ISSN 1 746-7233, England, UK
International Journal of Management Science and Engineering Management
Vol. 2 (2007) No. 3, pp. 197-220

Boating against the current and the market game strategies in new product development: Economic advance-retreat course analysis

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(*Received April 12 2007, Accepted June 27 2007*)

Abstract. Boating against the current is one image which generalizes many development processes in socio-economic fields. Its basic characteristic is that the human can design and implement his advancing strategies initiatively, and the comprehensive environment synthetized by various objective factors has to accept the human choice passively. But the environment can influence the human behaviors in a manner of pressure or resistance. This game between the human and the economic environment is different from the traditional game problems. The conceptual model of boating against the current is described in this paper. The basic concepts and characteristics of course (advance-retreat course, or rise-fall course) are summarized. Then the basic framework of course analysis is built preliminarily, and the application model of discrete course and the corresponding solving method are brought forward. Finally, two application examples of discrete course are studied. The analysis shows that the development of new products in the traditional industries and the newly arisen industries should adopt two completely different development strategies, and they are the "overall development, propulsion in the linear way" and the "gradual development, propulsion in the point way" respectively.

Keywords: Socio-economy, boating against the current, discrete ARC (discrete advance-retreat course), industrial strategy

1 Introduction

So far, the theoretical studies on economic development have gained many outstanding achievements, such as the cycle theory of Lucas[13], the real business cycle theory of Prescott[12], and the new growth theory of Romer[17, 18]. In recent years, economists pay more attention to improving the efficiency of economic development[3], exploring ways to make the national or regional economic development more balanced by the government economic aid[11], and ensuring the economic growth through the formulation and choosing of policies[9], the continued improvement of economic education system[10] and other methods.

Many economists study the issues of economic development in a variety of ways, such as using biological methods to study the ecomocic growth[1, 5, 8], and to choose rational economic measures[2, 4], the economic cluster evolution theory[14, 16], the application of game theory to solve economic problems[7], and so on.

These results are given to study the economic development in the classical, industrial, process and choosing view, which is very laudable. However, one kind of problems exists aboard in reality, which is different from the traditional game (having two wise and initiative actors)[15, 19]. That is the game between the human who are able to choose the strategies initiatively and the non-initiative environmental pressure. The result of the game will emerge only after some time. There are many examples of this game, such as the games between economic development and environmental pollution, enterprise management and market environment, policy

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making and the implementation environment, knowledge learning and the application environment, energy use and the resource environment, transportation and the traffic environment, exploitation of natural resources and the ecological environment, and so on. In these problems, the human as the intelligent and subjective actor will be free to choose and implement economic strategies, and the environment as non-wisdom objects only passively accept the choices of the human. But the environment can act to the human economic behaviors by pressures or resistances. The higher degree the economy develops to and the faster the economy grows, the greater the environmental pressure will be. This means that the process of economic development should have advance and retreat courses, or rise and fall courses. It is similar to the process of boating against the current. The socio-economic behaviors of the human can be various and the environmental pressure is always changing. Such processes known as advance-retreat courses[6] should not be overlooked.

In this paper, boating against the current is analyzed firstly, and the advance-retreat course (course for short) is discussed. The basic concepts, characteristics and model of the course are described. Then the basic theoretical framework and analytical foundation of the course are discussed. The application models and the existence of their solutions are studied, and the solving methods for the course are given out. What we done will lay the foundation for the following research of course and the development of the course analysis theory.

2 Boating against the current and its conceptual model

In this section, the process of boating against current is described, and some basic cases are given out. Then the basic characteristics of boating against the current are analyzed, and the basic model is built.

2.1 The description of boating against the current

There is a river with water, and someone is boating against the current. It is difficult for the boater to advance against the current, and easier to go with the current. The environment of boating is complicated, and the resistances or pressures from the current exist at any place at any time. The boater will succeed if he advances by overcoming difficulties, or else will fail and be eliminated if he always retreats. The further the boater goes ahead, the more income he gets, and the larger resistance he will face.

The boater could be a single person or persons. In the process of boating against the current, the resistances are called endogenous if they are caused by the changes in the internal environment, such as tired boater, wrong boating way, disordered organization, outdated equipment, and low income. And the rapid flow, the abominable terrain, the dense submerged rocks, the changeful weather, and the undulating slope of runway will be the resistances to advance too. Such resistances are called exogenous because they are caused by the changes in the external environment. In order to increase forward motivity, reduce the endogenous and exogenous resistances, and gain more income, by investment the boater can make the distribution of physical more reasonable, the boating way more scientific, the organization and management more effective, and the equipment more advanced. In the course analysis of boating against the current, the following questions will be worthy of attention.

- How far the boater will go ahead in the current circumstances?
- When will the benefit in return reach the greatest, and when will it begin to reduce?
- What is the relation between the forward motivity and the resistances?
- What kind of rules will the advance course change along with at different resistance level?
- What degree will the resistance reduce to, the boater can go forward against the current?
- What degree will the resistance increase to, it is necessary for the boater to invest to keep stabilized advancing?
- Should the investment be used first to increase the forward motivity, or to reduce the resistances?
- Should the boater first reduce the endogenous resistance or the exogenous resistance to reduce the whole resistance?

These questions seem insignificant. Howerver, since boating against the current is a summary of many socio-economic activities, and it is broadly representative, solving these problems will become unusual.

2.2 Cases of boating against the current

The following cases will tell us that boating against the current has broad background in reality, and could be a general model of many activities in socio-economic field.

2.2.1 Knowledge learning

In modern society, many new problems and new situations emerge in the society environment, and they form a variety of forward pressures. To survive and succeed, a person, an organization, a nation or a society needs to learn much necessary knowledge constantly. If one gives up and stops learning new knowledge, he will not be able to deal with the new emerging problems, and will gradually fall behind and be eliminated. Only by constantly learning and using new knowledge, one can obtain and maintain survival and development advantages. In the process of learning new knowledge, one will encounter many various difficulties, such as puzzles in understanding knowledge, scientific method to seek knowledge, effective ways to acquire knowledge and the time and fees for learning, and so on. These can be viewed as endogenous resistances to knowledge growth. The difference in science and technology background, the unperfect education system, the lack of investment in education, the lagging of education policies, and so on, will form the exogenous resistances to knowledge growth.

2.2.2 Laws and regulations formulation

In the social and economic development process, there are various issues need to be gradually solved, and at the same time new problems constantly emerge, the human need formulate and implement various laws and regulations to keep the society and economy in order. If at some moment the existing laws and regulations are stopped and the new laws and regulations will be constituted no more, many existing and emerging social-economic problems will disrupt social-economic status, and the social-economic order will be confused. Therefore, it is necessary to constantly update existing laws or introduce new regulations to maintain an orderly society and economy. In the process of laws and regulations formulation and implementation, government will encounter various difficulties, such as puzzles in the wording selection for the specific provisions, the confirmation of the duration and scope of the provisions, the way of letting civilians understand and recognize the meaning and effects of the provisions, and the consistency between the different provisions, and so on. These can be seen as the endogenous pressures on the formulation and implementation of laws and regulations. The difference between the new and the original laws and regulations system, the coordination between new laws and regulations and social-economic development, the new negative tendencies caused by new laws and regulations, the lagging inherent in the laws and regulations, and so on, will be regarded as exogenous r pressures on the process of formulating laws and regulations.

2.2.3 Enterprise development

In the process of enterprise management and development, many problems need to be solved constantly, such as making product development plan, organizing various production activities, understanding market and the changing of socio-economic environment, improving corporate management regulations, reasonably appointing managers and staffs, and so on. Once the enterperises stops doing these, their operating income will reduce and they will pay the due costs. Therefore, to maintain enterprises survival and development, managers must constantly open up markets, produce products, and gain production incomes.

Enterprises will encounter various difficulties in the process of management and development, such as what kind of products to produce, how to determine the price of products, how large the production scale to be, what kind of equipment to purchase, which management regulations to be formulated, what kind of staff to be employed, how to improve the basic quality and the overall capacity of the staffs, how to improve the level of internal organization and management, new product development, untimely development and production, unreasonable enterprise development strategies and implementation strategies. These may become the endogenous pressures on enterprise development. The difference between the developing trends of the

enterprises development and the economic development, imperfect market system, no necessary funds to support the enterprises developing, and product development lagging behind the market demand, and so on, may become the exogenous pressures on the enterprises development.

2.2.4 Transport development

The development of transport facilities is indispensable for the construction of a region (a state or a city), but the region has to face pressures from the continuously increasing of various vehicles at the same time. If a region stops expanding or developing transport facilities, it will be difficult to keep normal transportation when the existing facilities can not meet the demand, and the other developments and constructions in social-economic area will be seriously restricted. Therefore, only the continuous development and construction of transportation facilities could solve the new problems from the transport development, ensure the normal transportation, and promote social-economic development. In the development and construction process of transport system, we will encounter various problems, such as the roads are narrow and the total lengh of roads are not enough, the maintenance and upgrading of the roads affect transportation, there are too many troubles from the excessive growth of vehicles and the large amount of vehicles, the maintenance tasks of traffic control system at traffic intersections are too heavy, traffic management and control systems are not perfect. These can be regarded as the endogenous pressures on transport development. In addition, transport system can hardly meet the need of economic development, the traffic management regulations are outdated and imperfect, there is contradiction between the road expansion and the restricted land using, vehicle emissions pollute the environment, the education system for the use and management of vechicles is not perfect, funds for transport system development are lack and the management policies are relatively backward. All these constitute the exogenous pressure on the transport development.

2.2.5 Energy development

Without energy, it is impossible for the human to survive and have various production activities. With the continuous improvement of the production level and the continuous expansion of the production scales, the energy demand continuously increases too. As a result, the energy shortage has become more and more serious. Once the human stops the existing energy production and the development of new energy sources, not only is it difficult to keep various production activities on, but also the survival of humanity will be greatly threatened. Therefore, it is necessary for the human to keep energy production and to develop new energy constantly to sustain the development of society and economy, on which the human living depends. In the process of energy system development and construction, the human will encounter various problems, such as the improvement of life quality results in the increase of the domestic energy use, it is low efficient and wasteful in energy use, the energy research and development is laggard, the ups and downs of major energy (for example, coal, oil and so on) price bring instable effects on economic development, and so on. These can be seen as endogenous pressures on energy systems development and construction. The developments of transport, construction, communication, entertainment, catering and others result in the significant growth of energy demand for the socio-economic development. The amount of much non-renewable energy is decreasing gradually. The energy crisis may break out at any time. The gap between energy demand and energy output leads to the the uncertainty of energy development strategy. All these will be the exogenous pressures on the energy system development.

2.2.6 Pollution control

Many human activities, such as production, living, consumption, entertainment and so on, inevitably bring pollutions. These pollutions have been always threatening the survival environment of humanity. In order to maintain a good living and production environment, humanity has been committed to environment protection and pollution control. Once such efforts are stopped, the living and production environment for humanity will deteriorate rapidly. Therefore, the human must overcome various difficulties, and always try hard to control and reduce pollutions caused by industrial production, to ensure a good environment for mankind

living. In the process of pollution control we will encounter various issues, such as a large number of obsolete product packages, the physical environment pollution caused by garbages from daily life and industrial productions, the air pollution caused by exhaust emissions of various vehicles, the electromagnetic pollution results from the all-round development of communication means. All these are the endogenous pressures on pollution control. The significant difference in scientific and technological levels of different countries and regions results in the imbalance in the amounts of pollution. The developed nations and the less developed nations have very different attitudes and ability in the pollution control. There are inconsistencies in the various national and regional pollution control policies and systems. Every nation or region has different cognition on the the potentially serious consequences of pollution. All these will be the exogenous pressures on pollution control within the global scope.

All these processes mentioned above can come down to boating against the current problems, that is the human have to face various pressures and resistances, and only through efforts or investment of the human to strengthen the forward motivity or reduce the endogenous and exogenous pressures in advance, can the quality of human life and the living environment be gradually improved. In contrast, if mankind does not work hard to overcome difficulties, t hey will suffer more punishment. In fact, the process of the economic development, the society development and the human development can also be summarized as boating against the current. Of course, there are many other cases on boating against the current, which are no longer listed here.

2.3 Basic description of boating against the current

2.3.1 The rules, assumptions of boating against the current

Based on these above cases, we can sum up the basic rules and the general assumptions of boating against the current.

The basic rules

• The boater will gain return benefit if he overcomes difficulties and goes forward against the current.
• The boater has to pay the cost or fine if he goes back with the current.
• The boater will be eliminated if he is unable to pay the costs or fines.

The general assumptions

• In order to survive and develop, the boater will try his best to gian the largest return benefit.
• The farther the boater advances, the greater pressure he will encounter.
• The boater cann't stay in situ, and he can only move forward or backward.

2.3.2 Players in the game of boating against the current

There are two players in the boating against the current game, who are the player 1 and the player 2. Characteristics of each player can be expressed as the following.

Player 1: It is the subject of the game (boater). The basic characteristics of the subject are:

• Rationality. The subject will treat living and development rationally. He will not easily give up the chances to overcome all difficulties and pressures, and he will try his best to maximize his own benefit.
• Initiative. The subject can analyze and judge his states and situations initiatively, and select and implement strategies initiatively.
• Wisdom. The subject will use all possilble means and methods to enhance the forward motivity and reduce the pressures on advance.
• Strategy. The basic strategies of the subject are limited, including advance, stop or retreat. But the manners to implement these strategies are unlimited.

Player 2: It is the object of the game (boating environment), that is the system composed of all environmental elements which hinder the subject from advancing. The basic characteristics of the object are

• Existence. It is objective and inevitable for the existence of the object, which is in the manner of impacting on the subject advancing. The main role of the object is to hinder the subject form progress.
• Memorization. The object can memorize the advance distance of the subject, which is reflected in that with

the increasing of advance distance the resistance is also on the rise. That is the pressure factors and the pressure intensity all increase.

• Passivity. The object can not choose strategies initiatively, but it can react to any choice of the subject and implement pressures on the subject.

• Activity. The object only acts naturally and without wisdom. The activity of the object is represented as the increase or decrease of the pressures. But the manners to express these activities may be unlimited.

2.3.3 Description of boating against the current and analysis of the basic strategies for the subject

Here it is assumed that the boater has the necessary startup cost. In the game, the boating process is divided into two stages according to the process of the forward motivity increaing from small to large, namely the forward motivity has two different states, small and large, and the corresponding two values are u_1 and u_2 respectively. Then the pressure caused by boating environment impacts on the boater naturally and passively, which also has two different states, small and large, and the two values are $\bar{u}_1$ and $\bar{u}_2$ respectively. The following is the example of boating against the current with two states.

Example 1. According to the two values of the motivity and the pressure, small or large, the boater has four basic combinations of the motivity and the pressure and the four corresponding gain or loss states.

State 1. The motivity is small and the pressure is small,$(u_1, \bar{u}_1)$. Then the boater will gain 1 unit incremental benefit by the motivity, and the pressure will bring the boater 0 unit loss.

State 2. The motivity is large and the pressure is small, $(u_2, \bar{u}_1)$. Then the boater will gain 2 units incremental benefit by the motivity, and the pressure will bring the boater 1 unit loss.

State 3. The motivity is large and the pressure is large, $(u_2, \bar{u}_2)$. Then the boater will gain 1 unit incremental benefit by the motivity, and the pressure will bring the boater 1 unit loss.

State 4. The motivity is small and the pressure is large, $u_1, \bar{u}_2)$. Then the boater will gain 0 unit incremental benefit by the motivity, and the pressure will bring the boater 2 units loss.

The state 1 shows that it is beneficial to advance at the beginning of the game. The state 4 shows that when the resistance is large, it is difficult to gain benefit for the boater even he tries hard to advance. The four gain or loss states corresponding the four strategies can be expressed in a more visual manner, seen in Fig. 1.

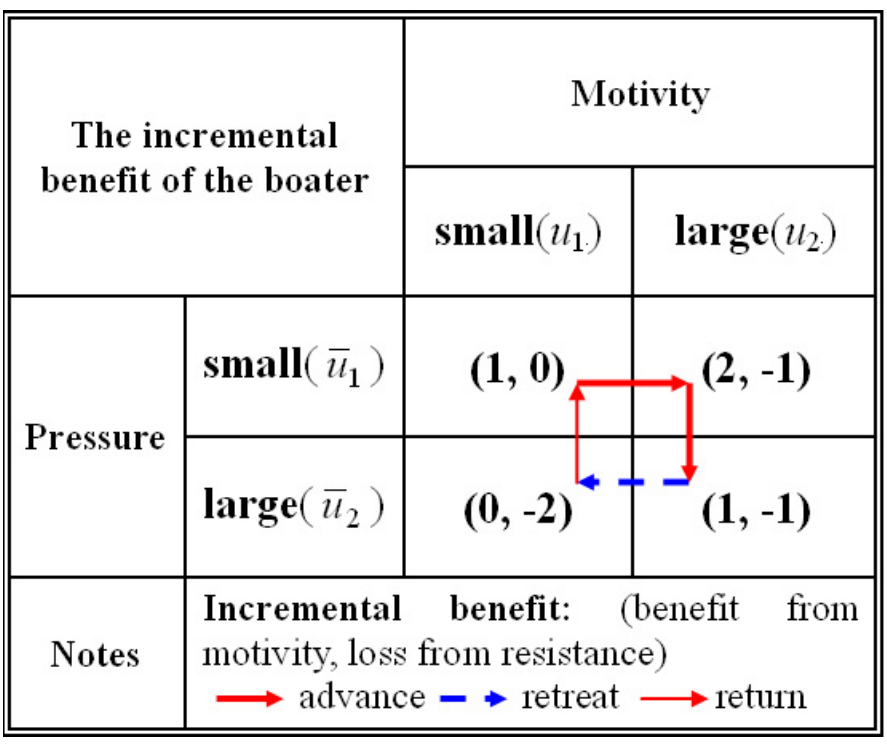

Fig. 1. Basic strategies of the subject

There are some explainations on example 1 which are described as the following.

• Incremental benefit. State 1, state 2, state 3 and state 4 are the gain or loss states of the boater, and the incremental benefits of them are (1, 0), (2 -1), (1, -1) and (0, 2) respectively.

• Equilibrium state. In the state 3, the boater has the same incremental benefit as incremental loss. This means that the motivity and the pressure are equal to form a balanced state. If the banlance is broke up, the process will enter the state 1, state 2 or state 4.

• Boating direction. The process of boating against the current will be pushed on in the sequence of state 1, state 2, state 3 and state 4.

• Advance course. The process of the state changing from state 1 to state 3 by state 2 is called as the advance course, which is shown in figure 1 as the real arrow lines.

• Retreat course. The process of the state changing from state 3 to state 4 is called the retreat course, which is shown in Fig. 1 as dotted arrow lines.

• Return course. Return from state 4 to state 1 again means the end of the current round of the boating against the current process. Another new process may begin and the gain or loss state may have the corresponding change. The return course is shown in figure 1 as thin real arrow lines.

We have the general assumption that in boating against the current, the boater will treat survival and development rationally, will not easily give up any chance to overcome all difficulties and pressures, and will try his best to gian the largest return benefit. In accordance with this assumption, the boater always tries his best to advance, and he will start and advance along the direction of increased motivity to obtain the maximum benefit. With the increasing of the motivity, the pressure increases too. Ultimately, under the strong pressure the boater has no other choice but to retreat in the direction of reduced motivity. After the failure the boater will try again in a new round. This process is shown in Fig. 1 as the cycle composed of arrow lines.

Throughout the advance course the boater will experience state 1 $\rightarrow$ state 2 $\rightarrow$ state 3, and will gain a accumulative total benefit of 4 units and have a accumulative total loss of 2 units. The total net benefit is 2 units. If the game continues, the boater has to experience the retreat course, state 3 $\rightarrow$ state 4. In this course the boater will lose a total net benefit of 2 units, and this is just equal to the net benefit gained in the advance course.

Of course, the boater can terminate the boating against the current proess initiatively at the appropriate time to ensure the maximum benefit. This state can be seen as the solution of boating against the current, for example, the state 3 in example 1.

2.3.4 Model of multi-state boating against the current

The boating against the current process with two states has been dissussed in the previous section, and the boating process in the two-state model has a single route. It is a very simple model of boating against the current. However, in the actual process of boating against the current, the forward motivity and the resistance often have more than two states. They may have three state values (small, medium, large), and correspondingly they will change from small to large by medium. Boating against the current may be a four-stage process (with four state values), five-stage process (with five state values), and so on. And the boating process may have a number of routes.

Generally, it is assumed that the process of boating against the current has $n(n > 1)$ stages, that is the forward motivity and the environmental pressure have n state values, u_i and $\bar{u}_i$, $i = 1, \cdots, n$, and $u_{i-1} < u_i, \bar{u}_{i-1} < \bar{u}_i,\ i = 2, \cdots, n$. When the boating process is pushed to the stage $(\bar{u}_i, u_j)$ in which the values of motivity and resistance are $\bar{u}_i$ and u_j, the gain or loss for the boater is

l_{ij}, the incremental benefit from motivity, $l_{ij} \geq 0$;

$\bar{l}_{ij}$, the loss of benefit caused by resistance, $\bar{l}_{ij} \leq 0$;

$\Delta l_{ij} = l_{ij} + \bar{l}_{ij}$, the net incremental benefit

where $i, j = 1, \cdots, n$.

The boater starts a game from the starting point and enters to the $(u_1, \bar{u}_1)$ state. Ensured that the motivity is no less than the pressure, the boating process experiences every state $(\bar{u}_i, u_j)$ $(j \geq i)$ without repeat and reaches the state $(u_n, \bar{u}_n)$. Such a route will be called as a feasible advance route.

Assuming that the boater always starts from the state $(u_1, \bar{u}_1)$, the accumulative benefit from advancing along with a feasible advance route in boating against the current process is

$$L_k(u,\bar{u}) = \left\{ \sum_{t=1}^{k} \sum_{s=j_t}^{j_{t+1}} \Delta l_{ts} + \sum_{s=k+1}^{n} \Delta l_{sn} + \sum_{s=1}^{n-1} \Delta l_{ns}, \quad 1 = j_1 < \cdots < j_k < j_{k+1} = n \right\} \tag{1}$$

where $u = (u_1, \cdots, u_n)^T$, $\bar{u} = (\bar{u}_1, \cdots, \bar{u}_n)^T$, $k = 1, \cdots, n-1$. Then the general problems of boating against the current can be described as

$$B(\sigma, L) = B(u, \bar{u}, L) \tag{2}$$

Where $\sigma = \sigma(u, \bar{u})$, $L = L(\sigma) = L(u, \bar{u}) = \{L_k(u, \bar{u}),\ 1 \le k \le n-1\}$, $L_k(u, \bar{u})$ is determined according to expression (1). The number of feasible advance routes, S, is got from the following expression (1).

$$S = 1 + \frac{(n-2)(n-1)}{2} \tag{3}$$

Example 2. A visual description of the three-state boating against the current problem is shown in Fig. 2. According to the expression (3) there are two feasible advance routes for this problem. In Fig. 2, the real arrow lines represent the feasible routes, the dotted arrow lines represent the retreat routes, and the thin real arrow lines represent the possible routes by which the game transits from the end point of the current process to the starting point of a a new round. Because the motivity and the pressure on the two routes are different, the accumulative benefits are usually not the same. Therefore, the boater needs to compare the accumulative benefit and make a choice of routes, to gain the maximum accumulative benefit.

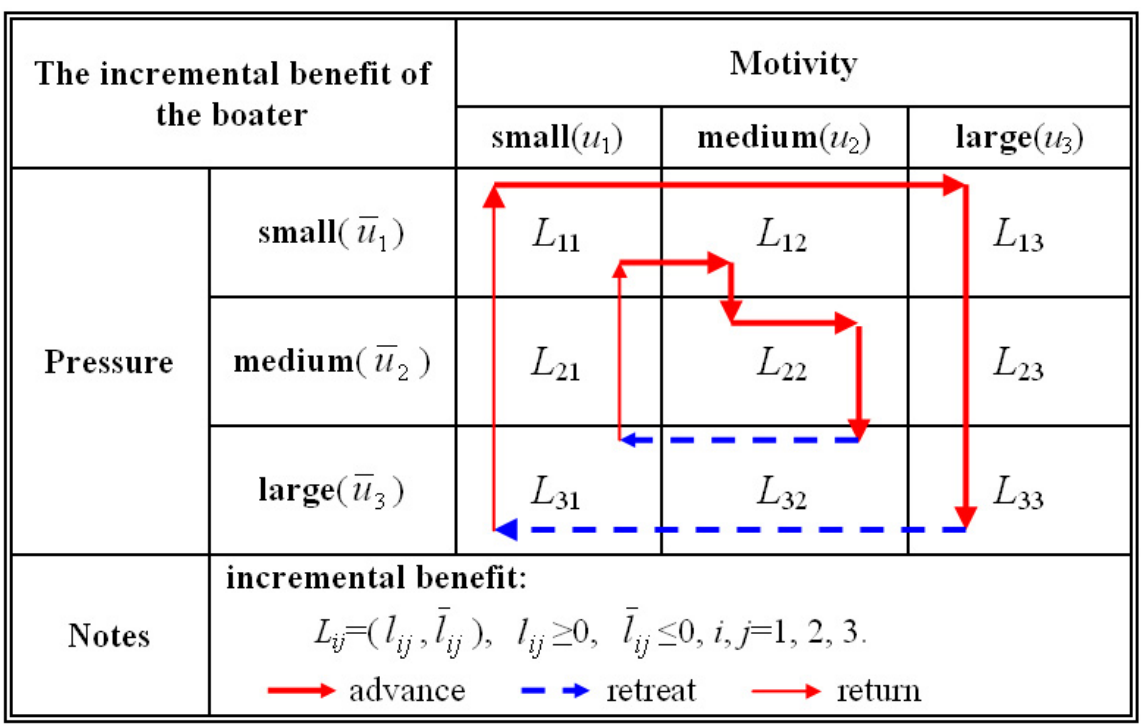

Fig. 2. Paths and strategis in the three-state game. The impetus and the pressure have three states (small, medium and large). There are two feasible advance routes. The boater can choose the more benefical route from the two.

In Fig. 2, the $L_{12} \to L_{22}$ course is still regarded as advance because the motivity is no less than the pressure. In the $L_{33} \to L_{31}$ course, under the strong pressure the forward motivity changes from large to small, this means that the boater has been in retreat. Therefore the boater always loses benefit in this subsection. At

this moment, the boater can adopt strategies initiately, for example, terminating the current process after L_{33} or starting a new boating process.

If the boater terminate the boating process at L_{33} with no other loss, his total net benefits of the different routes are respectively

$$L_1 = \sum_{i=1}^{3} \Delta l_{1i} + \sum_{i=2}^{3} \Delta l_{i3} \text{ and } L_2 = \sum_{i=1}^{2} \Delta l_{1i} + \sum_{i=2}^{3} \Delta l_{2i} + \Delta l_{33}, \text{ here } \Delta l_{ij} = l_{ij} + \bar{l}_{ij},\ i,j = 1,2,3.$$

In order to obtain the greatest total benefit the boater will choose $L^* = \max\{L_1, L_2\}$. L^* is the solution of the three-state boating against the current process. The further analysis shows that the solution has the following characteristics.

(i) Equilibrium. The boater terminates the current process at L_{33}, the current state of the motivity and the pressure is $(u_3, \bar{u}_3)$. This can be seen as an equilibrium point between the motivity and the pressure.

(ii) The greatest accumulative benefit. If the termination point of the current process is L_{33}, and assuming that there are no other loss, according to equation (1) the solution of this problem can be expressed as

$$L^* = \max_{1\le k\le 2} \left\{ \sum_{i=1}^{3} \Delta l_{1i} + \sum_{i=2}^{3} \Delta l_{i3}, \sum_{i=1}^{2} \Delta l_{1i} + \sum_{i=2}^{3} \Delta l_{2i} + \Delta l_{33} \right\}$$

For example 1, if the termination point is $(l_{22}, \bar{l}_{22})$, according to equation (1) the solution will be

$$L^* = l_{11} + \bar{l}_{11} + l_{12} + \bar{l}_{12} + l_{22} + \bar{l}_{22} = 2.$$

2.3.5 Solution of multi-state boating against the current and its existence

Similar to example 2, for the general problem (2), in the $(l_{nn}, \bar{l}_{nn}) \to (l_{n1}, \bar{l}_{n1})$ course, the forward motivity changes from large to small under the strong pressure, and the boater has been in retreat and always loses the benefit. At this moment, the boater can adopt strategies initiately to terminate the current process at $(l_{nn}, \bar{l}_{nn})$ to gain the greatest accumulative benefit. Then $\sum_{s=1}^{n-1} \Delta l_{ns}$ is removed from (1) and the remaining items are recorded as

$$L_k(u, \bar{u}) = \left\{ \sum_{t=1}^{k} \sum_{s=j_t}^{j_{t+1}} \Delta l_{ts} + \sum_{s=k+1}^{n} \Delta l_{sn}, \quad 1 = j_1 < \cdots < j_k < j_{k+1} = n \right\} \tag{4}$$

where $1 \le k \le n-1$. Then the problem (2) can be expressed in a more concise manner as

$$B(\sigma, L) \tag{5}$$

where $\sigma = \sigma(u, \bar{u})$, $L = L(\sigma) = L(u, \bar{u}) = \{L_k(u, \bar{u}),\ 1 \le k \le n-1\}$, and $L_k(u, \bar{u})$ is determined according to (4). The general solution of problem (5) is

$$L^* = \max_{1\le k\le n-1} L(u, \bar{u}) = \max_{1\le k\le n-1} \left\{ \sum_{t=1}^{k} \sum_{s=j_t}^{j_{t+1}} L_{ts} + \sum_{s=k+1}^{n} L_{sn}, \quad 1 = j_1 < \cdots < j_k < j_{k+1} = n \right\} \tag{6}$$

On the condition of no confusion, the following boating against the current means the model described by expression (5). It is necessary to point out

• The existence of the solution to problem (5) is easy to explain, because the feasible routes are limited and can be calculated according to (3). Then the accumulative benefit of each route can be got, and the greatest one will be the solution to problem (5).

• If the accumulative benefit gained by advancing along with the feasible route is viewed as the route length, the method used to find the longest route in network can be used to seek the solution to problem (5).

As the motivity and the resistance have the same state number, the problem is called as nomal boating against the current. In fact, there are cases that the numbers of motivity states and resistance states are different. That will be discussed in another paper.

To elevate boating against the current to a general theoretical level and to solve the broader advance-retreat problems, in the following section the advance-retreat problem will be discussed further.

3 The basic theory on advance-retreat course analysis

The growth and development process of anything includes two basic stages, advance (rise) stage and retreat (fall) stage, and the interactions between motivity and resistance must exsit in the process. Based on this view and the boating against the current problems dicussed before, if the subjects of the game are society, economy, enterprise or person, and the objects are socio-economic environmental factors, environmental factors in production and management of enterprises, or environmental factors in personal growth and development, which impact on the subjects advancing, such boating against the current model will have a more extensive description ability. Here, boating against the current is called as advance- retreat problem or course problem, and the related analysis and research is called as course analysis or advance-retreat analysis.

3.1 The concept system of advance-retreat analysis

Based on the abovementioned various cases, the basic elements, basic characteristics and types of countermeasures of advance-retreat analysis can be summed up from boating against the current problems. Then the basic concept system of advance-retreat problems can be synthesized.

3.1.1 The basic elements of advance-retreat course

The advance-retreat course consists of two pair basic elements, the initiative actors (subject for short) and the comprehensive environment (object for short) in advance course, and the forward motivity for the subject and the pressure from the object.

Definition 1. *(subject and object) The subject is the actor who takes the initiative to choose strategies and put the strategies into practice. The dominate behavior of the subject is moving in the direction of progressing and developing. The object refers to the objective things which hinder the subject from progress. The dominate behavior of the object is to passively resist on the subject's advancing.*

The subjects include all initiative actors, suah as govenment, organization, enterprise and person. The objects include all the environmental factors which may impact on the subject's advancing, such as socio-economic environmental prolems, which include policy, management, thchnology, production, operation, market, energy, transportation, employment, service and so on, natural environment problems, which include storm, tsunami, floods and droughts, rain and snow, temperature, and so on, and the living environment problems, such as disease, pollution, resources, species, and so on.

The various legal requirements of the subject are called the benefit of the subject, including requirements for capital, assets, resources, services, and wishes, and so on. The benefit of the subject is called as benefit for short. The net benefit is the remaining benefit after the loss is subtracted, and is called generally benefit for short.

Definition 2. *(motivity) The motivity for the subject is the power pushing the subject to advance, which is also called as the power for the subject. The motivity includes endogenous motivity and exogenous motivity. The endogenous motivity is from the subject itself and his internal environment, and the exogenous motivity is from the related environment elements outside the subject.*

The motivity derives from the necessity of survival, investment, competition and development, and the expectations for growth in benefit. The changes in management, capital and other changes in the environment may also become the motivity. The motivity includes physical, vigor, wisdom and capacity, demand and investment, etc.

Definition 3. *(pressure) The resistance from the object is the force that hinders the subject from advancing, and it is also called as environmental pressure or resistance, including endogenous pressure and exogenous pressure. The pressure caused by the changes in the internal environment which is composed of the subject's own factors, is called as endogenous pressure, and the pressure as a result of changes in the external environment which is composed of factors outside the subject, is called as exogenous pressure.*

The pressure derives from the environmental changes and environmental problems caused by the advancing process. It includes various environmental activities and effects, such as production costs, lack of resources, lack of energy, market competition and inappropriate policies, disordered management, outdated technology, high unemployment, sluggish traffic, disease spread, serious pollution. The pressure from the object means that the comprehensive environment has memories of the motivity.

The net forward motivity is the remaining part after the pressure is subtracted from the motivity. That is the net motivity for short.

Definition 4. *(advance, indirect advance and retreat) The movement along with the increasing of the net benefit for the subject is called as advance. The movement with the net benefit keeping unchanged is called as indirect advance, and the movement along with the decreasing of net benefit is called as retreat.*

3.1.2 The basic assumptions and characteristics about advance-retreat course

To study the advance-retreat course, the related basic rules and general assumptions need to be provided. Firstly, from definition 4, it is ruled that the increasing process of net benefit is regarded as advance and the the decreasing process of net benefit is regarded as retreat.

The basic rules There are three basis rules on advance-retreat course.

• The subject will gain incremental benefit if he overcomes difficulties and goes forward against the pressure.

• The subject has to pay the cost or fine if he stays in situ or retreats.

• The subject will be eliminated if his benefit is exhausted and he is unable to pay the costs.

The basic assumptions

• The goal of the advancing for the subject is to gain the maximum total benefit, which includes economic interests, social benefit and environmental benefit.

• The farther the subject advances, the greater pressure he will encounter.

• If the motivity is larger than the environmental pressure, the subject will gain incremental benefit. Instead, if the motivity is less than the environmental pressure, the subject will lose benefit.

The behaviors of subject get similar characters in section 2.3.1, such as rationality, initiative, wisdom and strategy. And object exists always and inevitably, though it (or they) can not make an initiative decision. object will make the natural and non-intellectual reaction to any choice of subject by the pressure or resistance as if remembering the distance of subject going ahead, and is represented in that the resistance become larger and larger along with the subject going ahead far and far.

The basic characteristics From the above rules and assumptions, it is easy to be seen that the advance-retreat course has three basic characteristics.

• The deterministic structure. Advance is the underlying trend (in particular for the overall socio-economic development), and the total benefit will increase with the advancing process.

• The stochastic environment. In every stage of the advancing process, some pressure factors may arise randomly or unexpected, and this will impact on the forward motivity. In addition, the forward motivity may change randomly with the environment changing. Therefore, the choice of advance or retreat for the subject will be affected by random factors to some degree.

• The uncertain outcome. The subject needs to remove all obstacles and pressures, and overcome various difficulties to gain benefit. Not all of actors will succeed and most of them will be eliminated. Therefore, is is uncertain who will be the victor.

3.1.3 The basic strategies for subject in advance-retreat process

In order to have an effective description on strategies in the advance-retreat course, the following marks are given.

u: The motivity to the subject advancing.

$\bar{u}$: The pressure encountered by the subject in advancing.

σ: The net motivity to the subject advancing. When $u > \bar{u},\ \sigma = \sigma(u, \bar{u}) > 0$; when $u = \bar{u}$, $\sigma = \sigma(u, \bar{u}) = 0$; when $u < \bar{u}$, $\sigma = \sigma(u, \bar{u}) < 0$.

L : The net benefit. $L = L(\sigma) = L(u, \bar{u})$.

Normally, there are three countermeasures for the subject in the advace-retreat course.

• When $u - \bar{u} > 0$, the net forward motivity $\sigma = \sigma(u, \bar{u}) > 0$, then the subject can choose advance, and the net benefit $L(u, \bar{u})$ will increase.

• When $u - \bar{u} = 0$, the net forward motivity $\sigma = \sigma(u, \bar{u}) = 0$, then the subject generally chooses to stay in situ, and net benefit $L(u, \bar{u})$ will keep no change (here the cost for staying in situs is ignored). The subject may also adopt measures to enhance motivity and to continue to advance, or choose temporary retreat under specific circumstances.

• When $u - \bar{u} < 0$, the net forward motivity $\sigma = \sigma(u, \bar{u}) < 0$, then it is feasible for the subject to choose retreat, and net benefit $L(u, \bar{u})$ will decrease. The subject may also adopt measures to enhance motivity and to continue to advance.

The subject can take the initiative to choose advance routes according to the states of the motivity u and the pressure $\bar{u}$, that is the state of net motivity $\sigma(u, \bar{u})$. Then there is the following definition.

3.2 The advance-retreat course and the existence of its solution

In this section the analytic definition of advance-retreat course and the existance theorem of its solution will be discussed. $C^{(i)}$ is a set of all i-order continous and differentiable functions, and $C^{(0)}$ is a set of all continuous functions. G is a set of all real-valued function, and A is a set of real numbers.

Definition 5. *(ζ—benefit utility) Assuming that $H(t) \in G$ and $H(t)$ is an increasing function,*

(i) if $\zeta(t) \in G$ and $\zeta(t) > 0$, $\sigma(t) \in C^{(0)}$ and $\zeta(t)\sigma(t)$ is integrable, $L[\sigma(t)] = H\left(\int_0^t \zeta(\tau)\sigma(\tau)d\tau\right)$ is defined as ζ- benefit utility of $\sigma(t)$.

(ii) if $\sigma = \{\sigma_1, \cdots, \sigma_n\}$, $\sigma_i, \zeta_i \in A$ and $\zeta_i > 0$, $L(\sigma_i) = H\left(\sum_{s=1}^{i} \zeta_s \sigma_s\right)$ is defined as ζ- benefit utility of σ, here $i = 1, \cdots, n$.

Consistency of ζ—benefit utility If $\sigma_1(t) \geq \sigma_2(t)$, $L[\sigma_1(t)] \geq L[\sigma_2(t)]$. In fact, as $\zeta(t) > 0$, when $\sigma_1(t) \geq \sigma_2(t)$, $\int_0^t \zeta(\tau)\sigma_1(\tau)d\tau \geq \int_0^t \zeta(\tau)\sigma_2(\tau)d\tau$. According to definition 5, $L[\sigma_1(t)] \geq L[\sigma_2(t)]$.

In the following discussion ζ-benefit utility is always used to measure the benefit level of the subject, that is $L[\sigma(t)] = H\left(\int_0^t \zeta(\tau)\sigma(\tau)d\tau\right)$. On the condition of no confusion, ζ-accumulative benefit utility $L[\sigma(t)]$ is called as benefit for short. When $\sigma(t)$ is the net motivity, $L[\sigma(t)]$ is called as net benefit.

Definition 6. *(course) Course is the shortened form of the advance-retreat course. It includes continous course and discrete course, and the definitions of them are*

(i) The net motivity $\sigma(t)$ and the net benefit $L[\sigma(t)]$ are continuous, $\sigma(0) > 0$ and $L[\sigma(0)] > 0$. If there is a moment $\bar{t}$ with $\bar{t} > 0$ or $\bar{t} = +\infty$, and when $t \in [0, \bar{t})$, $L[\sigma(t)] > 0$ and $L[\sigma(\bar{t})] = 0$, $L[\sigma(t)]$ $(t \in [0, \bar{t}])$ will be called as a continuous course.

(ii) For the net motivity σ_i $(i = 0, 1, \cdots, n)$ and the net benefit $L(\sigma_i)$, $U_0 > 0$ and $L[\sigma_0] > 0$. If there is $n_0 \in [0, n]$ at some moment, $L[\sigma_i] > 0$ $(i = 0, 1, \cdots, n_0 - 1)$ and $L[\sigma_{n0}] = 0$, $\{L[\sigma_i],\ i \in [0, n_0]\}$ will be called as a discrete course.

Referred to the model (5), the general model of course can be expressed as *ARC* $[\sigma(u, \bar{u}), L(u, \bar{u})]$, where, u is the motivity, $\bar{u}$ is the pressure, $\sigma = \sigma(u, \bar{u})$ is the net motivity, $L = L(\sigma) = L(u, \bar{u})$ is the net benefit.

Definition 7. *(solution of course) For the continuous course $L[\sigma(t)]$ $(t \in [0, \bar{t}]$, $\bar{t}$ is finite or $\bar{t} = +\infty)$, if there is $T \in (0, \bar{t})$, which satisfies $L[\sigma(T)] = \max_{0<t<\bar{t}}\{L[\sigma(t)]\}$ and $\sigma(T) = 0$, $L[\sigma(T)]$ will be a solution of the continuous course $L[\sigma(t)]$ $(t \in [0, \bar{t}])$. For the discrete course $L(\sigma_i)$ $(i = 0, 1, \cdots, n)$, if there is $T \in (0, n)$, which satisfies $L(\sigma_T) = \max_{0<i<n}\{L(\sigma_i)\}$ and $\sigma_T = 0$, $L(\sigma_T)$ will be a solution of the discrete course $L[\sigma_i]$ $(i = 0, 1, \cdots, n)$.*

Theorem 1. *(existance theorem on the solution of course)*

(i) For the continuous course $L[\sigma(t)]$ $(t \in [0, \bar{t}])$, $L[\sigma(t)] \in C^{(2)}$ *and* $\sigma(t) \in C^{(1)}$. *If there is* $T \in (0, \bar{t})$, *which satisfies* $\sigma(T) = 0$ *and* $\frac{d^2L(T)}{dt^2} < 0$, *the solution of the course* $L[\sigma(t)]$ *exists.*

(ii) If the discrete course $L(\sigma_i)$ *and* σ_i $(i = 0, 1, \cdots, n)$ *have the upper bound, that is there is* $T \in [0, n]$ *which satisfies* $L(\sigma_T) = \max\limits_{0<i<n}\{L(\sigma_i)\}$, *the solution* $L(\sigma_T)$ *of the discrete course* $L(\sigma_i)$ $(i = 0, 1, \cdots, n)$ *exists.*

Proof. (i) According to definition 5, $L[\sigma(t)] = F\left(\int_0^t \zeta(\tau)\sigma(\tau)d\tau\right) \in C^{(2)}$. Based on the conditions in theorem 1, $\frac{dL}{dt}\big|_{t=T} = F'\zeta(T)\sigma(T) = 0$ and $\frac{d^2L}{dt^2}\Big|_{t=T} = F'\zeta(t)\sigma'(t)|_{t=T} < 0$, so $L[\sigma(T)] = \max\limits_{0<t<\bar{t}}\{L[\sigma(t)]\}$ is the solution of the course.

(ii) According to definition 5, when $L(\sigma_i) = H\left(\sum\limits_{s=1}^{i}\zeta_s\sigma_s\right)$ and σ_i have upper bounds, $H(t)$ is an increasing function and $\zeta_i > 0$, so there is i_0 which satisfies $L(\sigma_{i_0}) = H\left(\sum\limits_{s=1}^{i_0}\zeta_s\sigma_s\right) = \max\limits_{1\le i<n}\left\{H\left(\sum\limits_{s=1}^{i}\zeta_s\sigma_s\right)\right\}$. If $\sigma_{i_0} = 0$, $L(\sigma_{i_0})$ is the solution of the course $L(\sigma_i)$ $(i = 0, 1, \cdots, n)$. If $\sigma_{i_0} \neq 0$, there should be $\sigma_{i_0} > 0$, or else $H\left(\sum\limits_{s=1}^{i_0}\zeta_s\sigma_s\right) < H\left(\sum\limits_{s=1}^{i_0-1}\zeta_s\sigma_s\right)$, it is inconsistent. When $\sigma_{i_0} > 0$, $\sigma_{i_0+1} \le 0$. Otherwise $H\left(\sum\limits_{s=1}^{i_0}\zeta_s\sigma_s\right) < H\left(\sum\limits_{s=1}^{i_0+1}\zeta_s\sigma_s\right)$, it is inconsistent. If $\sigma_{i_0+1} = 0$, $L(\sigma_{i_0+1})$ should be the solution of the course $L(\sigma_i)$ $(i = 0, 1, \cdots, n)$. Otherwise, if $\sigma_{i_0+k+1} = \sigma_{i_0+k}(k = 1, \cdots, n - i_0)$ and $\sigma_{i_0+1} = 0$, $L(\sigma_{i_0+1}) = L(\sigma_{i_0})$ is the solution of the course $L(\sigma_i)$ $(i = 0, 1, \cdots, n+1)$, and it is also the solution of the course $L(\sigma_i)$ $(i = 0, 1, \cdots, n)$.

3.3 Cases of the advance-retreat course

Example 3. If the net motivity $\sigma(t)$ is continuous and integrable, $\sigma(0) = \sigma_0$, $\sigma(T) = 0$, and $\zeta(t) > 0$ is a continuous function,

(i) $L[\sigma(t)] = L(\sigma_0)e^{\int_0^t \zeta(\tau)\sigma(\tau)d\tau}$ is a course, and its solution is $L[\sigma(T)]$.

(ii) $L[\sigma(t)] = L(\sigma_0) + \int_0^t \zeta(\tau)\sigma(\tau)d\tau$ is a course, and its solution is $L[\sigma(T)]$.

Specifically, if $\zeta(t) = \frac{1}{2}$, $u = 1 + \frac{8}{16+8t-t^2}$, $\bar{u} = \frac{2t}{16+8t-t^2}$, $\sigma(t) = u - \bar{u} = 1 + \frac{2(4-t)}{16+8t-t^2}$, and $L(\sigma_0) = 1$ in item (i), $L[\sigma(t)] = L(\sigma_0)e^{\int_0^t \zeta(\tau)\sigma(\tau)d\tau} = e^{\frac{t}{2}}\left(1 + \frac{t}{2} - \frac{t^2}{16}\right)$. The changing process of this course $L[\sigma(t)]$ is shown in Fig. 3. It can be seen from Fig. 3 that there are four basic stages in the development process.

Benefit accumulation stage. In the initial stage of the process, the subject will face many unprecedented difficulties. Therefore, the growth in net benefit is relatively slow, and it is in the difficult accumulation stage.

Benefit growth stage. After the initial benefit accumulation stage, the basic problems in prophase have been solved effectively, and every thing is in order, the net benefit begins to grow rapidly.

Benefit stagnant growth stage. After the rapid growth stage, new problems gradually become apparent. This leads to the economic development pressure increasing rapidly, and it is difficult to keep the net benefit on growing. Then the temporary peak of the benefit comes out.

Benefit decrease stage. With the end of benefit stagnant growth stage and the increasing of the environmental pressures, the net interest will decrease rapidly.

In practice, the development process of many things in socio-economic areas performs as this kind of process, such as the production and sale of products, the growth and development of enterprises, the emergence and development of industries, the occurrence and control of pollutions, the prevalence and control of diseases, the growth of the individual and organization, and so on. At the beginning, there is a period of gradually adapting, and it is often more difficult. If the difficulties can be overcomed, the process will enter into a regular and rapidly developing stage. This phase is characterized by the whole development and growth is in an orderly manner. Finally, many new problems which can not be solved by current methods, and the enormous pressures will lead the forward energy to crock up. At the same time, the net benefit will rapidly reduce and eventually reach to zero or below zero. In this last stage, if some positive measures are adopted to

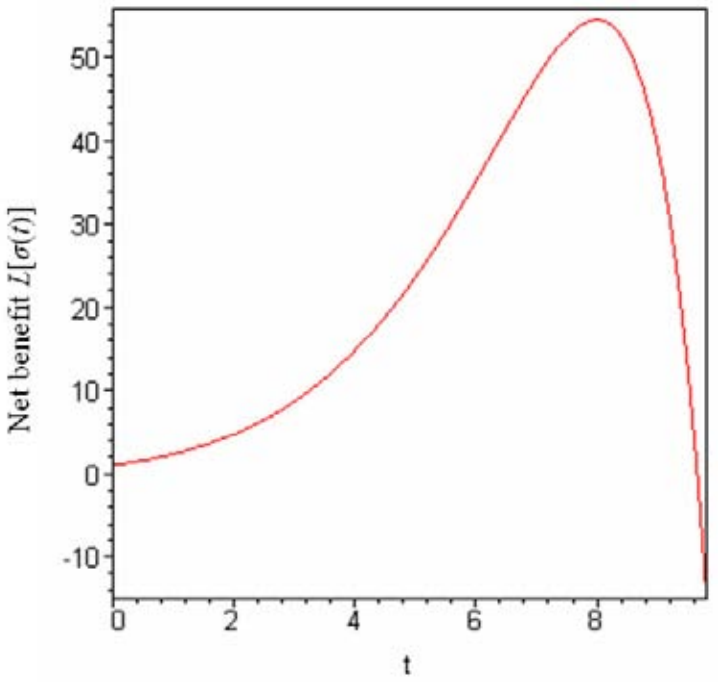

Fig. 3. The changing process of net benefit in advance-retreat course. At the beginning, the net benefit increases slowly and hardly. After this, the benefit increases quicker and smoothly. In the third stage, benefit increases very quickly. At the end of this process, the benefit decreases rapidly.

innovate effectively, the process may enter a new development stage; else it may come to the end, and cannot recover after a setback.

If the above process is of a specific industry, the industry development will pass through four basic processes, and they are

• In the process of the birth and development, the first to go through for the industry is a formidable process of carving out.

• After the carving out process, there will be a rapid expansion of the scale and the rapid growth of benefit for the industry.

• After the high-speed development of the industry, the development motivity is exhausted, then it is difficult to keep the benefit growing, and the benefit growth stagnates.

• After the stagnation of the benefit growth emerged in the industry, there may appear a drastic and rapid decline process of the benefit. At this time, it needs to take timely measures to control the process, and to further promote industrial growth process by investment, innovation or other suitable means. Then the development comes into a new process. Otherwise, the industry will eventually vanish.

This shows that the advance-retreat course can describe not only the advance-retreat phenomena in the development process of things, also the rise and fall processes in socio-economic realities.

Example 4. It is assumed that $L[\sigma(t)] = \mu(t) + h\sigma(t)$, $t \in [0, +\infty)$, $\mu(t)$ and $\sigma(t)$ are continous and differentiable, the constant $h > 0$, $\bar{t}$ is the root of equation $L[\sigma(t)] = 0$. If there is $T \in (0, \bar{t})$, which subjects to

$$\begin{cases} \sigma[T] = 0 \\ L'_t[\sigma(T)] = \mu'(T) + h\sigma'(T) = 0 \end{cases}$$

$L[\sigma(t)]$ $(t \in [0, \bar{t}])$ is a course and $L[\sigma(T)] = \max\limits_{t \geq 0}\{L[\sigma(t)] : \sigma(t) = 0\}$ is a solution of the course $L[\sigma(t)]$.

Example 5. Assuming that $L(U_i)$ is the close price (or average price) of a stock traded at the ith day, σ_i is the fluctuation range in the price of this stock at the ith day, and the stock is delisted at the n_0th day (that means the market price of the stock is zero), $L(\sigma_i)$ $(j = 0, 1, \cdots, n_0)$ is a discrete course.

4 The model and solving methods of discrete course

If the forward motivity u and the pressure $\bar{u}$ have limited n different states, $u = (u_1, \cdots, u_n)^T$, $\bar{u} = (\bar{u}_1, \cdots, \bar{u}_n)^T$, the combinations of motivity state and pressure state $(\bar{u}_i, u_j)$ TPF[2]. FPT are called as nodes, and (i, j) for short, $i, j = 1, \cdots, n$.

Definition 8. *(equilibrium point) The equilibrium appearing at the node (i, j) means that*

(i) if $u_j = \bar{u}_i$, the course is viewed as balanced in the net motivity at the node (i, j)(the motivity and the pressure are equal). At this moment, the node (i, j) is called as equilibrium point of net motivity.

(ii) if $\Delta l_{ij} = l_{ij} + \bar{l}_{ij} = 0$, the course is viewed as balanced in the net increased benefit at the node (i, j), both l_{ij} and $\bar{l}_{ij}$ are the same meaning in section 2.3.4. At this moment, the node (i, j) is called as equilibrium point of increased benefit.

According to definition 7 and definition 8, the solution of course can be interpreted as the value of the maximum net benefit under the net motivity equilibrium.

Assumption 1 In general, for $i, j = 1, \cdots, n$, it is assumed that

• Increasing states. The states of the motivity and pressure are listed from small to large, that is $u_{i+1} > u_i$, $\bar{u}_{i+1} > \bar{u}_i$.

• Equilibrium state. The values of the motivity and the pressure with the same list number are equal, it means that $u_i = \bar{u}_i$. If they are not equal, equalization can be realized by increasing motivity or pressure. Therefore, the node (i, i) is a net motivity equilibrium point.

• Linear motivity. When the subject advances to the node (i, j), the net motivity $\sigma_{ji} = \sigma(u_j, \bar{u}_i) = u_j - \bar{u}_i$.

• Linear benefit. When the subject advances to the node (i, j), the net incremental benefit brought by the motivity and the pressure is $L(\sigma_{ji}) = \Delta l_{ij} = l_{ij}(u_j) + \bar{l}_{ij}(\bar{u}_i)$, where $l_{ij}(u_j)$ is the incremental benefit brought by the motivity $u\mathrm{B}_{jB}$ under the pressure $\bar{u}_i$, $\bar{l}_{ij}(\bar{u}_i)$ is the the incremental benefit brought by the pressure $\bar{u}_i$ under the motivity u_j, and $l_{ij}(u_j) \geq 0$, $\bar{l}_{ij}(\bar{u}_i) \leq 0$. At the net motivity equilibrium point (i, i), the net incremental benefit $L(\sigma_{ii}) = 0$.

When $i \leq j$, $(\bar{u}_i, u_j)$ is called as an advance node, which means that the net forward motivity σ_{ji} is large than zero or equal to zero, and it is possible for the subject to advance. When $i > j$, $(\bar{u}_i, u_j)$ is called as retreat node, which means that the net forward motivity σ_{ji} is small than zero, and the subject has to retreat.

4.1 The model and solution of the discrete course without oblique line

The course without oblique line means that, in the discrete course, the advance and retreat have the following manners at the node $(i, j), i, j = 1, \cdots, n$.

(i) Direct advance is the movement along with the motivity increasing under the condition that the pressure keeps no change, that is $(i, j-1) \to (i, j)$, and it is called as advance for short.

(ii) Indirect advance is the movement along with the pressure increasing under the condition that the motivity keeps no change and is large than the resistance, that is, when $i \leq j$, $(i-1, j) \to (i, j)$.

(iii) Direct retreat is the movement along with the pressure decreasing under the condition that the motivity keeps no change, that is $(i, j) \to (i-1, j)$, and it is called retreat for short.

(iv) Indirect retreat is the movement along with the motivity decreasing under the condition that the pressure keeps no change, that is $(i, j) \to (i, j-1)$.

If the goal of direct advancing is to gain incremental benefit, the indirect advance is to adapt to the new environmental changes.

Because in the above advance-retreat course there are no the oblique advance $(i-1, j-1) \to (i, j)$ and the oblique retreat $(i, j) \to (i-1, j-1)$, the course is called as a course without oblique line.

[2] TPPT The order of motivity and pressure in node $(\bar{u}_i, u_j)$ is (pressure, motivity), not the (motivity, pressure). Such expression being adopted is just for the sake of convenient discussion behind.

4.1.1 Gain or loss model of the discrete course without oblique line

As mentioned above, when the subject advances or indirectly advances to the node $(i,\ j)$, the current benefit of the subject has changed, and it includes the following two aspects.

(i) Gain or loss from advance. The advance movement $(i, j-1) \to (i, j)$ will result in the enhanced motivity and bring in the following incremental benefit

$$\Delta l_{ij} = l_{ij}(u_j) + \bar{l}_{ij}(\bar{u}_i) = l_{ij} + \bar{l}_{ij}$$

where, $l_{ij} \geq 0$ is the incremental benefit brought in by the forward motivity, and $\bar{l}_{ij} \leq 0$ is the loss caused by the forward pressure.

When the net incremental benefit $\Delta l_{ij} \geq 0$, the subject should consider advancing to gain more net benefit. But when $\Delta l_{ij} < 0$, in general the subject should suspend advancing or choose to retreat, otherwise any effort will not bring any net benefit, even will reduce the net benefit.

(ii) Gain or loss from indirect advance. In the indirectly advancing, because the pressure is no larger than the motivity, the subject will advance and gain incremental benefit.The subject will face greater pressure and this will result in the loss of the benefit. When $i \leq j$, the indirect advance $(i-1, j) \to (i, j)$ will enhance the forward motivity and bring the following benefit changes for the subject.

$$l_{ij}^0 = l_{ij}^+ + l_{ij}^-$$

where, $l_{ij}^+ \geq 0$ the incremental benefit brought in by the forward motivity in the indirectly advancing, and $l_{ij}^- \leq 0$ is the benefit loss caused by the pressure in the indirectly advancing.

Since the indirect advance is a process with a progressive accumulation of the forward motivity in nature, the net incremental benefit is usually small. This means $|l_{ij}^0| < \varepsilon, \varepsilon > 0$. In general, it is set that $l_{ij}^0 = 0$.

The purpose of advance-retreat course analysis is to discuss how to advance for the subjet to achieve the greatest benefit, but not how to retreat. Accordingly, the following discussion will focus on the benefit changing with the advancing.

4.1.2 The recursive algorithm for benefit scale and the solution of course

Definition 9. *(scale for benefit) The benefit scale of the node (i, j) refers to the the maximum benefits from the routes from the initial node (1, 1) to the node (i, j) , expressed as $d_{ij} = d(\bar{u}_i, u_j)$.*

According to definition 9 and the above measure method on gain or loss in the discrete course without oblique line, the following can be gotten.

Algorithm 1 Denoting $\Delta l_{ij} = l_{ij} + \bar{l}_{ij}$, $l_{ij}^0 = l_{ij}^+ + l_{ij}^-$, $i, j = 1, \cdots, n$, the recursive algorithm for the benefit scales in the discrete course without oblique line is: for $i, j = 1, \cdots, n$

(i) $d_{11} = \Delta l_{11}$.

(ii) $d_{1j} = d_{1j-1} + \Delta l_{1j-1}$.

(iii) $i = j$, $d_{ij} = d_{i-1j} + l_{i-1j}^0$.

(iv) $i < j$, $d_{ij} = \max\{d_{i,j-1} + \Delta l_{i,j-1}, d_{i-1,j} + l_{i-1,j}^0\}$.

According to the algorithm 1, the solution of the discrete course without oblique line is $L(u, \bar{u}) = d_{nn}$.

4.2 The model and solution of the discrete course with oblique line

Based on the discrete course without oblique line, for $i, j = 1, \cdots, n-1$, if the subject can advance along the oblique line $(i, j) \to (i+1, j+1)$ and retreat along with the oblique line $(i, j) \to (i-1, j-1)$, the course is called as a discrete course with oblique line. Then, at the node (i, j), the gain or losse vector from the subject advancing is $L_{ij} = (l_{ij}, \bar{l}_{ij}, l_{ij}^p, \bar{l}_{ij}^p, l_{ij}^+, l_{ij}^-)$, where, l_{ij}, $\bar{l}_{ij}$, l_{ij}^+, l_{ij}^- is same as those in section 4.1.1, and

l_{ij}^p : the incremental benefit from the oblique advance motivity, $l_{ij}^p \geq 0$.

$\bar{l}_{ij}^p$: the loss caused by oblique advance pressure, $\bar{l}_{ij}^p \leq 0$.

According to the above measure method on gain or loss of benefit in the discrete course with oblique line, the following algorithm 2 can be gotten.

Algorithm 2 Denoting $\Delta l_{ij} = l_{ij} + \bar{l}_{ij}$, $\Delta l^p_{ij} = l^p_{ij} + \bar{l}^p_{ij}$, $l^0_{ij} = l^+_{ij} + l^-_{ij}$, $i, j = 1, \cdots, n$, the recursive algorithm for benefit scales in the discrete course with oblique line, for $i, j = 2, \cdots, n$, is

(i) $d_{11} = \Delta l_{11}$.

(ii) $d_{1j} = d_{1j-1} + \Delta l_{1j-1}$.

(iii) $i = j$, $d_{ij} = \max\{d_{i-1,j-1} + l^p_{i-1j-1}, d_{i-1,j} + l^0_{i-1,j}\}$, $d_{ij} = d_{i-1j} + l^0_{i-1j}$.

(iv) $i < j$, $d_{ij} = \max\{d_{i,j-1} + \Delta l_{i,j-1}, d_{i-1j-1} + \Delta l^p_{i-1j-1}, d^+_{i-1,j} l^0_{i-1,j}\}$.

According to the algorithm 2, the solution of the discrete course with oblique line is $L(u, \bar{u}) = d_{nn}$.

Normally the advance at the biginning should bring in more incremental benefit, so there is the following assumption.

Assumption 2 On the incremental gain and loss, there are the following reasonable assumptions.

(i) With the resistance keeping no change, the greater motivity will bring more direct incremental benefits, that is $\Delta l_{i,j-1} < \Delta l_{ij}$.

(ii) When $i < j$, the current incremental benefit from oblique and direct advance is no less than that from the indirect advance, that is $\Delta l_{ij} > l^0_{ij}$ and $\Delta l^p_{ij} > l^0_{ij}$.

When the motivity is larger than the pressure, there is net motivity space for the subject to indirectly advance. But when the motivity is less than or equal to the pressure, the subject will face greater resistance and have to retreat. Therefore there is the following assumption 3.

Assumption 3 When the net motivity is larger than zero, the subject can indirectly advance. But when the motivity is equal to the pressure (net motivity is in equilibrium), the subject should not indirectly advance.

5 Course analysis on production innovation for enterprises

It is assumed that an enterprise plans to develop a series of 5 products typed G (the development here refers to the developing, design, production and sell process, similar in the following). These five kinds of products gradually upgrade in turn of type G_1, G_2, G_3, G_4, G_5. With the increasing of type grade, the research and development technology improves gradually and the sale of the production becomes more and more difficult. Assuming that G_5 is the high-end product of this series of this type, at this moment, if the enterprise wants to continue to develop, it must develop a replacement product with entirely new technology. In the following, the advance-retreat course is ARC for short.

5.1 Analysis of product innovation force

Here, the denotations which will be used are given out. For $i = 1, \cdots, 5$,

u_i: Integrated motivity for enterprise to developing new product G_i, motivity for short.

$\bar{u}_i$: Integrated pressure for enterprise on developing new product G_i, resistance for short.

δ: Endogenous pressure coefficient of enterprise developing new products, $\delta > 0$.

g_i: Market environment for enterprise to develop new product G_i, environment for short.

r: Exogenous pressure coefficient of enterprise developing new products, $r¿0$.

Assuming that interest rates, inflation rates, unemployment rates, economic policies and other factors remain unchanged, the the product innovation force for enterprise can be analyzed from the following three aspects.

(i) **The net motivity to product innovation for enterprise.**

The product innovation force includes the integrated motivity u_i and the integrated pressure $\bar{u}_i$. When the motivity is u_j and the resistance is $\bar{u}_i$, the net motivity to developing new product Gj for the enterprise is $\sigma_{ji} = u_j - \bar{u}_i$.

(ii) **The integrated motivity to product innovation for enterprise.**

According to definition 2, the motivity to developing new products for enterprise includes endogenous motivity and exogenous motivity. Here, the endogenous motivity refers to the demand and impulse force of the enterprise for developing new products to seek greater and more comprehensive development, and to

gain more benefit. The exogenous motivity is the promoting force for enterprise to develop new products, which is from related productions in other enterprises and development of products market environment in various aspects. To facilitate the analysis, a more detailed decomposition of the integrated motivity will not be described here.

(iii) **The integrated pressure on product innovation for enterprise.**

According to definition 3, the integrated pressure on the enterprise developing new products includes endogenous resistance and exogenous resistance.

• The endogenous pressure refers to the technical difficulties and equipment pressures and other relevant factors encountered by the enterprise to develop and produce new products. Usually, the development and production of new products need higher technology and advanced equipment. Therefore, the greater the motivity is, the greater the endogenous pressure will be. It is assumed that the endogenous pressure on the development of new product G_i is in linear correlation with the motivity u_i, this means that the endogenous pressure is δu_i.

• The exogenous pressure refers to external pressures from opening up the markets, market competition and other environmental pressures on new products sales for the enterprise. If the external market environment of the new products sales includes the production, transport, sales and other relevant factors of all the other similar products, and the market environment is measured by the related market facilities and the total amount of environment assets, the greater the environment assets are, the more exogenous pressure will be broght by the assets. The sales of new products need to develop deeper and broader market space based on that of the original products, and will face more fierce market competition, and encounter greater exogenous pressure. It is assumed that the exogenous pressure from the development of new product G_i is in linear correlation with the market environment g_i, this means that the exogenous resistance is rg_i.

Thus, the integrated pressure on developing new products G_i for enterprise is $\bar{u}_i = \delta u_i + rg_i$.

Assumption 4 There are the following assumptions for enterprises.

(i) The enterprise has the cost to develop new products.

(ii) In any market, the competitors of the enterprise are always developing, so the integrated pressure faced by the enterprise will continuously grow.

(iii) When the development motivity remains unchanged, the net development motivity will gradually reduce with the gradually increased pressure. Therefore, if we do not develop new products to strengthen the development motivity, the benefit level will fall too.

According to the assumption 4, the enterprise will constantly develop products of new types. At the same time, there is the following assumption.

Assumption 5 The forward motivity will bring in more net incremental benefit when the enterprise develops products with higher type, which is $l_{i,j-1} < l_{ij}$. But the loss caused by the advance pressure will not reduce when the enterprise develops products with higher type, that is $\bar{l}_{i,j-1} \leq \bar{l}_{ij}$.

In the following analysis of the examples, it is assumed that the process to develop new products for enterprise submits to a discrete ARC without oblique line, and the dimension of measuring benefits is one unit of product. From the analysis of the following two cases, it can be seen that if the goal of the direct advance is to gain incremental benefit, the indirect advance is to adapt to the new changes in environment.

5.2 Example on enterprise development strategies in the traditional industries

To use analysis method for discrete process to discuss the dominant strategies for new product development in the traditional industries and the traditional product markets, the assumptions on the changes of the motivity and pressure on the new products development are given out in the following.

(i) Under the same pressure $\bar{u}_i$, the loss caused by the pressure remains constant in the development of a series of new products G_1, G_2, G_3, G_4 and G_5.

(ii) When the pressure state $\bar{u}_i$ changes, the loss of benefit will gradually increase with the pressure $\bar{u}_i$ $(i = 1, \cdots, 5)$ changing from small to large.

(iii) The incremental gain or loss at the node (i, j) in this discrete course is $L_{ij} = (l_{ij}, l^0_{ij}, \bar{l}_{ij})$, where, $l_{ij} \geq 0$, $l^0_{ij} = 0$, $\bar{l}_{ij} \leq 0$, $l_{11} = \bar{l}_{11} = 0$, $i, j = 1, \cdots, 5$. As there is no indirect advance under the pressure $\bar{u}_1, l^0_{1j}$ does not exist.

5.2.1 Content of the example

In particular, the gain or loss state $(l_{ij}, l_{ij}^0, \bar{l}_{ij})$ of the new products development for the enterprise is shown in Fig. 4, here $l_{ij} \geq 0$, $l_{ij}^0 = 0$, $\bar{l}_{ij} \leq 0$, $i, j = 1, \cdots, 5$.

Gain or loss from new products development and the benefit scales		New products and the motivity states				
		G_1 (u_1)	G_2 (u_2)	G_3 (u_3)	G_4 (u_4)	G_5 (u_5)
Pressure on new products development	$\bar{u}_1$	(0, 0) **d_{11}=0**	(2, -1) **d_{12}=1**	(3, -1) **d_{13}=3**	(4, -1) **d_{14}=6**	(5, -1) **d_{15}=10**
	$\bar{u}_2$	(1,0,-2)	(2,0,-2) **d_{22}=1**	(3,0,-2) **d_{23}=3**	(4,0,-2) **d_{24}=6**	(5,0,-2) **d_{25}=10**
	$\bar{u}_3$	(1,0,-3)	(2,0,-3)	(3,0,-3) **d_{33}=3**	(4,0,-3) **d_{34}=6**	(5,0,-3) **d_{35}=10**
	$\bar{u}_4$	(1,0,-4)	(2,0,-4)	(3,0,-4)	(4,0,-4) **d_{44}=6**	(5,0,-4) **d_{45}=10**
	$\bar{u}_5$	(1,0,-5)	(2,0,-5)	(3,0,-5)	(4,0,-5)	(5,0,-5) **d_{55}=10**
Notes	1. The gain or loss at the node (i,j) in the course is $(l_{ij}, l_{ij}^0, \bar{l}_{ij})$, where, $l_{ij} \geq 0$, $l_{ij}^0 = 0$, $\bar{l}_{ij} \leq 0$, $i, j = 1, \cdots, 5$. $l_{11} = \bar{l}_{11} = 0$, l_{1j}^0 does not exist. 2. The benefit scale at the node (i,j) is $d_{ij}, j \geq i, i, j = 1, \cdots, 5$. → the best advance route - - → retreat route ⟶ return route					

Fig. 4. Analysis on gains or losses and the routes in the new products development ARC in traditional industries

It can be seen from the Fig. 4 that

• When $j \geq i$, the forward motivity is larger than the forward pressure, and the node (i, j) is an advance node, because the enterprise has the net motivity to developing new products.

• When $j < i$, the forward motivity is less than the forward pressure, and the node (i, j) is a retreat node (the darken section in the Fig. 4), because the enterprise has no net motivity to developing new products and has to retreat.

• According to algorithm 1 and algorithm 2, the scales of nodes in Fig. 4, $d_{ij} (j \geq i, i, j = 1, \cdots, 5)$ are computed. There are the following results about the node scales.

• When $j < i$, that is at the retreat node, the benefit scale d_{ij} is not computed, because at this time the development of new products will result in continuous loss. Therefore, the enterprise normally should not develop new products in this situation.

• When $j \geq i$, that is at the advance node, the benefit scale, d_{ij} is given out. As $d_{55} = 10$, the solution of this discrete ARC is $L(u, \bar{u}) = d_{55} = 10$.

• The best route for enterprise to develop new products is (1, 1) → (1, 2) → (1, 3) → (1, 4) → (1, 5). Developing new products along this line will bring in the largest benefit and give the solution of the new product development ARC, $L(u, \bar{u}) = 10$.

5.2.2 Feature analysis on the routes in ARC

On the basis of the above example, we can analyze the routes in the whole process of new products development as the following.

(i) At the node $(1, j)$, there is no indirect advance route, and the gain or loss state is $(l_{ij}, \bar{l}_{ij}), j = 1, \cdots, 5$.

(ii) The route (1, 1) → (1, 2) → (1, 3) → (1, 4) → (1, 5) is an advance path. In the advancing process, the enterprises continue to study new technologies, develop new products, and fully prepare for the new products to enter the market. The research work is carried out mainly in the enterprise, so the environmental pressure changes little.

(iii) The route (1, 5) → (2, 5) → (3, 5) → (4, 5) → (5, 5) is the process of producing and selling new products, adapting to the market changes and gaining sales profit for enterprises. In this process, the market competition gradually becomes intense, and the development pressure gradually increases, eventually the process reaches the equilibrium point (5, 5) with the largest motivity and the largest benefit.

(iv) The route (5, 5) → (5, 4) → (5, 3) → (5, 2)→ (5, 1) is the retreat process after being prosperous, that is the process the enterprises usually experience after finishing all new products development and having gained market success. In this route, the enterprise needs to develop new generation product with new technology types, else it will eventually lose all benefits gained when to the node (5, 1).

(v) When the enterprise retreat to the node (5, 1), it will fall from then on, or experience the returning line (5, 1) → (4, 1) → (3, 1) → (2, 1) → (1, 1) by hard work and exploration, and select to develop again and start a new process after cumulating forward motivity.

5.2.3 The analysis on the development strategies for enterprises in traditional industries

Further, the practical background and basis of the above assumptions and computation results can be interpreted as that the traditional industries or traditional product markets have the following characteristics.

Higher stability In the traditional industries or traditional product markets, the overall situation is relatively stable. The commodity production and market operation are more regular and standard. The emergence of new products will not change the existing stability of the market in a short time. The reaction to the new products of the market environment will appear after a certain period of time.

Lower tolerance Due to the relative stability of the overall market situation and the higher monopoly by the large enterprises, it is easy for a series of new products with foundation to meet the comprehensive needs of various consumers in the markets, but it is more difficult for the single new product to enter the market and be accepted by the market. For the large enterprises, the environmental pressure on developing new products is relatively stable. After the new products have entered the market broadly, the competitive pressures and environmental pressures start to gradually increase.

Higher controllability The mainstream enterprises have more capability of controling the new products development and the industrial market development. After the new products have entered the market, even if the other enterprises react in a short time, it is difficult for them to form an essential competitive threat. However, with the time going by, this competition forces will gradually increase, result in intensely competitive environment and competitive situation, and bring more and more loss of benefit caused by environmental pressure.

These conclusions are consistent with the actual situation in traditional industries and traditional products market. So there is the following strategy 1 on the traditional industries and traditional product markets.

Enterprise Development Strategy 1 (the development strategy for enterprises in traditional industries). In the traditional industries and traditional product markets, the dominant strategy on new products development for enterprises should firstly focus on rapid developing new serial technologies and products, then pushing the product series whole into the market, in order to bring resounding and continuous market effect. This is the strategy of the overall development and the propulsion in linear way. Then the substitute product with new technologies type should be developed timely.

5.3 Example on enterprise development strategies in the newly arisen industries

To use analysis method for discrete process to discuss the dominant strategies for new product development in the newly arisen industries and the newly arisen product market, the assumptions on the motivity and pressure changes in the new products development are given out in the following.

(i) Under the same pressure $\bar{u}_i$, because of the new technologies and their faster renovation, the loss caused by the pressure gradually increase in the development of a series of new products G_1, G_2, G_3, G_4 and G_5.

(ii) At the equilibrium point (i, j) $(i = j)$ and in the advance course of new product development process, the loss caused by pressure is small at the beginning and gradually increases after then.

(iii) By adapting to market environment and analyzing changes in market, the enterprise can find a more favorable breakthrough point for product development. Therefore, indirect advance will enable the loss of benefit caused by the new products development to become smaller.

(iv) The incremental gain or loss at the node (i, j) in this discrete course is $L_{ij} = (l_{ij}, l^0_{ij}, \bar{l}_{ij})$, where, $l_{ij} \geq 0$, $l^0_{ij} = 0$, $\bar{l}_{ij} \leq 0$, $l_{11} = \bar{l}_{11} = 0$, $i, j = 1, \cdots, 5$. As there is no indirect advance under the pressure $\bar{u}_1$, l^0_{1j} does not exist.

5.3.1 Content of the example

In particular, the gain or loss state $(l_{ij}, l^0_{ij}, \bar{l}_{ij})$ of the new product development for the enterprises is shown in Fig. 5, here $l_{ij} \geq 0$, $l^0_{ij} = 0$, $\bar{l}_{ij} \leq 0$, $i, j = 1, \cdots, 5$.

Gain or loss from new products development and the benefit scales		**New products and the impetus states**				
		G_1 (u_1)	G_2 (u_2)	G_3 (u_3)	G_4 (u_4)	G_5 (u_5)
Pressure from new products development	$\bar{u}_1$	(0, 0) **d_{11}=0**	(2, -1) **d_{12}=1**	(3, -2) **d_{13}=2**	(4, -3) **d_{14}=3**	(5, -4) **d_{15}=4**
	$\bar{u}_2$	(1,0,-2)	(2,0,-2) **d_{22}=1**	(3,0,-1) **d_{23}=3**	(4,0,-2) **d_{24}=5**	(5,0,-3) **d_{25}=7**
	$\bar{u}_3$	(1,0,-3)	(2,0,-3)	(3,0,-3) **d_{33}=3**	(4,0,-1) **d_{34}=6**	(5,0,-2) **d_{35}=9**
	$\bar{u}_4$	(1,0,-4)	(2,0,-4)	(3,0,-4)	(4,0,-4) **d_{44}=6**	(5,0,-1) **d_{45}=10**
	$\bar{u}_5$	(1,0,-5)	(2,0,-5)	(3,0,-5)	(4,0,-5)	(5,0,-5) **d_{55}=10**
Notes		1. The gain or loss state at the node (i, j) in the course is $(l_{ij}, l^0_{ij}, \bar{l}_{ij})$, where, $l_{ij} \geq 0$, $l^0_{ij} = 0$, $\bar{l}_{ij} \leq 0$, $i, j = 1, \cdots, 5$. $l_{11} = \bar{l}_{11} = 0$, l^0_{1j} does not exist. 2. The benefit scale at the node (i, j) is $d_{ij}, j \geq i, i, j = 1, \cdots, 5$. → the best advance route - - ▸ retreat route ⟶ return route				

Fig. 5. Analysis on gain or loss states and the routes in the new products development course in newly arisen industries

It can be seen from the Fig. 5 that

• When $j \geq i$, the forward motivity is larger than the forward pressure, and the node (i, j) is an advance node, because the enterprise has the net motivity on developing new products.

• When $j < i$, the forward motivity is less than the forward pressure, and the node (i, j) is a retreat node (the darken section in the Fig. 5), because the enterprise has no net motivity on developing new products and has to retreat.

• According to algorithm 1 and algorithm 2, the scale of every node in Fig. 5, $d_{ij}(j \geq i,\ i, j = 1, \cdots, 5)$ is gotten. About the node scale d_{ij}, there are the following computing results.

• When $j < i$, that is at the void node, the benefit scale d_{ij} is not be computed, because at this time the development of new products will result in continuous loss. Therefore, the enterprises normally should not develop new products in this situation.

• When $j \geq i$, that is at the effective node, the benefit scale d_{ij} is given out. As $d_{55} = 10$, the solution of this discrete ARC is $L(u, \bar{u}) = d_{55} = 10$.

• The best route for enterprise to develop new products is (1, 1) → (1, 2) → (2, 2) → (2, 3) → (3, 3) → (3, 4) → (4, 4) → (4, 5) → (5, 5). Developing new products along this line will bring in the largest net benefit and give the solution of the new product development ARC, $L(u, \bar{u}) = 10$.

5.3.2 Analysis on features of the routes in ARC

On the basis of the above example, we can analyze the routeshs in the whole process of product development as the following.

(i) At the node $(1, j)$, there is no indirect advance course, and the gain or loss state is $(l_{ij}, \bar{l}_{ij})$, $j = 1, \cdots, 5$.

(ii) The route (1, 1) → (1, 2) → (1, 3) → (1, 4) → (1, 5) is an advance path. In this path, the enterprises continue to study new technologies and develop new products. This makes the forward motivity and the incremental benefit gradually increase. At the same time, the environmental pressures gradually increase too, resulting in that the loss also gradually increases.

(iii) The route (1, 1) → (1, 2) → (2, 2) → (2, 3) → (3, 3) → (3, 4) → (4, 4) → (4, 5) → (5, 5) is the process of developing, producing and selling new products by turns. In this process, the enterprises adapt to the market and environment changes while they develop new products, and realize the maximum benefit growth. Eventually the process reaches the equilibrium point (5, 5) with the largest motivity and the largest benefit.

(iv) The route (5, 5) → (5, 4) → (5, 3) → (5, 2) → (5, 1) is the recession process after being in fashion for a period, which is the process the enterprises usually experiences after finishing all new products development and having gained market success. In this route, the enterprises need to develop new generation product with new technology types, else they will eventually lose all gained benefit when reach to the node (5, 1).

(v) When the enterprises retreat to the node (5, 1), they will fall from then on, or experience the returning line (5, 1) → (4, 1) → (3, 1) → (2, 1) → (1, 1) by hard work and exploration, and select to develop again and start a new process after cumulating forward motivity.

5.3.3 The analysis on the development strategies for enterprises in newly arisen industries

Further, the practical background and basis of the above assumptions and computation results can be interpreted as that the newly arisen industries or product markets have the following characteristics.

Lower stability In the newly arisen industries or product markets, the overall situation will change at any time. The relative stability of the commodity production and market structure is lower. The emergence of a new product may affect the existing stability of the market in short period of. This means that the market environment will soon react to the appearance of a new product.

Higher tolerance Because in the newly arisen industries, the monopoly level of the market is lower and there are relatively few products to replace the existing product, it is easier to be accepted by the market for the new product entering the market, as long as it has good performances. Therefore, the loss of benefit caused by product development pressure is less at the beginning.

Strong timeliness After a new product has entered into the market, the other enterprises will respond quickly, and then a lot of similar products will emerge rapidly. In addition, the technology updates rapidly. All these lead to the intensely competitive environment. Therefore, the loss of benefit caused by pressure will increase in a short time.

These conclusions are consistent with the actual situation of newly arisen industries and product markets. Therefore, there is the following strategy 2 for the enterprises in newly arisen industries and product markets.

Enterprise Development Strategy 2 (the development strategy for enterprises in newly arisen industries). In the newly arisen industries and product markets, the dominant strategy on new products development for enterprises should firstly rapidly and effectively develop new technologies and products, and push the new products quickly into the market. Based on the market reaction, it needs to further develop the next new product until the product development of this type reaches its technology limits. This is the strategy of gradually development and the propulsion in point way. Then the substitute product with new technology type should be developed timely.

6 Conclusions

By summarizing the practical development processes of many things in the economic field, the model of boating against the current is proposed in this paper, the examples are given, the corresponding concept model is described and a preliminary analysis is given. Based on these, the ARC problem (i.e. advance-retreat course) is put forward, and the theoretical foundation and the analytical framework of ARC are discussed.

The core of the ARC problems is the relation between the advancing of the subject and the obstacles from the objects. The subjects which have the ability to take the initiative to implement strategy and the objects which can only passively response by pressures are always in the mutual gaming process. The subjects need to constantly advance in order to survive and progress, and the objects will form or make pressures on the advance through various environmental factors. These relationships directly impact on the strategies for the subjects to gain the benefit. In fact, the concept of ARC can not only be used to describe the advance-retreat phenomena in the process of things development, but also be used to describe the rise and fall process in socio-economic realities. The ARC analysis is the system of theories and research methods on ARC problems (that is the advance-retreat relation between the subjects and the objects, or the rise and fall course of the subjects).

Specifically, this paper has completed the following work:

(i) Based on the summary of many reality problems, the analysis on boating against the current is proposed, which can be used to describe many basic phenomena in social-economic areas.

(ii) The conceptal system and the analytical model of boating against the current are established, and the existence of solutions and the solving methods are discussed. All these are important for using the model of boating against the current to solve practical problems.

(iii) The advance-retreat course (ARC) and its related concepts are defined. The theoretical system and the basic framework of the ARC analysis are described. The basic models of the continuous ARC and the discrete ARC are built. Then the existence of the solutions to the models is discussed. All these lay a theoretical foundation for the further development of the ARC analysis.

(iv) The practical model of the discrete course with oblique line and that without oblique line are established separately, and the solving algorithms are given.

(v) Through two examples, the characteristics of the course and the dominant strategies are analyzed for the new products development in the traditional industries and in the newly-arisen industries separately, and the two different development strategies are concluded. These two strategies can make the enterprises in the traditional industries and in the newly-arisen industries maximize their benefits.

The interesting results from the analysis in this paper are

• The growth and development process of many things in the socio-economic field can be approximately divided into four basic stages, which is the difficult carving out and starting stage, the rapid expanding and developing stage, the brief stagnantly developing stage and the recession stage with collapsibility.

• When the manufacturing enterprises in the traditional industries develop new product series, they should adopt a basic strategy, comprehensive development and linear propulsion, that is firstly focusing on the development of new product series and then pushing the whole products into the market.

• When the manufacturing enterprises in the newly-arisen industries develop new product series, they should adopt a basic strategy, gradually development and point propulsion. That is pushing a new product to market quickly once it has been produced. After observing and reviewing its market effects, the follow-up upgraded products will be developed and pushed into the market.

Of course, there are many issues on the ARC worth to be solved, such as
(i) the further improvement of the basic theories on ARC analysis.
(ii) the deep exploration of more effective and rich model of ARC analysis.
(iii) the comprehensive practical verify and the full empirical study on various ARC model.

At the end, authors hope that the work in this paper will play a positive role in the development of theories and methods on ARC analysis, especially in the the actual economic development.

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