

An approximate condition for supply chain coordination with risk-averse agents

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Abstract. Numerous scholars investigate the supply chain coordination, but most of them assume the agents are risk-neutral. We apply utility function to character risk-averse agents, give a definition of coordination, then show an approximate condition of coordination with this definition. After that, we apply the approximate condition in three special utility functions to show an easy way to get the coordination solution. We also design a contract under the coordination condition; show two classes of utility functions that suit for our contract. In addition, we analyze the properties of parameters in the model, which provides a better understand of coordination condition and contracts.

Keywords: supply chain coordination, risk-averse, align, expected utility

1 Introduction

In the last two decades, academicians have shown much interest on supply chain management. Numerous areas are studied, such as pricing policy, inventory strategy, coordination contracts, bullwhip effect, *et al.* In this paper, we focus on the coordination contracts. Although a completely integrated solution may result in optimal system performance, it is not always in the best interest of every individual member. Usually, the individual member is much keen in optimizing their individual objectives. In this way, double marginalization^[14] takes place and affects the performance of supply chain system. In order to minimize the double marginalization, the agents usually recur to coordination contracts. These contracts align the objectives of individual supply chain members and inspire them to improve the efficiency of supply chain.

Many supply chain coordination contracts have been developed in the literature. Such as buyback contract^[12], quantity discounts contract^[17], quantity flexibility contract^[7], revenue sharing contract^[2], sales rebate contract^[15]. All of these contracts can align the objectives of individual parties with that of supply chain system. In particular, they shift the risk and the revenue between the members. These contracts and other possible contracts usually use in supply chain system with risk-neutral agents. But in the real business world, the active agents usually show conservative character when they face risk. So it may be more reasonable if the agents are considered as risk-averse. Fortunately, quite a few scholars consider this situation, and our paper also try to investigate it.

In economics and finance, the individual party is usually considered as risk-averse. Literature about these two areas proposes some useful methods to reflect the risk-averse character. These methods include: mean-variance, utility function, value at risk, condition value at risk, downside risk constraint *et al.* Among them, the mean-variance and utility function are two important and popular approaches to deal with risk concerns. In recent years, these two approaches have been applied in supply chain model. Such as Lau and Lau^[11], Gan, Sethi and Yan^[8], Choi *et al.*^[5]Choi, Li, Yan^[4]. Because the mean-variance penalizes both the upside and

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downside deviations from the mean and only works best when the agents' profit are normally distributed. So in this paper, we apply utility function approach to construct supply chain model and propose an approximate condition to make the supply chain coordination.

In the reminder of this paper, we first give a literature review about newsvendor and supply chain with risk-averse agents. Then we show the approximate condition and prove it. After that, we apply the approximate condition in three commonly used utility functions. Next, we design a contract with the approximate condition, investigate the supply chain model under this contract. Finally, we summarize the conclusion and have a discussion for the future research.

2 Literature review

In this section, we first give a brief review about newsvendor model then focus on supply chain management, both of them with the risk-averse individual parties.

Although newsvendor problem has existed for several tens of years, scholars consider the individual party's risk-averse character till the eighties of 19 century. Lau^[10] uses two new objectives instead of the maximizing the expected profit. One objective is maximizing expected utility; the other objective is maximizing the probability of achieving a budgeted profit. Then compare the optimal order quantity with the one in the original newsvendor model. Eeckhoudt *et al*^[6] assume that the risk-averse newsvendor has a second order chance if the demand exceeds his first order. After analysis, they show that the optimal order quantity is less than the optimal risk-neutral order quantity and it is a decrease function of the risk aversion. Chen and Federgruen^[3] revisit several basic inventory models and show that a mean-variance tradeoff analysis can be carried out efficiently they also compare their model with the standard treatment of inventory problem and show the difference. Agrawal and Seshadri^[1] investigate a newsvendor model with the assumption that the demand function depends on selling price. They consider the expected utility maximization instead of expected profit and show the impact of uncertainty and risk aversion on price and order quantity in newsboy problem.

Comparing with newsvendor problem, supply chain system includes two individual parties at least, and they usually have their own objectives, these make supply chain problem always more difficult than newsvendor problem. Next, we have a review about the supply chain management with risk-averse agents. Lau and Lau^[11] model the problem with mean-variance function, then use numerical examples to analysis the pricing strategy and return policy. They show that the return-credits can benefit the manufacture better than the retailer without special condition. Tsay^[16] analyzes the impact of risk on manufacturer and retailer under various scenarios of strategic power. He shows that the penalty for ignoring risk sensitivity can be substantial. Gan *et al* [8] apply group decision theory to definite the coordination of supply chain with risk-neutral or risk-averse agents. They develop coordinating contracts in three special cases, give the method to find the pareto-optimal solutions. Gan *et al*^[9] analyze the supply chain formed by a risk-neutral supplier and a risk-averse retailer. After demonstrating three common contracts (wholesale, buyback and revenue sharing) could not coordinate supply chain channel, they design a risk-sharing contract to coordinate the channel. The effect of this contract is offering the downside protection to the retailer. Shi and Chen^[13] analyze a decentralized supply chain model, both the supplier's and the retailer's objectives are to maximize the probability of achieving a predetermined target profit. They investigate the model with linear tariff contracts and buyback contracts, show that only the wholesale contracts can coordinate the supply chain channel. Choi, *et al*^[5] propose the mean-variance to capture the risk preference of each individual agent in supply chain. After investigating the wholesale policy, they show that a lightly risk-averse supply chain coordinator can coordinate with a slightly risk-prone retailer but not a very risk-averse retailer. Choi, Li, Yan^[4] carry out a mean-variance analysis of supply chain, illustrate how a return policy affects the channel coordination and risk control in both the centralized and decentralized supply chain.

3 An approximate condition

All of us know that in a decentralized supply chain system, individual agents maximize their own objectives. Double marginalization takes place, decreasing the supply chain's efficiency. To improve supply chain's

performance, the agents usually recur to coordination contracts. But just as Gan *et al*^[8] said that we could not always get the explicit coordination solutions of supply chain with risk-averse agents in general situation. So in this section, we will show an approximate condition to coordinate the supply chain with risk-averse agents.

We consider a decentralized supply chain system consisting of one supplier and one retailer. Their utility functions are $U_s(x)$ and $U_r(x)$ respectively, both of them are increasing and concave functions. In our model, market demand is the only random variable. So we can describe the profit of supplier and retailer as a fixed-profit and a risk-profit. The fixed-profit isn't affected by the market demand; the risk-profit is the function of market demand. We set $\pi_s = m_s + v_s$, $\pi_r = m_r + v_r$ (the letters π , m and v denote total profit, fixed-profit and risk-profit respectively, the subscripts s and r denote supplier and retailer respectively). In this paper, we only consider the situation that m_r is a linear function of m_s . This is reasonable while the market demand is the only random variance.

There is a participation constraint of each agent, a contract is feasible only when all the agents' participation constraints are satisfied, which means the payoffs of each agent with this contract is larger than the reservation level. For simplicity, the reservation level is set at zero. So in the remainder of this paper, we ignore the participation constraint. We know that in a decentralized supply chain system, individual parties have their own objectives. It makes the double marginalization take place, and the double marginalization is the key drawback affecting the channel coordination. So if we can align the individual parties' objectives, we can coordinate the channel. Before investigating how to coordinate the supply chain, we give the follow definition.

Definition 1. *Supply chain coordination. A contract is said to coordinate the supply chain if all the agents' optimal actions under this contract satisfy the follow condition:*

1. All the agents' participation constraints are satisfied.
2. The optimal actions of each agent will align their objectives with the supply chain system's objective.

By comparing with the definition of Gan *et al* [8], we find that theirs recurs to group decision theory, gives a precise mathematics description, Ours describes the coordination in the sense of operation management. Using our definition, we will get an approximate coordination condition of supply chain with risk-averse agents.

Proposition 1. *In a decentralized supply chain system, if an action pair of the supplier and the retailer leads to their profits satisfy:*

$$(a) R_s(m_s)v_s = R_r(m_r)v_r, \quad \left(R_s(x) = -\frac{U_s''(x)}{U_s'(x)}, R_r(x) = -\frac{U_r''(x)}{U_r'(x)} \right) \quad (1)$$

$$(b) \frac{U_r'(m_r)}{U_s'(m_s)} = ct, \quad (ct \text{ is a constant}) \quad (2)$$

Then this action pair coordinates the supply chain.

Proof. While

$$v_s \times R_r(m_r) \neq 0, \quad \text{we set } \frac{v_r}{v_s} = \frac{R_s(m_s)}{R_r(m_r)} = \beta \quad (\beta \geq 0). \quad (3)$$

Then we can get

$$\beta U_s'(m_s)U_r''(m_r) = U_r'(m_r)U_s''(m_s) \quad (4)$$

Rewrite equation (2), we get that

$$U_r'(m_r) = ct \times U_s'(m_s) \quad (5)$$

Integrating both sides with respect to m_s , get

$$U_r'(m_r) = ct \times \frac{dm_r}{dm_s} U_s(m_s) + ct_0 \quad (6)$$

where ct_0 is a constant.

Taking derivative both sides of equation (2) with respect to m_s , we get

$$\frac{dm_r}{dm_s} U'_s(m_s) U''_r(m_r) - U'_r(m_r) U''_s(m_s) = 0 \quad (7)$$

Combining with equation (4), we get $\beta - \frac{dm_r}{dm_s} = 0$, so $m_r = \beta m_s - T$ (T is a constant). We denote $E(x)$ as the expectation of x and perform a Taylor-series expansion of the supplier's and the retailer's expected utility functions around m_s and m_r respectively.

$$EU_s(m_s + v_s) = U_s(m_s) + EU'_s(m_s)v_s + EU''_s(m_s)v_s^2/2 + EU'''_s(m_s)v_s^3/6 + \dots \quad (8)$$

$$EU_r(m_r + v_r) = U_r(m_r) + EU'_r(m_r)v_r + EU''_r(m_r)v_r^2/2 + EU'''_r(m_r)v_r^3/6 + \dots \quad (9)$$

It is common practice to truncate this function to make estimation tractable. Restricting the utility function to depend on only the first and second order terms, we can write an approximation for the expected utility function as follows:

$$EU_s(m_s + v_s) \approx U_s(m_s) + EU'_s(m_s)v_s + EU''_s(m_s)v_s^2/2 \quad (10)$$

$$EU_r(m_r + v_r) \approx U_r(m_r) + EU'_r(m_r)v_r + EU''_r(m_r)v_r^2/2 \quad (11)$$

Substituting equations (3) ~ (6) into the expectation utility function of supplier, we get

$$\begin{aligned} EU_s(m_s + v_s) &\approx U_s(m_s) + EU'_s(m_s)v_s + EU''_s(m_s)v_s^2/2 \\ &= \frac{U_r(m_r) - ct_0}{\beta \times ct} + \frac{U'_r(m_r)E v_r}{\beta \times ct} + \frac{U''_r(m_r)E v_r^2}{\beta \times ct} \\ &= \frac{EU_r(m_r + v_r)}{\beta \times ct} - \frac{ct_0}{\beta \times ct} \end{aligned} \quad (12)$$

So the total utility of the supply chain system is

$$\begin{aligned} \lambda_s EU_s(\pi_s) + \lambda_r EU_r(\pi_r) &\approx \lambda_s EU_s(\pi_s) + \lambda_r \beta \times ct \times EU_s(\pi_s) + \lambda_r \times ct_0 \\ &= (\lambda_s + \lambda_r \beta \times ct) EU_s(\pi_s) + \lambda_r \times ct_0 \end{aligned} \quad (13)$$

where $\lambda_s + \lambda_r = 1$, $\lambda_s \geq 0$, $\lambda_r \geq 0$ According to our definition, the supply chain is coordination.

From proposition 1, we get the equation $R_s(m_s)v_s = R_r(m_r)v_r$ that shows the relation between the absolute risk-averse measure and the risk-profit: The higher of the agent's absolute risk-averse measure, the lower of risk-profit he gets. For describing this relation clearly, we consider two extreme situations with only one risk-averse agent. While the retailer is risk-averse: from equality $R_s(m_s)v_s = R_r(m_r)v_r$, we know that $v_r = 0$, this means that the risk-neutral supplier bears all the risk caused by the demand uncertain. The retailer gets fixed-profit without risk. While the supplier is risk-averse: $v_s = 0$, the risk-neutral retailer faces all the risk, the supplier gets fixed-profit.

If we investigate the equation $R_s(m_s)v_s = R_r(m_r)v_r$ again, rewrite it as $\frac{v_r}{v_s} = \frac{R_s(m_s)}{R_r(m_r)} = \beta$, we will find that: while the supplier and the retailer are Constant Absolute Risk Averse(CARA), the ratio of the supplier's risk-profit to the retailer's risk-profit is constant, it is independent on the fixed-profit m_r , m_s . The value of β is affected by the absolute risk measure of supplier and retailer.

Because in the proof of proposition 1, we only use the first and second order terms of Taylor-series expansion, ignore the higher order terms. In this case, the error usually takes place. That is why we call our coordination condition an approximate condition. However, the error will disappear while the agents have special shapes of utility function. In next section we will show three kinds of utility functions without error.

4 Special shapes of utility function

Best to our knowledge, there is not literature that shows a contract to coordinate the supply chain with risk-averse agents adopting general utility functions. In last section, we give an approximate condition that coordinates the supply chain. In this section, we will investigate three commonly used utility functions, demonstrate the method to solve the coordinate problem with the help of approximate condition.

4.1 Quadratic function

Because we truncate terms of degree higher than 2 while we use Taylor-series expansion to prove the proposition 1. We can easily find that a quadratic utility function will satisfy the proof without error. Now, we will show this situation and give the optimal solution. We assume that the supplier's and the retailer's utility function are $U_s(x) = a_s x^2 + x$ and $U_r(x) = a_r x^2 + x$ respectively. Because both the supplier and the retailer are risk-averse, so $a_s < 0, a_r < 0$. Using the condition of the proposition 1, we can get that: $T = 1/(2a_r) - \beta/(2a_s)m_r = \beta m_s - T v_r = \beta v_s$. Then we substitute them into the total utility of the supply chain system.

$$\begin{aligned} & \max [\lambda_s EU_s(\pi_s) + \lambda_r EU_r(\pi_r)] \\ & \text{s.t. } \pi_s + \pi_r = \pi \end{aligned} \quad (14)$$

We know that the problem of maximizing the expected quadratic utility can be reduced to one of maximizing a mean-variance model, so we assume $\lambda_s = \lambda_r$. This assumption makes it easy to compare with the mean-variance case of Gan *et al*^[8]. Simplifying expression (14), we find that the optimal value of satisfies

$$\beta^* = \arg \max \left[\frac{a_s}{(1+\beta)^2} + \frac{\beta^2 a_r}{(1+\beta)^2} \right] \quad (15)$$

Then, we can get

$$\beta^* = a_s/a_r. \quad T = 1/(2a_r) - \beta(2a_s) = 0.$$

So

$$\pi_s = \frac{a_r}{a_s + a_r} \pi, \quad \pi_r = \frac{a_s}{a_s + a_r} \pi.$$

Comparing with the mean-variance case of Gan *et al* [8], we find that the difference is the side payment, in the quadratic utility case, side payment is zero. This difference is caused by the reason: in the quadratic utility case, we assume the agents' reservation payoffs are zero; in mean-variance case, the side payment exists to make the agents' payoffs satisfy the reservation level.

4.2 Exponential function

In this subsection, we consider the situation that the supplier's and the retailer's utility functions are exponential functions. Set $U_s(x) = 1 - e^{-\rho_s x}$, $U_r(x) = 1 - e^{-\rho_r x}$, so the absolute risk-averse measures of the supplier and the retailer are ρ_s and ρ_r respectively. Set $\pi_r + \pi_s = \pi$. By using the proposition 1, we can get:

$$\begin{aligned} \beta &= \rho_s/\rho_r, \\ ct &= U_r'(m_r)/U_s'(m_s) = \exp(\rho_r T)/\beta, \\ ct_0 &= U_r(m_r) - \exp(\rho_r T)U_s(m_s) = 1 - \exp(\rho_r T). \end{aligned}$$

Then we solve the optimal objective functions:

$$\begin{cases} \max EU_r(\pi_r) = \max \left[1 - \exp\left(\frac{\rho_r T}{1+\beta}\right) E \exp\left[\frac{-\beta \rho_r \pi}{1+\beta}\right] \right] \\ \max EU_s(\pi_s) = \max \left[1 - \exp\left(\frac{\rho_s T}{1+\beta}\right) E \exp\left[\frac{-\beta \rho_s \pi}{1+\beta}\right] \right] \end{cases} \quad (16)$$

Set

$$Y(T) = \lambda_r \exp\left(\frac{\rho_r T}{1+\beta}\right) + \lambda_s \exp\left(\frac{\rho_s T}{1+\beta}\right),$$

the optimal solution of $\min Y(T)$ is

$$T^* = \frac{1 + \beta}{\rho_s + \rho_r} \ln \frac{\lambda_s \rho_s}{\lambda_r \rho_r}.$$

Combining with the equality $\pi_r = \beta \pi_s - T$, we get the supplier's and the retailer's profit:

$$\pi_s = \frac{\rho_r}{\rho_r + \rho_s} \pi + \frac{1}{\rho_r + \rho_s} \ln \frac{\lambda_s \rho_s}{\lambda_r \rho_r}, \quad \pi_r = \frac{\rho_s}{\rho_r + \rho_s} \pi - \frac{1}{\rho_r + \rho_s} \ln \frac{\lambda_s \rho_s}{\lambda_r \rho_r}$$

This solution is consistent with the theorem 5.1 of Gan *et al*(2004).

4.3 Power function

Now, we consider the third kind of utility function-power function, assume that the supplier and the retailer have the same utility function: $U_r(x) = U_s(x) = \frac{x^g}{g}$, where $0 < g < 1$. Using the condition of proposition 1, we get that $T = 0$, so $\pi_r = \beta \pi_s$. The objectives of the individual agent in the decentralized supply chain are given by

$$\begin{cases} \max EU_s(\pi_s) = \max E \left(\frac{\pi_s^g}{g} \right) = \max \frac{1}{g(1+\beta)^g} E \pi^g \\ \max EU_r(\pi_r) = \max E \left(\frac{\pi_r^g}{g} \right) = \max \frac{\beta^g}{g(1+\beta)^g} E \pi^g \end{cases} \quad (17)$$

Set

$$Y(\beta) = \frac{\lambda_s}{g(1+\beta)^g} + \frac{\lambda_r \beta^g}{g(1+\beta)^g},$$

we solve the objective function "max $Y(\beta)$ ", get the optimal solution $\beta^* = \left(\frac{\lambda_s}{\lambda_r} \right)^g$.

In this section, we consider three different kinds of utility functions, show how to apply the proposition 1 to solve the coordination problem. Although with the approximate condition of proposition 1, we can not get the precise solution always, it does show an easy understanding and calculation method to get an approximate solution. What worth mentioning is the error disappears sometimes. At the background that we can not get a precise coordination solution in supply chain with risk-averse agents while the agents have general utility functions, proposition 1 shows an easy way to coordinate the supply chain.

5 Design a contract

In this section we design a coordination contract under the condition of proposition 1. We consider a two echelon supply chain system consisting of one supplier and one retailer, both of them are risk-averse. We assume the supplier's manufacture cost is c , sell it to the retailer at a wholesale price w . The retailer has only one chance to order, his order quantity is q . Then he sales the product to the market at a fixed price p , the product remains at the end of selling season can return to the supplier at a price b . The marker demand is uncertain, $f(x)$ is the density function of demand. $F(x)$ is the cumulative distribution function of demand. In this contract, we set

$$w = w_0 + kt/q \quad (w_0 = k(p - c) + c, \quad 0 < k < 1, \quad t \geq 0), \quad b = kp.$$

So the profit of supplier and retailer are given by:

$$\begin{aligned} \pi_s &= (w - c)q - b(q - D)^+ \\ &= k(p - c)q + kt - kp(q - D)^+ \\ &= m_s + v_s \end{aligned} \quad (18)$$

$$\begin{aligned}
 \pi_r &= (p - w)q - (p - b)(q - D)^+ \\
 &= (1 - k)(p - c)q - kt - (1 - k)p(q - D)^+ \\
 &= m_r + v_r \\
 &= \frac{1 - k}{k}m_s - t + \frac{1 - k}{k}v_s \\
 &= \beta m_s + \beta v_s - t
 \end{aligned}
 \tag{19}$$

Where $\beta = \frac{1-k}{k}$, $m_s = k(p-c)q+kt$, $v_s = -kp(q-D)^+$, $m_r = (1-k)(p-c)q-kt$, $v_r = -(1-k)p(q-D)^+$. While $t = 0$, this contract degenerates as a buyback contract which induces the similar result with revenue sharing contract. To get more details, you can refer to Cachon and Lariviere^[2]. The expected utility functions of the supplier and the retailer are given by, respectively.

$$EU_s = \int_0^q U_s[k(p - c)q + kt - kq(q - x)]f(x)dx + \int_q^\infty U_s[k(p - c)q + kt]f(x)dx \tag{20}$$

$$EU_r = \int_0^q U_r[k(p - c)q + kt - kq(q - x)]f(x)dx + \int_q^\infty U_r[k(p - c)q + kt]f(x)dx \tag{21}$$

To simplify the equations, we set

$$\begin{aligned}
 \pi_{s-} &= k(p - c)q + kt - kp(q - x), \\
 \pi_{s+} &= k(p - c)q + kt, \\
 \pi_{r-} &= (1 - k)[(p - c)q - p(q - x)] - kt, \\
 \pi_{r+} &= (1 - k)(p - c)q - kt.
 \end{aligned}$$

The first order derivative of EU_r with respect to q is given by

$$\begin{aligned}
 \frac{\partial EU_r}{\partial q} &= [(1 - k)(p - c) - (1 - k)p] \int_0^q U'_r(\pi_{r+})f(x)dx + (1 - k)(p - c) \int_q^\infty U'_r(\pi_{r+})f(x)dx \\
 &= (1 - k)(p - c) \left[(1 - \xi) \int_0^q U'_r(\pi_{r-})f(x)dx + \int_q^\infty U'_r(\pi_{r+})f(x)dx \right]
 \end{aligned}
 \tag{22}$$

Where $\xi = \frac{p}{p-c}$. After simply calculation, we can easy know that expected utility function of retailer is a concave function with respect to his order quantity. So we can use the first order condition to get the optimal order quantity. The optimal order quantity q_r^* will satisfy

$$\xi = 1 + \int_{q_r^*}^\infty U'_r(\pi_{r+})f(x)dx / \int_0^{q_r^*} U'_r(\pi_{r-})f(x)dx;$$

if the retailer is risk-neutral, the optimal order quantity q_n^* satisfies $\xi = 1 + \frac{1-F(q_n^*)}{F(q_n^*)}$. Because the utility function of risk-averse agent satisfies $U''_r(x) < 0$, we can get $\xi < 1 + \frac{1-F(q_n^*)}{F(q_n^*)}$ so $q_n^* > q_r^*$. This is a general result when we deal with inventory model with risk-averse agent.

Proposition 2. *With this contract we design above, while k and t are given. $q_r^*(t)$ arg max $EU_r(\pi_r)$ is the coordination solution when the supplier's and the retailer's utility function satisfy the condition:*

$$\frac{U'_r(\beta x - t)}{U'_s(x)} = ct_1 \quad (ct_1 \text{ is a constant}) \tag{23}$$

Proof. Because $q_r^*(t)$ arg max $EU_r(\pi_r)$, so it satisfies the first order condition of retailer's expected utility function:

$$(1 - \xi) \int_0^{q_r^*} U'_r(\pi_{r-})f(x)dx + \int_{q_r^*}^\infty U'_r(\pi_{r+})f(x)dx = 0.$$

The derivative of the supplier’s expected utility with respect to is given by

$$\begin{aligned}
 \frac{\partial EU_s}{\partial q_r^*} &= [k(p - c) - kp] \int_0^{q_r^*} U'_s(\pi_{s-})f(x)dx + k(p - c) \int_{q_r^*}^{\infty} U'_s(\pi_{s+})f(x)dx \\
 &= k(p - c) \left[(1 - \xi) \int_0^{q_r^*} U'_s(\pi_{s-})f(x)dx + \int_{q_r^*}^{\infty} U'_s(\pi_{s+})f(x)dx \right] \\
 &= k(p - c) \times ct_1 \times \left[(1 - \xi) \int_0^{q_r^*} U'_r(\pi_{r-})f(x)dx + \int_{q_r^*}^{\infty} U'_r(\pi_{r+})f(x)dx \right] \\
 &= 0
 \end{aligned}
 \tag{24}$$

Because $\frac{\partial^2 EU_s}{\partial (q_r^*)^2} < 0$, this means that the expected utility function of supplier is a concave function with respect to retailer’s order quantity. From the equation (24), we know $q_r^*(t) \arg \max EU_s(\pi_s)$. So we can get the result that the supply chain channel’s objective $\lambda_r EU_r(\pi_r) + \lambda_s EU_s(\pi_s)$ is a concave function with respect to retailer’s order quantity. That means $q_r^*(t) \arg \max [\lambda_r EU_r(\pi_r) + \lambda_s EU_s(\pi_s)]$, so $q_r^*(t)$ is the coordination solution of supply chain.

In this proposition, we assume the condition $\frac{U'_r(\beta x - t)}{U'_s(x)} = ct_1$ is satisfied. Comparing with the condition of proposition 1, this condition is more rigorous, but we can find that two classes of utility functions will satisfy the rigorous condition besides they satisfy the condition of proposition 1. The first class of utility function is the exponential function; the second class is that the utility functions of retailer and supplier have a linear relation. This means that our contract can coordinate supply chain with these two classes of utility functions without error.

If the condition of proposition 2 is satisfied, we can easily get the optimal value of t that maximizes the supply chain’s expected utility.

Proposition 3. *The supply chain’s expected utility function is a concave function with respect to t , and the optimal value of t satisfies*

$$\frac{\partial [\lambda_s EU(\pi_s) + \lambda_r EU(\pi_r)]}{\partial t} = 0$$

Proof. The proof is easy and omitted.

Proposition 4. *If $q_r^*(t) \arg \max EU_r(\pi_r)$, we can get the relation of $q_r^*(t)$ and t as follows:*

(1) if $R'_r(x) = 0$, then $\frac{\partial q_r^*(t)}{\partial t} = 0$; (2) if $R'_r(x) > 0$, then $\frac{\partial q_r^*(t)}{\partial t} > 0$; (3) if $R'_r(x) < 0$, then $\frac{\partial q_r^*(t)}{\partial t} < 0$

Proof. Set

$$\varphi(q_r^*, t) = (1 - \xi) \int_0^{q_r^*} U'_r[\pi_{r-}]f(x)dx + \int_{q_r^*}^{\infty} U'_r[\pi_{r+}]f(x)dx.
 \tag{25}$$

Applying the implicit function theorem, we can get

$$\frac{\partial q_r^*(t)}{\partial t} = - \frac{\partial \varphi(q_r^*, t)}{\partial t} / \frac{\partial \varphi(q_r^*, t)}{\partial q_r^*}
 \tag{26}$$

Because $EU_r(\pi_r)$ is a concave function with respect to q_r , so $\frac{\partial \varphi(q_r^*, t)}{\partial q_r^*} < 0$

$$\begin{aligned}
 \frac{\partial \varphi(q_r^*, t)}{\partial t} &= -k \left[(1 - \xi) \int_0^{q_r^*} U''_r(\pi_{r-})f(x)dx + \int_{q_r^*}^{\infty} U''_r(\pi_{r+})f(x)dx \right] \\
 &= k \left[(1 - \xi) \int_0^{q_r^*} R_r(\pi_{r-})U'_r(\pi_{r-})f(x)dx + \int_{q_r^*}^{\infty} R_r(\pi_{r+})U'_r(\pi_{r+})f(x)dx \right]
 \end{aligned}
 \tag{27}$$

We know $\pi_{r-} \leq \pi_{r+}$, while $R'_r(x) = 0$, $R_r(\pi_{r-}) = R_r(\pi_{r+})$ apply the integral median theorem, we get $\frac{\partial \varphi(q_r^*, t)}{\partial t} = 0$, so $\frac{\partial q_r^*(t)}{\partial t} = 0$, if $R'_r(x) > 0$, then $R_r(\pi_{r-}) \leq R_r(\pi_{r+})$, so $\frac{\partial \varphi(q_r^*, t)}{\partial t} > 0$, $\frac{\partial q_r^*(t)}{\partial t} > 0$; if $R'_r(x) < 0$, then $R_r(\pi_{r-}) \geq R_r(\pi_{r+})$, so $\frac{\partial \varphi(q_r^*, t)}{\partial t} < 0$, $\frac{\partial q_r^*(t)}{\partial t} < 0$.

From proposition 4, we know that when the utility function of retailer is CARA, the retailer's optimal order quantity is independent of t . If the utility function is Increasing Absolute Risk Aversion (IARA), the retailer's optimal order quantity increases as t increases. If the utility function is Decreasing Absolute Risk Aversion (DARA), the retailer's optimal order quantity decreases as t increases. Usually, we consider that the optimal order quantity decreases as t increases, because a lower t makes a lower wholesale price, a lower wholesale price will inspire the retailer to order more. But the real relation between them is more complex. The proposition 4 help us better understand the effects of absolute risk averse measure on the parameters of the supply chain model.

In proposition 2, we set that k and t are fixed. After that we investigate the effects of t on the supply chain's expected utility and optimal order quantity. Now, we will analyze the effects of k on retailer's expected utility.

The derivative of the retailer's expected utility by k is given as:

$$\begin{aligned} \frac{\partial EU_r(k, q_r^*)}{\partial k} &= - \left\{ (p\varepsilon + t - cq_r^*) \int_0^{q_r^*} U'_r(\pi_{r-}) f(x) dx + [(p - c)q_r^* + t] \int_{q_r^*}^{\infty} U'_r(\pi_{r+}) f(x) dx \right\} \\ &< -q_r^* \left[-c \int_0^{q_r^*} U'_r(\pi_{r-}) f(x) dx + (p - c) \int_{q_r^*}^{\infty} U'_r(\pi_{r+}) f(x) dx \right] \\ &= 0 \end{aligned} \quad (28)$$

where $q_r^* > \varepsilon > 0$.

The retailer's expected utility decreases as k increases. This confirms the conclusion that: there is no such thing as something for nothing. When the retailer's expected utility is high, the value of k is small. From $\beta = \frac{1-k}{k}$, we know the value of β is high. So the retailer's risk-profit is high, that means the retailer bears a large share of risk.

6 Conclusion

In this paper we give a definition of supply chain coordination, and then show an approximate condition that could coordinate the supply chain under the frame of our definition. After that we apply the approximate condition in three commonly used utility functions. We found that the proposition 1 performs perfectly in these three cases. It shows an easy way to get the coordination solution of supply chain. Subsequently, we design a contract satisfying the condition of proposition 1. With this contract, we investigate the relation among the coordination solution, the parameters value and the expected utility. We find that: it is able to get the coordination solution when the agents' utility functions are exponential function or there is a linear relation between them; the relation between the parameter and the retailer's optimal order quantity q_r^* is affected by the character of retailer's absolute risk averse measure; the retailer's expected utility is an increase function of β . Although our coordinate condition is not perfect all the time, it does show the intrinsic relation of the action policies. Through splitting the profit into a fixed-profit and a risk-profit, we can explain more clearly how the risk-averse agents make decisions, and how their decisions affect the profit split.

The supply chain coordination with risk-averse agents is more complex than the risk-neutral problem. Although we give an approximate condition, there are many places to be expanded. First, we can analyze a supply chain model with multi-agent, it is more close to the business world. Second, we can investigate the approximate condition in other shapes of utility function besides the three kinds we analysis in this paper. Third, we can compare our results with the ones gotten from supply chain model using other methods to character the risk-averse agents.

References

- [1] V. Agrawal, S. Seshadri. Impact of uncertainty and risk aversion on price and order quantity in the newsvendor problem. *Manufacturing and Service Operations Management*, 2000, **2**(4): 410–423.
- [2] G. Cachon, M. Lariviere. Supply chain coordination with revenue sharing: strengths and limitations. *Management Science*, 2005, **51**(1): 30–44.
- [3] F. Chen, A. Federgruen. *Mean-variance analysis of basic inventory models*. Ph.D. Thesis, Columbia University, New York, New York, 2000.
- [4] T. Choi, D. Li, H. Yan. Mean-variance analysis of a single supplier and retailer supply chain under a returns policy. *European Journal of Operational Research*, 2008, **184**(1): 356–376.
- [5] T. Choia, D. Li, etc. Channel coordination in supply chains with agents having mean-variance objectives. *Omega*, 2008, **36**(4): 565–576.
- [6] L. Eeckhoudt, C. Gollier, H. Schlesinger. The risk averse (and prudent) newsboy. *Marketing Science*, 1995, **41**(5): 786–794.
- [7] G. Eppen, A. Iyer. Backup agreements in fashion buying - the value of upstream flexibility. *Management Science*, 1997, **43**(11): 1469–84.
- [8] X. Gan, S. Sethi, H. Yan. Coordination of supply chains with risk-averse agents. *Production and Operations Management*, 2004, **13**(2): 135–149.
- [9] X. Gan, S. Sethi, H. Yan. Channel coordination with a risk-neutral supplier and a downside risk-averse retailer. *Production and Operations Management*, 2005, **14**(1): 80–89.
- [10] H. Lau. The newsboy problem under alternative optimization objectives. *Journal of the Operations Research Society*, 1980, **31**(5): 393–403.
- [11] H. Lau, A. Lau. Manufacturer's pricing strategy and return policy for a single-period commodity. *European Journal of Operation Research*, 1999, **116**: 291–304.
- [12] B. Pasternack. Optimal pricing and returns policies for perishable commodities. *Marketing Science*, 1985, **4**(2): 166–176.
- [13] C. Shi, B. Chen. Pareto-optimal contracts for a supply chain with satisficing objectives. *Journal of the Operational Research society*, 2007, **58**(6): 751–759.
- [14] J. Spengler. Vertical integration and antitrust policy. *Journal of Political Economy*, 1950, **58**(4): 347–352.
- [15] T. Taylor. Supply chain coordination under channel rebates with sales effort effect. *Management Science*, 2002, **48**(8): 992–1007.
- [16] A. Tsay. Risk sensitivity in distribution channel partnerships: Implications for manufacturer return policies. *Journal of Retailing*, 2002, **78**(2): 147–160.
- [17] Z. Weng. Channel coordination and quantity discounts. *Management Science*, 1995, **41**(9): 1509–1522.