

## Time-delayed Feedback Control of the Energy Resource Chaotic System

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(Received 28 February 2006 , accepted 30 March 2006 )

**Abstract:** In this paper, the control of energy resource chaotic system is investigated by time-delayed feedback control (TDFC) method. The controllability and the stability of the equilibriums of the system are verified. Several stable periodic orbits are got in the numerical simulations, which shows the effectivity of TDFC method.

**Keywords:** energy resource chaotic system; time-delayed feedback control; stability

### 1 Introduction

Chaos is sometimes undesirable, because a nonlinear system in the chaotic state is very sensitive to its initial condition and chaos causes often irregular behaviors in practical systems. Thus one may wish to avoid and eliminate such behaviors. Chaos control has attracted a great deal of attention from various fields since Huber published the first paper on chaos control in 1989 [1]. Over the last decades, many methods and techniques have been developed, such as OGY method [2], active control [3], observer-based control [4], feedback and nonfeedback control [5-8], inverse optimal control [9], adaptive control [10-11], etc.

Recently, time-delayed feedback control (TDFC) has provoked a renewal of interest within the context of chaotic dynamical systems [12]-[15].

We found an energy resource chaotic attractor which is a third order autonomous system; it displays very complex dynamical behaviors. We achieved chaotic synchronization for the energy resource system by applied the modified adaptive synchronization. Chaos in energy resource chaotic system is controlled to equilibrium points or periodic orbits by using feedback control and adaptive control [16-19].

This paper attempts to continue the study of chaos control problems of the energy resource chaotic system, by a time-delay feedback control technique. This paper is organized as follows. Section 2 briefly introduced the energy resource chaotic system. Time-delayed feedback control of the energy resource chaotic system is investigated in Section 3. Numerical simulation is presented in Section 4. The brief conclusions are finally given in Section 5.

### 2 Energy resource chaotic system

Energy resource chaotic system [16] is described by the following system of differential equations:

$$\begin{cases} \frac{dx}{dt} = a_1x \left(1 - \frac{x}{M}\right) - a_2(y + z) \\ \frac{dy}{dt} = -b_1y - b_2z + b_3x [N - (x - z)] \\ \frac{dz}{dt} = c_1z (c_2x - c_3) \end{cases} \quad (1)$$

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where  $x(t)$  is the energy resource shortage in  $A$  region,  $y(t)$  is the energy resource supply increment in  $B$  region,  $z(t)$  is the energy resource import in  $A$  region;  $a_i, b_i, c_i, M, N$  are positive real constants. This system has three equilibria:

$$O(0, 0, 0), S_1 = (x_1, y_1, z_1), S_2 = (x_2, y_2, z_2),$$

where

$$x_1 = \frac{a_2 b_3 M N - a_1 b_1 M}{a_2 b_3 M - a_1 b_1}, y_1 = \frac{a_1 b_3 (M - N) (a_2 b_3 M N - a_1 b_1 M)}{(a_2 b_3 M - a_1 b_1)^2}, z_1 = 0;$$

$$x_2 = \frac{c_3}{c_2}, y_2 = \frac{\left[ \frac{a_1}{a_2} \left( 1 - \frac{x_2}{M} \right) (b_3 x_2 - b_2) - b_3 x_2 + b_3 N \right] x_2}{b_1 + b_3 x_2 - b_2}, z_2 = \frac{\frac{a_1 b_1}{a_2} x_2 \left( 1 - \frac{x_2}{M} \right) - b_3 N x_2 + b_3 x_2^2}{b_1 + b_3 x_2 - b_2}.$$

From [16], this system has a chaotic attractor when  $a_1 = 0.09, a_2 = 0.15, b_1 = 0.06, b_2 = 0.082, b_3 = 0.07, c_1 = 0.2, c_2 = 0.5, c_3 = 0.4, M = 1.8, N = 1$  and initial condition  $[0.82, 0.29, 0.48]$ . The value of Lyapunov exponents of this system is obtained as  $(0.068, 0.016, -0.016)$ . We obtain the equilibrium points  $O(0, 0, 0), S_1(0.68, 0.2539, 0), S_2(0.8, 0.1255, 0.1412)$ , which are unstable.

### 3 Time-delayed feedback control of the energy resource chaotic system

We consider a general continuous-time nonlinear dynamical system

$$\dot{x} = f(x, t), x(t_0) = x_0 \in R^n \tag{2}$$

Suppose that this system has an (unstable) periodic solution  $\bar{x}(t)$  and  $x(t)$  is currently in a chaotic state. The task is to find a TDFC (with a proper delay-time  $\tau > 0$ )

$$u(t) = K(x(t) - x(t - \tau)) \tag{3}$$

to be added to  $f(x, t)$  so that the controlled system orbit can track the target:

$$\lim_{t \rightarrow \infty} \|x(t) - \bar{x}(t)\| = 0 \tag{4}$$

where  $\|y(t)\| = [y^T(t)y(t)]^{1/2}$  is the Euclidean norm, which is a function of time. The design problem is then to determine the control gain matrix  $K$  to achieve the goal (4).

Since the controlled system is

$$\dot{x} = f(x, t) + K(x(t) - x(t - \tau)) \tag{5}$$

and the periodic orbit is a solution of the original system, namely,

$$\dot{\bar{x}} = f(\bar{x}, t) \tag{6}$$

subtracting (5) from (6) yields the error dynamical system

$$\dot{e} = F(e, t) + K(x(t) - x(t - \tau)) \tag{7}$$

where  $e = x - \bar{x}$  and  $F(e, t) = f(x, t) - f(\bar{x}, t) = f(e + \bar{x}, t) - f(\bar{x}, t)$ .

**Proposition** <sup>[15]</sup>: Suppose that the given system (2) is autonomous. Let  $J(x) = f'(x) = \frac{\partial f(x)}{\partial x}$  be the system Jacobian and assume that  $J(x)$  is uniformly bounded by a constant matrix  $J_0$ , namely,  $J(x) \leq J_0$  for all  $x(t)$  and all  $t : t_0 \leq t < \infty$ . Then there is always a constant control gain matrix  $K$  such that the controlled orbit of system (5) is driven to approach a limit set in the phase space.

**Proof:** According to the fundamental theory of differential equations, the solution of the linearized system of (5) is

$$x(t) = e^{(t-t_0)[J(x)+K]}x_0 + \int_{t_0}^t e^{(t-s)[J(x)+K]}K(x(s) - x(s - \tau)) ds$$

which satisfies

$$\|x(t)\| \leq e^{(t-t_0)[J(x)+K]} \|x_0\| + \left\| \int_{t_0}^t e^{(t-s)[J(x)+K]} K(x(s) - x(s-\tau)) ds \right\|$$

where both  $x(t)$  and  $x(t-\tau)$  are chaotic, hence, are uniformly bounded in the phase space so that the above integral term converges. So is bounded. Since the real parts of all eigenvalues of  $[J_0 + K]$  can be made negative by a suitable  $K$ , the first term on the right hand side of the above inequality tends to zero as  $t \rightarrow \infty$ . This implies that the controlled orbit is always bounded and is always pointing inward in the phase space. It then follows the extended Poincare-Bendixson Theorem [20] that the controlled orbit approaches a limit set in the phase space.

Let system (1) be controlled by a time-delayed feedback control of the form:

$$\begin{cases} \frac{dx}{dt} = a_1x \left(1 - \frac{x}{M}\right) - a_2(y+z) - k_a[x(t) - x(t-\tau)] \\ \frac{dy}{dt} = -b_1y - b_2z + b_3x[N - (x-z)] \\ \frac{dz}{dt} = c_1z(c_2x - c_3) \end{cases} \quad (8)$$

(i) The Jacobian matrix of the system (8) at the point  $O(0, 0, 0)$  is

$$J_0 + K_a = \begin{bmatrix} a_1 - k_a & -a_2 & -a_2 \\ b_3 & -b_1 & -b_2 \\ 0 & 0 & -c_1c_3 \end{bmatrix}.$$

It has the characteristic equation

$$f(\lambda) = (\lambda + c_1c_3)(\lambda^2 + (k_a + b_1 - a_1)\lambda + a_2b_3 + b_1k_a - a_1b_1) = 0 \quad (9)$$

According to Proposition, it is sufficient to determine a control gain matrix  $K_a = \begin{bmatrix} -k_a & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ , so

that the matrix  $J_0 + K$  has all eigenvalues with negative real parts. When  $a_1 = 0.09$ ,  $a_2 = 0.15$ ,  $b_1 = 0.06$ ,  $b_2 = 0.082$ ,  $b_3 = 0.07$ ,  $c_1 = 0.2$ ,  $c_2 = 0.5$ ,  $c_3 = 0.4$ ,  $M = 1.8$ ,  $N = 1$ , by calculations, we can obtain  $k_a > 0.03$ , which satisfies the required condition in Proposition. Therefore, it achieves the intended chaos control to  $O(0, 0, 0)$  or a periodic orbit.

(ii) The Jacobian matrix of the system (8) at the point  $S_1 = (x_1, y_1, z_1)$  is

$$J_1 + K_a = \begin{bmatrix} a_1 \left(1 - \frac{2x}{M}\right) - k_a & -a_2 & -a_2 \\ b_3(N - 2x) & -b_1 & -b_2 + b_3x \\ 0 & 0 & c_1(c_2x - c_3) \end{bmatrix}$$

When we fix parameters as above, we obtain the equilibrium point  $S_1(0.68, 0.2539, 0)$ . By calculations, we can obtain the characteristic equation of the Jacobian matrix of the system (8) at  $S_1$

$$(\lambda + 0.012)(\lambda^2 + (k_a + 0.038)\lambda + 0.06k_a - 0.0051) = 0.$$

According to Proposition and Routh-Hurwitz criteria, the equilibrium point  $S_1$  is asymptotically stable if the following conditions are satisfied:

$$\begin{cases} k_a + 0.038 > 0 \\ 0.06k_a - 0.0051 > 0 \end{cases}$$

We can obtain  $k_a > 0.085$ , which satisfies the required condition in Proposition. Therefore, it achieves the intended chaos control to  $S_1$  or a periodic orbit.

(iii) The Jacobian matrix of the system (8) at the point  $S_2(x_2, y_2, z_2)$  is

$$J_2 + K_a = \begin{bmatrix} a_1 \left(1 - \frac{2x}{M}\right) - k_a & -a_2 & -a_2 \\ b_3(N - 2x + z) & -b_1 & -b_2 + b_3x \\ c_1c_2z & 0 & 0 \end{bmatrix}$$

When we fix parameters as above, we obtain the equilibrium point  $S_2(0.8, 0.1255, 0.1412)$ . By calculations, we can obtain the characteristic equation of the Jacobian matrix of the system (8) at  $S_2$

$$\lambda^3 + (k_a + 0.05)\lambda^2 + (0.06k_a - 0.0033)\lambda + 0.000072 = 0.$$

According to Proposition and Routh-Hurwitz criteria, the equilibrium point  $S_2$  is asymptotically stable if the following conditions are satisfied:

$$\begin{cases} k_a + 0.05 > 0 \\ 0.06k_a - 0.0033 > 0 \\ (k_a + 0.05)(0.06k_a - 0.0033) > 0.000072 \end{cases}$$

We can obtain  $k_a > 0.0654$ , which satisfies the required condition in Proposition. Therefore, it achieves the intended chaos control to  $S_2$  or a periodic orbit. Completely similarly, let system (1) be controlled by a time-delayed feedback control of the form:

$$\begin{cases} \frac{dx}{dt} = a_1x \left(1 - \frac{x}{M}\right) - a_2(y + z) \\ \frac{dy}{dt} = -b_1y - b_2z + b_3x [N - (x - z)] - k_b [y(t) - y(t - \tau)] \\ \frac{dz}{dt} = c_1z (c_2x - c_3) \end{cases} \quad (10)$$

(iv) The Jacobian matrix of the system (10) at the point  $O(0, 0, 0)$  is

$$J_0 + K_b = \begin{bmatrix} a_1 & -a_2 & -a_2 \\ b_3 & -b_1 - k_b & -b_2 \\ 0 & 0 & -c_1c_3 \end{bmatrix}$$

It has the characteristic equation

$$f(\lambda) = (\lambda + c_1c_3)(\lambda^2 + (b_1 - a_1 + k_b)\lambda + a_2b_3 - a_1b_1 - a_1k_b) = 0$$

When we fix parameters as above, we can obtain the characteristic equation of the Jacobian matrix of the system (10) at  $O(0, 0, 0)$

$$(\lambda + 0.08)(\lambda^2 + (k_b - 0.03)\lambda + 0.0051 - 0.09k_b) = 0$$

By calculations, we can obtain  $k_b \in (0.03, 0.0567)$ , satisfies the required condition in Proposition. Therefore, it achieves the intended chaos control to  $O(0, 0, 0)$  or a periodic orbit.

(v) The Jacobian matrix of the system (10) at the point  $S_1(x_1, y_1, z_1)$  is

$$J_1 + K_b = \begin{bmatrix} a_1 \left(1 - \frac{2x}{M}\right) & -a_2 & -a_2 \\ b_3(N - 2x) & -b_1 - k_b & -b_2 + b_3x \\ 0 & 0 & c_1(c_2x - c_3) \end{bmatrix}$$

When we fix parameters as above, we obtain the equilibrium point  $S_1(0.68, 0.2539, 0)$ . By calculations, we can obtain the characteristic equation of the Jacobian matrix of the system (10) at  $S_1$

$$(\lambda + 0.012)(\lambda^2 + (k_b + 0.038)\lambda - (0.022k_b + 0.0051)) = 0.$$

According to Proposition and Routh-Hurwitz criteria, the equilibrium point  $S_1$  is asymptotically stable if the following conditions are satisfied:

$$\begin{cases} k_b + 0.038 > 0 \\ -0.022k_b - 0.0051 > 0 \end{cases}$$

Therefore, for any  $k_b$  and  $\tau$ , the system (10) is impossible to be controlled to  $S_1$ .

(vi) The Jacobian matrix of the system (10) at the point  $S_2(x_2, y_2, z_2)$  is

$$J_2 + K_b = \begin{bmatrix} a_1 \left(1 - \frac{2x}{M}\right) & -a_2 & -a_2 \\ b_3(N - 2x + z) & -b_1 - k_b & -b_2 + b_3x \\ c_1c_2z & 0 & 0 \end{bmatrix}$$

When we fix parameters as above, we obtain the equilibrium point  $S_2(0.8, 0.1255, 0.1412)$ . By calculations, we can obtain the characteristic equation of the Jacobian matrix of the system (10) at  $S_2(x_2, y_2, z_2)$

$$\lambda^3 + (0.05 + k_b)\lambda^2 - (0.0032994 + 0.01k_b)\lambda + 0.000072 + 0.002118k_b = 0.$$

According to Proposition and Routh-Hurwitz criteria, the equilibrium point  $S_2$  is asymptotically stable if the following conditions are satisfied:

$$\begin{cases} k_b + 0.05 > 0 \\ -(0.0032994 + 0.01k_b) > 0 \\ 0.000072 + 0.002118k_b > 0 \\ -(k_b + 0.05)(0.0032994 + 0.01k_b) > 0.000072 + 0.002118k_b \end{cases}$$

Therefore, for any  $k_b$  and  $\tau$ , the system (10) is impossible to be controlled to  $S_2$ .

### 4 Numerical simulations

Numerical experiments are carried out to integrate the controlled system by the MATLAB. The parameters are chosen as  $a_1 = 0.09, a_2 = 0.15, b_1 = 0.06, b_2 = 0.082, b_3 = 0.07, c_1 = 0.2, c_2 = 0.5, c_3 = 0.4, M = 1.8, N = 1$  to ensure the existence of chaos in the absence of control. Let initial states  $x = 0.6, y = 0.16, z = 0.07$ , when  $k_a = 0.04, \tau = 30$  achieves the intended chaos control to  $O(0, 0, 0)$  as shown in Fig.1; when  $k_a = 0.04, \tau = 39.8$ , chaos of the controlled system (8) is suppressed to a limit cycle as shown in Fig.2; when  $k_b = 0.05, \tau = 4.8$ , chaos of the controlled system (10) is suppressed to a limit cycle as shown in Fig.3.

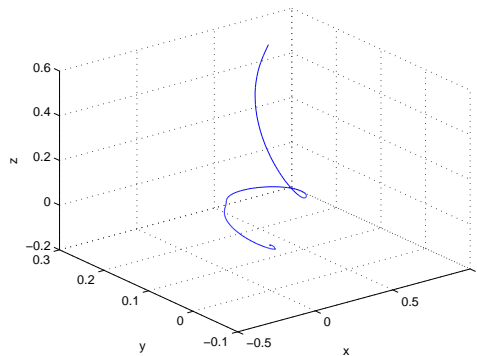


Figure 1: The equilibrium point of the system (8) is stabilized for  $k_a = 0.04, \tau = 30$ .

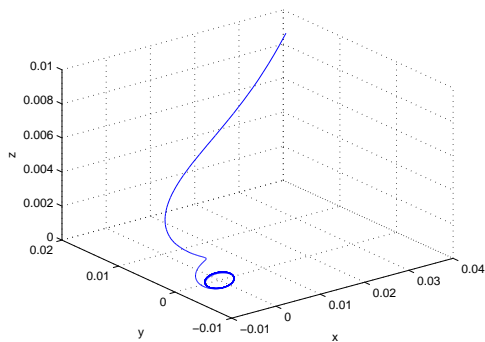


Figure 2: The limit cycles of the controlled system (8) for  $k_a = 0.04, \tau = 39.8$ .

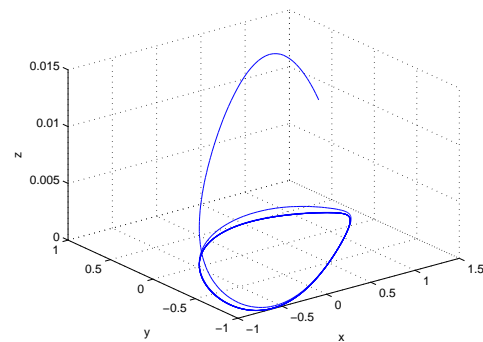


Figure 3: The limit cycles of the controlled system (10) for  $k_b = 0.05, \tau = 4.8$ .

## 5 Conclusions

The control problem of energy resource chaotic system is investigated. We construct the time-delayed feedback controller which can render the energy resource chaotic system asymptotically stable. The several stable periodic orbits are obtained in the numerical simulations. The results showed the validity of the proposed controllers.

### Acknowledgements

Research was supported by the National Nature Science Foundation of China (No. 90210004), National Social Science Foundation of China (No. 02BJY051) and Nature Science Foundation of Jiangsu Province of China (No. 05KJB110018).

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