixed credit period is permitted. Liao et al.[17] developed an EOQ model for stock-dependent demand rate when a delay in payment is permissible. Abad and Jaggi considered a seller-buyer problem in which the end demand is price sensitive and the seller may offer trade credit, characterized by delay in payments, to the buyer. Shinn[1] and Whang[23] have considered order size dependent delay in payment and have developed a model for determination of retailer's optimal price and order size simultaneously. Chen and Kang considered a permissible delay in payments in the integrated model from the viewpoint of the vendor-buyer cooperation in the supply chain management. Luo[2, 18] studied and analyzed coordinated strategy between vendor and buyer through the use of credit period. Sarmah et al.[21] developed a manufacturer and a buyer coordination mechanism through credit option where both parties have a certain amount of target profit from the business. Sarmah et al.[22] also developed a coordination model with credit option in a single-manufacturer and multiple heterogeneous buyers' situation to determine the optimum production.

In this paper, we consider the credit period as a mechanism to develop coordination between a single manufacturer and a single buyer for a single product. The objective of our model is to view the system as an integrated whole and to determine the optimum production and a suitable credit period that minimize the total cost system. Our model reveals that there is an upper bound and a lower bound of credit period that each of the parties can offer to the other. The interrelationship of the opportunity costs of the manufacturer and the buyer is used as one of the bounds for the credit period to optimize the global system performance. Extra savings due to coordination can be shared between the two parties and this coordination can also be a win-win proposition for both parties.

The remainder of the paper is organized as follows. In Section 2, we outline the model assumptions and establish the basic model to illustrate the individual optimal strategies of inventories. In section 3, we provide the necessary conditions to implement a credit period policy. In Section 4, we discuss how to establish and realize the coordination mechanism. In Section 5 an extension numerical study is carried out to evaluate the effectiveness of the proposed coordinated strategies. Finally, in Section 6, we conclude and discuss future research.

2 Model development

We consider a single-manufacturer, single-buyer inventory control problem where the manufacturer delivers the finished goods to the buyer. Our objective is to develop an economic lot size model to minimize the integrated supply chain costs.

The following notation and assumptions are made in the development of the integrated production inventory model.

Notations concerning manufacturer and buyer:

D : Demand rate, units/year;

P : Manufacture's production rate, units/year;

P_1 and P_2 denote the delivered unit price paid by the manufacturer and the buyer, respectively, \$/unit;

A : Buyer's ordering cost per order, \$/order;

S : Manufacturer's production setup cost per batch, \$/order;

i_1 and i_2 denote the manufacturer and the buyer's cost of capital or opportunity cost in annual percentage (decimal), respectively;

$h_1^{'}$ and $h_2^{'}$ denote the manufacturer and the buyer's unit variable holding cost excluding the cost of capital, respectively. h_1 and h_2 denote the manufacturer and the buyer's unit variable holding cost including the cost of capital, respectively. Obviously,
$h_1 = h_1^{'} + P_1 i_1$ and $h_2 = h_2^{'} + P_2 i_2$;

Assumptions of the model:

(1) Demand rate is known and constant.
(2) Production rate, P, is known and constant.
(3) Shortages are not allowed.
(4) Lead time is zero and the replenishment rate is infinite.
(5) The manufacturer's credit period begins at the time when the ordered quantity is delivered.

2.1 The optimal strategy for buyer and manufacturer in traditional business mode

Fig. 1 depicts the time-weighted inventory (TWI) of the manufacturer and buyer's inventory level. The manufacturer produces a production lot size of mQ_0 and delivers m times of the stock, Q_0 to the buyer's warehouse.

Since the buyer makes order decisions in term of a simple EOQ, the buyer's optimal ordering quantity is $Q_0 = \sqrt{2AD/h_2}$, and the buyer's minimized annual cost is:

$$F_{b1}(Q_0) = \sqrt{2ADh_2} \tag{1}$$

Thus, the manufacturer's order size should be some integer multiple of Q_0 since he is faced with a stream of demands, each with an order size Q_0 and at fixed intervals Q_0/D.

The manufacturer's time-weighted inventory is as follows:

$$\frac{1}{2}\left[(m-1)\left(1-\frac{D}{P}\right)+\frac{D}{P}\right] \tag{2}$$

For proof the above equation, see Lee[16].

So the total annual cost for the manufacturer is given by:

$$F_{M1}(m) = \frac{SD}{mQ_0} + h_1\frac{Q_0}{2}\left[(m-1)\left(1-\frac{D}{P}\right)+\frac{D}{P}\right] \tag{3}$$

Let m^* be the optimum solution of $\min_{m\geq 1} F_{M1}(m)$. As $F_{M1}(m)$ is strictly convex in m for $m \geq 0$, thus we obtain:

$$\begin{aligned} m^* &= \max\left\{\min\left\{m \,|\, F_{M1}(m+1) \geq F_{M1}(m),\, 0\right\}\right\} \\ m^{\cdot} &= \left\lceil \sqrt{\frac{Sh_2}{Ah_1\left(1-\frac{D}{P}\right)}+\frac{1}{4}}-\frac{1}{2} \right\rceil \end{aligned} \tag{4}$$

where $\lceil x \rceil$ is the least integer greater than or equal to x.

Hence, the manufacturer's current order size is $m^*\sqrt{2DA/h_2}$ and places $D/(m^*\sqrt{2DA/h_2})$ orders each year with an interval of $m^*\sqrt{2DA/h_2}/D$ throughout that period. The manufacturer's minimized total cost is $F_{M1}(m^*)$, which implies the assumption $F_{M1}(m^*) \leq (p_2 - p_1)D$ made in this paper. The above manufacturer's ordering policy is based on minimizing the buyer's total cost, or maximizing its profits.

3 The coordination optimal strategy in centralized system supply chain

In the traditional EPQ model, it is assumed that the buyer must pay to the manufacturer for the items as soon as the items are received. The manufacturer usually hopes the buyer to change his order quantity. In practice, the manufacturer is willing to offer the buyer a certain credit period without interest during the permissible delay period to promote market competition. In other words, the manufacturer would like to consider the credit period only when his cost doesn't increase. Similarly, although the credit period can reduce the buyer's cost, its inventory holding cost will increase at the same time. So, the buyer would accept the manufacturer's credit period policy only when his cost doesn't increase. Thus, the first problem we should consider is: whether can we find the limiting conditions of the credit period policy that both sides will accept?

Furthermore, if a credit period policy just reduces the cost of one party, it cannot guarantee that the other party has enough enthusiasm to take this policy. Thus, the second problem we should consider is whether there is a credit period policy which can improve the global system performance.

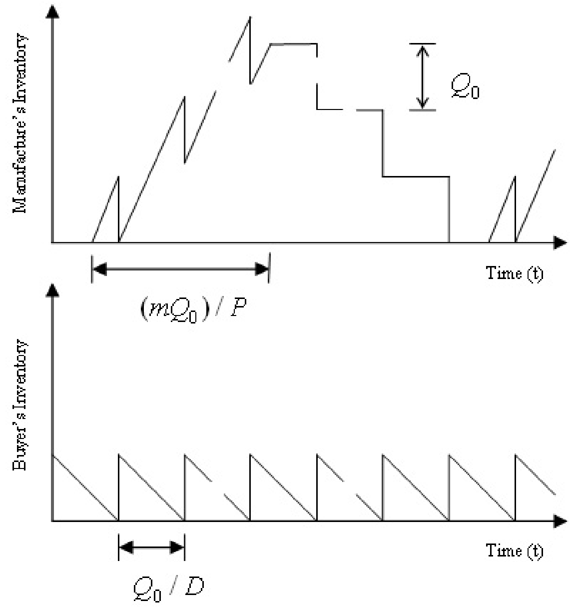

Fig. 1. The distributor's inventory level and the manufacturer's time-weighted inventory

3.1 The necessary conditions to implement a credit period policy

Now consider the case that the manufacturer induces the buyer to increase his order quantity through credit period. Assume that the manufacturer requires the buyer to change his order size from Q_0 to KQ_0, where $K > 0$, on which condition he will provide the credit period. Let be the length of the credit period, and the manufacturer's order size be nKQ_0, since the buyer's new order size is fixed at KQ_0, where n is a positive integer. The buyer's total interest saving on the money payable during the credit period is P_2DTi_1.

3.1.1 Conditions for the buyer to change the order size

The condition under which the buyer accepts this arrangement will hold only when his cost does not increase. We get the buyer's new annual cost function as follows:

$$F_{b2}(KQ_0, T) = DA/(KQ_0) + KQ_0h_2/2 - P_2DTi_2 \tag{5}$$

So if the manufacturer wants the buyer to change his order quantity, the condition $F_{b2}(KQ_0, T) \leq F_{b1}(Q_0)$ must have been satisfied. This means that:

$$T \geq \frac{DA/(KQ_0) + KQ_0h_2/2 - \sqrt{2DAh_2}}{P_2Di_2} \tag{6}$$

Substituting $Q_0 = \sqrt{2AD/h_2}$ for Q_0 in Eq. (6), we get:

$$T \geq \frac{(K + 1/K - 2)\sqrt{DAh_2/2}}{P_2Di_2} \tag{7}$$

In fact, Eq. (7) implies that the implemented credit period policy has a lower bound, $T_{\min}$, which can be expressed as:

$$T_{\min} = \frac{(K + 1/K - 2)\sqrt{DAh_2/2}}{P_2Di_2} \tag{8}$$

3.1.2 Conditions for the manufacturer to change buyer's order size

We can see from the above analysis that the credit period must not be less than a value determined by Eq. (8) in order that it can be accepted by the buyer. Similarly, there should exist a higher bound of T which guarantees that the manufacturer would not increase cost. Consider the same case as above, that is, the manufacturer requires the buyer to change his order quantity from Q_0 to KQ_0 and promises that he will provide the credit period. We get the manufacturer's new annual cost function as follows:

$$F_{M2}(n, KQ_0, T) = DS/(nKQ_0) + \frac{KQ_0 h_1}{2}\left[(n-1)\left(1-\frac{D}{P}\right)+\frac{D}{P}\right] + P_2DTi_1 \tag{9}$$

If the manufacturer wants to reduce his cost after taking benefit of the credit period policy, it must satisfy the restriction $F_{s2}(n, KQ_0, T) \leq F_{s1}(m, Q_0)$. In other words:

$$T \leq \frac{(DS/Q_0)(1/m - 1/(nK)) + (Q_0h_1/2)\left[\left(1-\frac{D}{P}\right)\{(m-1)-K(n-1)\}+\frac{D}{p}(1-K)\right]}{P_2Di_1} \tag{10}$$

Substituting $Q_0 = \sqrt{2AD/h_2}$ for Q_0 in Eq. (10), we get:

$$T \leq \frac{S\sqrt{\frac{Dh_2}{2A}}(1/m - 1/(nK)) + h_1\sqrt{\frac{DA}{2h_2}}\left[\left(1-\frac{D}{P}\right)\{(m-1)-K(n-1)\}+\frac{D}{p}(1-K)\right]}{P_2Di_1} \tag{11}$$

In fact, Eq. (11) implies that the implemented credit period policy has an upper bound, $T_{\max}$ which can be expressed as:

$$T_{Max} = \frac{S\sqrt{\frac{Dh_2}{2A}}(1/m - 1/(nK)) + h_1\sqrt{\frac{DA}{2h_2}}\left[\left(1-\frac{D}{P}\right)\{(m-1)-K(n-1)\}+\frac{D}{p}(1-K)\right]}{P_2Di_1} \tag{12}$$

4 Establishment and achievement of the coordination mechanism

From Section 3, we can see that in the traditional supply chain mode, each party makes decisions only from the viewpoint of their own optimal cost. In this section, further analysis will focus on whether we can find a strategy to decrease both sides' costs.

4.1 The optimal strategy under joint decision situation

The joint cost of the entire supply chain in this situation should equal to the sum of both parties' costs. Let $F_{total2}(n, KQ_0, T)$ denote the joint cost of the entire supply chain, we then have:

$$\begin{aligned} F_{total2}(n, KQ_0, T) &= F_{M2}(n, KQ_0, T) + F_{b2}(KQ_0, T) \\ &= \frac{D}{KQ_0}(A + S/n) + \frac{KQ_0}{2}\left(h_2 + h_1\left[(n-1)\left(1-\frac{D}{P}\right)+\frac{D}{P}\right]\right) + P_2DT(i_1 - i_2) \\ &= \frac{1}{K}\sqrt{\frac{Dh_2}{2A_2}}(A+S/n) + K\sqrt{\frac{DA}{2h_2}}\left(h_2 + h_1\left[(n-1)\left(1-\frac{D}{P}\right)+\frac{D}{P}\right]\right) + P_2DT(i_1 - i_2) \end{aligned} \tag{13}$$

Thus the problem can be formulated as in the following mathematical programming:

$$\begin{aligned} &Min\ F_{total2} \\ &S.t: \begin{cases} T_{\min} \leq T \leq T_{\max} & (a) \\ K \geq 0 & (b) \\ n \geq 0,\ and\ is\ an\ \text{int}eger & (c) \end{cases} \qquad \text{Problem A} \end{aligned}$$

By substituting equation Eq. (13) in problem A, F_{total2} is minimized to set optimal quantity T^* from the equation below:

$$\begin{cases} if\ i_1 \geq i_2 \Rightarrow\ T^* = T_{\min} \\ if\ i_1 \leq i_1\ \Rightarrow\ T^* = T_{\max} \end{cases}$$

Note that after calculating optimal quantity K^*, N^* for problem A, if constraint (a) is not satisfied, then the problem will not have a feasible solution. In other words, this coordination mechanism is not usable for every quantity of credit period.

Theorem 1. *In problem (A), the optimal quantity K, n will be the same under both conditions $T = T_{\min}$ or $T = T_{\max}$.*

Proof. See Appendix A.

$$\text{If} \quad T = T_{\min} \quad \text{then}$$

$$F_{total2}\left(n, KQ_0, T\right) = F_{M2}\left(n, KQ_0, T\right) + F_{b2}\left(KQ_0, T\right) = \frac{1}{K}\left(S\sqrt{Dh_2/2A}/n + i_1\sqrt{DAh_2/2}/i_2\right) + K\left(h_1\sqrt{DA/2h_2}\left[(n-1)\left(1-\frac{D}{P}\right)+\frac{D}{P}\right] + i_1\sqrt{DAh_2/2}/i_2\right) + (1 - i_1/i_2)\sqrt{2ADh_2}$$

For any given n, the derivative of F_{total2} with respect to K can be derived as:

$$\frac{\partial F_{total2}}{\partial K} = 0$$

$$\frac{-1}{K^2}\left(S\sqrt{Dh_2/2A}/n + i_1\sqrt{DAh_2/2}/i_2\right) + h_1\sqrt{DA/2h_2}\left[(n-1)\left(1-\frac{D}{P}\right)+\frac{D}{P}\right] + i_1\sqrt{DAh_2/2}/i_2 = 0$$

$$K^*(n^*) = \sqrt{\frac{S\sqrt{Dh_2/2A}/n + i_1\sqrt{DAh_2/2}/i_2}{h_1\sqrt{DA/2h_2}\left[(n-1)\left(1-\frac{D}{P}\right)+\frac{D}{P}\right] + i_1\sqrt{DAh_2/2}/i_2}} \tag{14}$$

Since the second derivative of F_{total2} with respect to K is

$$\frac{\partial^2 F_{total2}}{\partial K^2} = \frac{1}{K^3}\left(S\sqrt{Dh_2/2A}/n + i_1\sqrt{DAh_2/2}/i_2\right) > 0$$

Therefore, F_{total2} is convex of K for any given positive integer n. Thus, the optimal value of K is $K^*(n)$. After substituting Eq. (14) into Eq. (13), the manufacturer's annual total cost for any given n becomes:

$$F_{total2} = 2\sqrt{\left(S\sqrt{Dh_2/2A}/n + i_1\sqrt{DAh_2/2}/i_2\right)\left(h_1\sqrt{DA/2h_2}\left[(n-1)\left(1-\frac{D}{P}\right)+\frac{D}{P}\right] + i_1\sqrt{DAh_2/2}/i_2\right)} - \frac{i_1}{i_2}\sqrt{2DAh_2} \tag{15}$$

Thus, our problem is concluded to consist of the following optimization problem:

$$\min_{n\geq 0} \quad F_{total}$$

Now, we resolve Eq. (15), the optimal value of n for Eq. (13) can be determined as (see Appendix B for proof).

$$n^* = \begin{cases} \left\lceil \sqrt{\frac{S}{A\left(1-\frac{D}{p}\right)}\left(\frac{h_2}{h_1}-\frac{i_2}{i_1}\right)+\frac{1}{4}} - \frac{1}{2} \right\rceil & if\ \frac{S}{A\left(1-\frac{D}{p}\right)}\left(\frac{h_2}{h_1}-\frac{i_2}{i_1}\right) \geq 2 \\ 1 & otherwise \end{cases} \tag{16}$$

4.2 Designing the implementation strategy

The above section showed that using optimal quantity K^*, n^* and $T^* = T_{\min}$ or $T^* = T_{\max}$ conditions can optimize the joint cost of the entire supply chain; however, it is not enough to motivate both parties of the supply chain to coordinate only by this conclusion, since if the whole cost decrease is obtained by one party (under $T = T_{\min}$ and $T = T_{\max}$ conditions), the other party may refuse to coordinate. Thus, it is necessary to consider a suitable way to share the total cost decrease between the two parties. Coordination can be achieved only in the case where both parties can enjoy lower costs than in the case without coordination. Thus, the process to achieve coordination should be set along the following lines: first, calculate the optimal quantity K^*, n^* and T^* obtained in Section 3, and then design a dividing procedure to guarantee that both parties will get an appropriate share of the cost decrease.

Note that there are two optimal quantities, $T_{\min}$ and $T_{\max}$, for T^* when $i_1 = i_2$ that yield identical values of cost saving for the supply chain. Thus, depending on their power of bargaining, either party considers one of these two quantities. But optimal credit period quantity, under $i_1 > i_2$, is $T_{\min}$ and when $i_1 < i_2$, is $T_{\max}$.

Now we design a way to implement the sharing out of the reduced cost to make both the buyer and manufacturer satisfied. Assume that the buyer gains the fraction $\alpha(0 < \alpha < 1)$ of the decreased cost and the manufacturer the $(1 - \alpha)$ fraction.

Notice that when $T^* = T_{\min}$, the quantity $\alpha\%$ of the total reduced costs of sharing by manufacturer are paid from manufacturer to buyer and similarly when $T^* = T_{\max}$, the quantity $(1 - \alpha\%)$ of the total reduced costs of sharing by buyer are paid from buyer to manufacturer.

Finally, it should be pointed out that the value of provides a method of sharing out the increased profit between the two parties. In real business practice, it may be determined by the party with the stronger power in the supply chain.

5 Numerical example

The algorithm used for solving the problem is summarized as follows:

Step 1. Input the initial value D, P, P_1, S, A, $h_1^{'}$, $h_2^{'}$, i_1, i_2, α.

Step 2. Set $h_1 = h_1' + P_1 i_1$ and $h_2 = h_2' + P_2 i_2$

Step 3. Set m^* as in Eq. (4), and compute $F_{M_1}(M^*)$ as in Eq. (3)

Step 4. If $\frac{S}{A(1-D/P)}\left(\frac{h_2}{h_1} - \frac{i_2}{i_1}\right) \geq 2$,then $n^* = \left\lceil \sqrt{\frac{S}{A(1-D/P)}\left(\frac{h_2}{h_1} - \frac{i_2}{i_1}\right) + \frac{1}{4}} - \frac{1}{2} \right\rceil$; otherwise, set $n^* = 1$

Step 5. Compute $K^*(N^*)$ as in Eq. (14).

Step 6. Compute $T_{\min}$ as in Eq. (8) and $T_{\max}$ as in Eq. (12).

If $T_{\max} < T_{\min}$, then, $T^* = 0, K^* = 1, n^* = m^*$ and stop; otherwise, go to next step.

Step 7.

$$\begin{cases} if\ i_1 \geq i_2 \Rightarrow\ T^* = T_{\min} \\ if\ i_1 \leq i_1\ \Rightarrow\ T^* = T_{\max} \end{cases}$$

Step 8. Compute:

$$F_{b1}(Q_0), F_{M1}(m^*),\ F_{total1}(m^*, Q_0) = F_{b1}(Q_0) + F_{M1}(m^*),$$
$$F_{b2}\left(K^* Q_0, T^*\right), F_{M2}\left(n^*, K^* Q_0, T^*\right),\ F_{total2}\left(n^*, K^* Q_0, T^*\right) = F_{M2}\left(n^*, K^* Q_0, T^*\right) + F_{b2}\left(K^* Q_0, T^*\right)$$

Some numerical experiments have been carried out to illustrate the performance of our model. Applying the algorithm yields the results shown in Tab. 1. In this Table, columns of cost savings are defined as below:

$$\text{Savinga}^a(\%) = \alpha\left(F_{total1}(m^*, Q_0) - F_{total2}\left(n^*, K^*Q_0, T^*\right)\right)/F_{b1}(Q_0) \tag{17}$$

$$\text{Savinga}^b(\%) = (1-\alpha)\left(F_{total1}(m^*, Q_0) - F_{total2}\left(n^*, K^*Q_0, T^*\right)\right)/F_{M1}(m^*) \tag{18}$$

$$\text{Savinga}^c(\%) = \left(F_{total1}(m^*, Q_0) - F_{total2}\left(n^*, K^*Q_0, T^*\right)\right)/F_{total1}(m^*, Q_0) \tag{19}$$

The results from the examples presented in Tab. 1 indicate that there is a decrease in channel and individual costs for both the manufacturer and the buyer when the credit period is offered. These results also exhibit what our expectation that there are two optimal quantities, $T_{\min}$ and $T_{\max}$, for T^* when $i_1 = i_2$ that yield identical cost saving for the supply chain. But the optimal value for the credit period when $i_1 > i_2$ is $T_{\min}$ but $T_{\max}$ when $i_1 < i_2$.

6 Conclusion

This paper studied an integrated inventory control model consisting of a single manufacturer and a single buyer for a single product. Our objective was to minimize the total cost of the supply chain per unit time. This model used credit period policy to achieve the supply chain coordination. The range of credit period over which coordination between the two members can take place and the channel cost can be reduced. Our analyses showed that coordination and joint decision improve the performance of the entire supply chain. We calculated the reduced cost in this situation and designed an implementation method to divide it between the buyer and the manufacturer, so that both of them could decrease their own costs beyond those in the traditional business relationship.

The supply chain disruption management is a meaningful and interesting field. There are still many questions that need to be studied. The model can be extended to the case in which there are more competing entities than one at each level of the supply chain.

Table 1. Sample computational results when: $D = 1000$ units year^{-1}, $P = 6000$ units year^{-1}, $P1 = \$200\text{unit}^{-1}$, $P2 = \$500\text{unit}^{-1}$, $h_1^{'} = \$100\text{unit}^{-1}\text{year}^{-1}$, $h_2^{'} = \$150\text{unit}^{-1}\text{year}^{-1}$, $\alpha = 0.5$.

A_1	A_2	i_1	i_2	T^*	Savings in percentage under our proposed strategy		
					Buyer's savings^a	Manufacturer's savings^b	System savings^c
20000	5000	0.2	0.05	0.010	1.672	1.194	1.393
20000	5000	0.05	0.05	0.079 or 0.133	1.625	1.307	1.449
20000	5000	0.05	0.2	0.771	29.550	27.617	28.551
20000	5500	0.2	0.05	0.008	1.223	0.934	1.060
20000	5500	0.05	0.05	0.066 or 0.136	1.996	1.688	1.829
20000	5500	0.05	0.2	0.751	27.683	27.426	27.554
20000	6000	0.2	0.05	0.006	0.899	0.729	0.805
20000	6000	0.05	0.05	0.056 or 0.144	2.390	2.112	2.242
20000	6000	0.05	0.2	0.737	26.500	27.636	27.056

Appendix A

If $T = T_{\min}$, then

$$\begin{aligned}\underset{K,n}{Min}\ F_{total2}(T_{\min}) = K\left(h_1\sqrt{DA/2h_2}\left[(n-1)\left(1-\tfrac{D}{P}\right)+\tfrac{D}{P}\right]+i_1\sqrt{DAh_2/2}/i_2\right) \\ +\tfrac{1}{K}\left(S\sqrt{Dh_2/2A}/n+i_1\sqrt{DAh_2/2}/i_2\right)+(1-i_1/i_2)\sqrt{2ADh_2}\end{aligned}$$

$$\cong \underset{K,n}{Min}\ (i_1/i_2)\left[K\left(h_1 i_2\sqrt{DA/2h_2}\left[(n-1)\left(1-\tfrac{D}{P}\right)+\tfrac{D}{P}\right]/i_1+\sqrt{DAh_2/2}\right)\right.$$
$$\left.+\tfrac{1}{K}\left(Si_2\sqrt{Dh_2/2A}/(ni_1)+\sqrt{DAh_2/2}\right)\right]$$
$$\cong \underset{K,n}{Min}\ \ K\left(h_1 i_2\sqrt{DA/2h_2}\left[(n-1)\left(1-\tfrac{D}{P}\right)+\tfrac{D}{P}\right]/i_1+\sqrt{DAh_2/2}\right)$$
$$+\tfrac{1}{K}\left(Si_2\sqrt{Dh_2/2A}/(ni_1)+\sqrt{DAh_2/2}\right)$$
$$+(1-i_2/i_1)\left[\tfrac{SD}{mQ_0}+h_1\tfrac{Q_0}{2}\left[(m-1)\left(1-\tfrac{D}{P}\right)+\tfrac{D}{P}\right]\right]=\underset{K,n}{Min}\,F_{total2}(T_{\max})$$

Appendix B

Let n^* be the optimal solution of Eq. (13), then

$$\begin{cases} F_{total2}(n^*)-F_{total2}(n^*-1)\le 0 \\ F_{total2}(n^*)-F_{total2}(n^*+1)\le 0 \end{cases}$$

Since:

$$F_{total2}(n^*)-F_{total2}(n^*-1)\le 0$$

$$2\sqrt{\left(S\sqrt{Dh_2/2A}/n+i_1\sqrt{DAh_2/2}/i_2\right)\left(h_1\sqrt{DA/2h_2}\left[(n-1)\left(1-\tfrac{D}{P}\right)+\tfrac{D}{P}\right]+i_1\sqrt{DAh_2/2}/i_2\right)}\le$$
$$2\sqrt{\left(S\sqrt{Dh_2/2A}/(n-1)+i_1\sqrt{DAh_2/2}/i_2\right)\left(h_1\sqrt{DA/2h_2}\left[(n-2)\left(1-\tfrac{D}{P}\right)+\tfrac{D}{P}\right]+i_1\sqrt{DAh_2/2}/i_2\right)}$$

By simple computation, we have

$$(n-\frac{1}{2})^2\le\frac{S}{A\left(1-\frac{D}{p}\right)}\left(\frac{h_2}{h_1}-\frac{i_2}{i_1}\right)+\frac{1}{4} \tag{20}$$

Similarly, by

$$(n+\frac{1}{2})^2\le\frac{S}{A\left(1-\frac{D}{p}\right)}\left(\frac{h_2}{h_1}-\frac{i_2}{i_1}\right)+\frac{1}{4} \tag{21}$$

Therefore, we can obtain n^* by Eq. (20) and Eq. (21),

$$n^*=\begin{cases}\left[\sqrt{\frac{S}{A\left(1-\frac{D}{p}\right)}\left(\frac{h_2}{h_1}-\frac{i_2}{i_1}\right)+\frac{1}{4}}-\frac{1}{2}\right] & if\ \frac{S}{A\left(1-\frac{D}{p}\right)}\left(\frac{h_2}{h_1}-\frac{i_2}{i_1}\right)\ge 2\\ 1 & otherwise.\end{cases}$$

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