ISSN 1749-8023 (print), 1749-8031 (online)
International Journal of Magnetic Resonance Imaging
Vol. 01, No. 01, 2007, *pp*. 027-032

Regularized Computed Tomography using Complex Wavelets

V.Thavavel [1] and R. Murugesan [2 +]

[1] Department of Applied Sciences, Sethu Institute of Technology, Kariapatti, India
[2] Department of Physical Chemistry, Madurai Kamaraj University, Madurai, India

(*Received 10 October 2006, accepted 20 December 2006*)

Abstract. Reconstructing low-dose computed tomography (CT) images is an unstable inverse problem, due to the presence of noise. To address this problem, we propose a new regularized reconstruction method that combines features from the Filtered Back-Projection (FBP) algorithm and regularization theory. The filtering part of FBP comprises Fourier-domain inversion followed by noise suppression based on thresholding procedure in complex wavelet domain. The proposed method exploits the properties of dual tree complex wavelet transform (DT-CWT) to remove blurring and noise without the need for assuming a specific noise model. Furthermore, it uses an adaptive shrinkage function based on median, mean and standard deviation of wavelet coefficients to suppress noise while preserving the sharpness of the reconstructed image. The efficacy of the proposed method was assessed with projections simulated from Shepp-Logan Phantom. Simulation results confirm that the proposed method produces consistently good reconstruction in terms of suppressing noise and preserving resolution in the reconstructed images.

Keywords: tomography reconstruction, FBP, regularized inverse filter, dual tree complex wavelet, wavelet denoising.

1. Introduction

Low-dose CT imaging is clinically desired and has been under investigation in the last decade [1]. Reconstructing the low-dose (either low mA or shorter acquisition period) CT images in the presence of additive noise is an ill-posed inverse problem. This requires a regularization of the reconstructed noise component, to achieve images with high spatial resolution and acceptable signal-to-noise ratio (SNR). Standard regularization methods include FBP with non-linear filtering corrections, expectation-maximization (EM) and maximum a posteriori (MAP) estimators [2-4]. Most of these existing techniques uses a *priori* information of random image field (RIF) and operates on the entire sinogram, rather than on projections individually. To address these limitations in terms of poor performance, instability and computational complexity, recent work has focused on a new family of regularizing methods based on wavelet analysis [5-7]. The main drawback with these existing wavelet based regularized methods is that the frequency resolution is constrained to octave bandwidth for sufficient implementation [8]. We propose here a reconstruction method which is particularly well adapted to this situation. It takes the advantage of Fourier-domain regularization bespoke to the convolution system to control the noise but uses it sparingly to keep the accompanying smearing distortions to the minimum required. The bulk of the noise removal and signal estimation is achieved using complex wavelet shrinkage. The present work also exploits the properties of DT-CWT viz. excellent directionality and explicit phase information to remove blur and noise without the need for assuming a specific noise model. By means of projections corrupted with different noise levels, the proposed method was assessed for simulated Shepp-Logan phantom reconstruction. The results demonstrate that even under high noisy condition, the present method resulted in an image with suppressed noise, as well with compromised spatial resolution.

+ Corresponding author. *E-mail address*: vthavamurugesan@yahoo.co.in.

2. A New Reconstruction Methodology

2.1. Theory of FBP

Radon transform is often used to model tomographic projection process for deterministic reconstruction of medical images [9]. The Radon transform of a two-dimensional function $f(x,y)$ is given by the collection of its projections, each ray is indexed by its distance to the origin and its angle [10], defined as

$$p(\xi,\varphi)=\int f(x,y)\delta((x\cos\varphi+y\sin\varphi)-\xi)dxdy \tag{1}$$

where δ is the Dirac delta function. An attractive feature of this transform is that it has an exact inversion formula. The digital implementation of (1) leads to the standard filtered back-projection (FBP) algorithm [11], which is carried out in two-steps. The first step, filtering part, multiplies the projections with filter kernel in frequency domain given by

$$P(\xi,\varphi)=FT^{-1}[FT[p(\xi,\varphi)]*H(\xi)] \tag{2}$$

where $H(\xi)$ denotes the filter function. The second step, the back projection part, propagates the measured sinogram back into the image space along the projection paths given by

$$F_{FBP}(x,y)=\int P(x\cos\varphi+\sin\varphi,\varphi)d\varphi \tag{3}$$

The basic blur function required for deconvolution results from the backprojection step. The backprojection blur function for large number of projections is approximately r^{-1} or the inverse of the spatial polar co-ordinate radius. Consequently, back-projecting in the presence of additive noise is an ill-posed inverse problem, which means that regularization has to be incorporated in the reconstruction procedure.

2.2. Regularization in FBP using Complex Wavelets

Wavelets have been previously introduced in tomography by a large number of researchers. The most widespread application of wavelets in tomography is local reconstruction [12, 13]. The standard implementation of FBP using wavelet transform was proposed in [14]. Few other authors have used wavelet methods to implement a post-filtering of a reconstructed image after it was reconstructed by a standard algorithm. Recently, the wavelet-vaguelette decomposition (WVD) [15] was applied to regularize FBP [16-18]. The main drawback with WVD method is that the noise variance becomes large when the system function contains zeros; making the method ill-posed. These problems are solved effectively in the present work by applying complex wavelet transform (CWT). In CWT, filters have complex coefficients as well generates complex output samples. However, a further problem arises in achieving perfect reconstruction for complex wavelet decomposition beyond level 1. To overcome this, Kingsbury [19] have recently developed the DT-CWT, which allows perfect reconstruction while still provides the other advantages of complex wavelets. Hence, the present work applies DT-CWT to regularize the filtering part of FBP and is summarized below:

1. *Regularization by complex wavelet denoising*: The actual denoising is achieved by thresholding

the coefficients with thresholds that are scale-wise adaptive, depending on standard deviation (σ), absolute mean (μ) and absolute median (M) of wavelet coefficients of the scale. To compute a complex threshold, the method has been extended to both real and imaginary domain, as in [20]. The value of threshold for real part is calculated as

$$T_{real} = \frac{1}{2}\frac{\sigma_{real}}{\mu_{real}} M_{real} \tag{4}$$

The threshold value for imaginary part is calculated as

$$T_{imag} = \frac{1}{2}\frac{\sigma_{imag}}{\mu_{imag}} M_{imag} \tag{5}$$

2. *Wavelet Filtering*: After computing the complex threshold values, the wavelet coefficients are filtered in wavelet domain to give real and imaginary part of wavelet coefficients for deblurred image as follows

$$W_{d\_real} = W_{o\_real} \frac{T_{real}^2}{T_{real}^2 + \sigma_n^2} \tag{6}$$

$$W_{d\_imag} = W_{o\_imag} \frac{T_{imag}^2}{T_{imag}^2 + \sigma_n^2} \tag{7}$$

3. Results and Discussion

We carried out tests with the Shepp-Logan phantom consisting of 10 superimposed ellipses [21]; a model used in tomography for evaluating properties of reconstruction algorithms. The phantom was discretized into a 128×128 image. Projection data were simulated by applying radon transform to the phantom, using 60 angles and 185 radial samples of projections. To evaluate the performance of the proposed approach under noisy condition, the simulated projection data were contaminated by multiplicative noise with different SNR.

The applicability of the proposed algorithm for tomographic reconstruction as well the metrical performance in terms of signal-to-noise ratio is exhibited in Fig.1. To obtain comparably best possible PSNR, the filter for FBP reconstruction and thresholding strategy for WVD reconstruction are optimized. Despite that the PSNRs of the FBP-reconstructed image and the WVD-reconstructed image are respectively 32.1 dB and 31.4 dB, the PSNR of the proposed algorithm is 35.6 dB. With visual inspection of Fig.1, we can observe that the FBP-reconstructed image is corrupted by a significant amount of noise which cannot be reduced unless the reconstructed image becomes extremely smoothed. As well, the WVD-reconstructed images are corrupted by artifacts and their smaller structures are more difficult to detect than in image reconstructed by proposed algorithm.

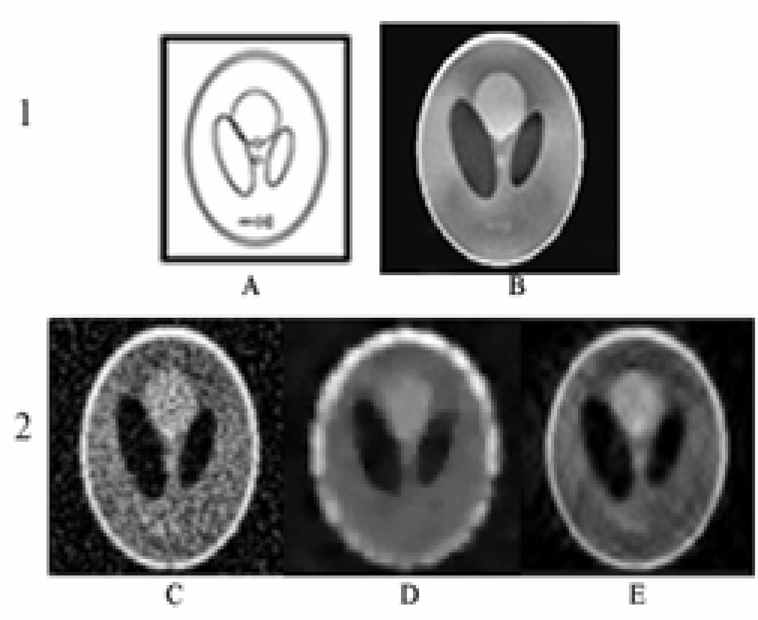

Fig.1: Reconstruction from noisy projections of Shepp-Logan phantom. The original phantom image (A) resembling the human brain features along with its cranial contour (B) is shown in row 1. The head phantom images reconstructed using standard FBP (C), WVD (D) and proposed algorithm (E) in row 2.

From the results depicted in Fig.1, we can infer that the standard FBP responds to noise and shows degradation in reconstruction quality under noisy condition. This may pose problems if the spin probe localizes selectively in a particular organ while whole-body imaging is performed. On other hand, WVD outperforms FBP in suppressing the noise but produces strip artifacts all over the ROI in the images reconstructed. However, the reconstruction with proposed algorithm offered better image quality in terms of SNR.

4. Conclusions

We have presented a new regularization method for tomography which combines features from the FBP algorithm and regularization theory. The filtering part of FBP performs Fourier-domain inversion followed by noise suppression in complex wavelet domain to provide a good estimate of the original image even with ill-conditioned systems. The preliminary results shown in this paper are extremely encouraging, and seem to indicate that a substantial improvement of the stability of the reconstruction and of quality of the image can be expected, especially under low dose imaging.

5. Acknowledgements

R. Murugesan thanks Department of Science and Technology, New Delhi, India for providing a major research grant. A part of this work was carried out under the UGC sponsored Center for Potential in Genomic Sciences Programme. Support in part by the Scientist Exchange Program of the Office of International Affairs, National Cancer Institute, NIH, DHHS, USA is also gratefully acknowledged.

6. References

[1] O. W. Linton, A. Fred, and F. A. Mettler, National conference on dose reduction in CT, with an emphasis on pediatric patients, *Amer. J. Roentgenol.*, 181, 2003, pp.321-329.

[2] L. A. Shepp and Y. Vardi, Maximum likelihood reconstruction for emission tomography, *IEEE Trans. Med. Imag.*, MI-1, 1982, pp.113-122.

[3] T.H. Farquhar, A. Chatziioannou, G. Chinn, M. Dahlbom and E. J. Hoffman, An investigation of filter choice for filtered back-projection reconstruction in PET, *IEEE Trans on Nuclear Science*, 45, 1998, pp.1133-1137.

[4] H. M. Hudson and R. S. Larkin, Accelerated image reconstruction using ordered subsets of projection data, *IEEE Trans. Med. Imag.*, 13, 1994, pp.601-609.

[5] A. Raheja, T. F. Doniere, and A. P. Dhawan, Multiresolution expectation maximization reconstruction algorithm for positron emission tomography using wavelet processing, *IEEE Transactions on Nuclear Science*, 46, 1999, pp.594-602.

[6] P. Kisilev, M. Jacobson, and Y. Y. Zeevi, Wavelet domain ML reconstruction in positive emission tomography, *Proceedings of 22nd Annual International Conference of the IEEE Engineering in Medicine and Biology Society*, pp.90-93, 2000.

[7] B. A. Mair, R. B. Carroll, and J. M. M. Anderson, Filter banks and the EM algorithm, *Proceedings of Nuclear Science Symposium*, pp.1747-1751, 1996.

[8] Elsa D. Angelini, Jérôme Kalifa and Andrew F. Laine, Harmonic multiresolution estimators for denoising and regularization of SPECT-PET data, *IEEE International Symposium on Biomedical Imaging*, U.S.A, 2002.

[9] S. R. Deans, *The Radon Transform and Some of its Applications*, Wiley Press, New York, 1983.

[10] F. Natterer, *The Mathematic of Computerized Tomography*, Wiley Press, Chicester, 1986.

[11] G.N. Ramachandran and A.V. Lakshminarayanan, 3D reconstructions from radiographs and electron micrographs: appl. of convolution instead of Fourier Trans., *Proc. Nat. Acad. Sci.*, 68, 1971, pp.2236-2240.

[12] T. Olson and J. DeStefano, Wavelet localization of the radon transform, *IEEE Trans. Image Processing*, 42, 1994, pp.2055-2067.

[13] F. Rashid-Farrokhi, K. Liu, C. Berenstein, and D. Walnut, Wavelet-based multiresolution local tomography, *IEEE Trans.Image Processing*, 22, 1997, pp.1412-1430.

[14] S. Zhao, Wavelet filtering for filtered backprojection in computer tomography, *J. Appl. Comput. Harmonic Anal.*, 6, 1999, pp.346-373.

[15] D. Donoho, Nonlinear solution of linear inverse problems by waveletvaguelette decompositions, *J. Appl. Comput. Harmonic Anal.*, 2(2), 1995, pp.101-126.

[16] Y. Choi, J. Y. Koo, and N. Y. Lee, Image reconstruction using the wavelet transform for positron emission tomography, *IEEE Trans Med Imag*, 20, 2001, pp.1188-1193.

[17] N. Lee and B. Lucier, Wavelet methods for inverting the radon transform with noisy data, *IEEE Trans. Image Processing*, 10, 2001.

[18] Jérôme Kalifa, Andrew Laine and Peter D. Esser, Regularization in tomographic reconstruction using thresholding estimators, *IEEE Trans Med Imag*, 22(3), 2003, pp.351-359.

[19] N. G. Kingsbury, Image processing with complex wavelet, *Phil. Trans. Roy Soc. London A*, 357, 1999, pp.2543-2560.

[20] A. Khare and U. S. Tiwary, A new method for deblurring and denoising of medical images using complex wavelet transform, *Proceedings of the IEEE*, pp.1897-1900, 2005.

[21] L.A Shepp and B.F. Logan, The Fourier reconstruction of a head section, *IEEE Transactions on Nucler Science*, NS-21, 1974.