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Dynamics of Periodic Delayed Planar Neural Networks

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Abstract: This paper formulates and studies a model of periodic delayed planar systems. The model can well describe many practical architecture of delayed neural networks, which is generation of some existing neural such as delayed cellular networks(DCN) and periodic delayed neural networks(PDNNS). The existence and global exponential stability of PDPNN's periodic solutions are investigated, without assuming the smoothness, monotonicity and boundedness of the activation function.

Key words: neural networks; delayed; periodic

1 Introduction

Recently, delayed neural networks (DNNS) have abstracted increasing interest in both theoretical studies and engineering applications. They have been successfully applied in signal processing, pattern recognition, optimization and associative memories, especially in various types of motion-related application such as speed detection of moving objects, processing of moving images, and pattern classification.

As we all know, neural networks are complex and large-scale nonlinear dynamics, which the dynamics of the delayed neural network are even richer and more complicated. Dynamics of the delayed neural network have been studied in [3, 5, 14]. The delayed neural network models with two neurons have been investigated, see [7, 10, 14, 15]. In[10], Zhou considered the periodic delayed neural networks(PDNNS).

$$\dot{x}_i(t) = -c_i(t)x_i(t) + \sum_{j=1}^{n} a_{ij}(t)f_j(x_j(t)) + \sum_{j=1}^{n} a^{\tau}_{ij}(t)f_j(x_j(t-\tau_{ij})) + u_i(t) \quad i = 1,2,3\cdots n, \tag{1}$$

or

$$\dot{x}(t) = -c(t)x(t) + A(t)f(x(t)) + A^{\tau}(t)f(x(t-\tau)) + u(t),$$

where $c(t) = diag(c_1(t),\cdots,c_n(t))$, $A(t) = (a_{ij}(t))_{n\times n}$, $A^{\tau}(t) = (a^{\tau}_{ij}(t))_{n\times n}$ and $u(t) = (u_1(t),\cdots,u_n(t))^T$ are the continuous ω-periodic matrix-valued functions and ω-periodic functions with respect to the time variable t.

Recently, Huang considered the dynamics of planar systems with tine-varying delays:

$$\begin{cases} \dot{x}_1(t) = -a_1(t)x_1(t) + b_1(t)f_1(x_1(t-\tau_{11}(t)), x_2(t-\tau_{12}(t))) + I_1(t) \\ \dot{x}_2(t) = -a_2(t)x_2(t) + b_2(t)f_2(x_1(t-\tau_{21}(t)), x_2(t-\tau_{22}(t))) + I_2(t) \end{cases} \tag{2}$$

where $a_i \in (R,(0,\infty)), b_i, I_i \in c(R,R), i = 1,2$ are periodic with a common period $w > 0$, $f_i \in c(R^2,R)$. $\tau_{ij} \in c(R,(0,\infty))$, $i,j = 1,2$ being ω-periodic.

Motive by above mentioned, in this paper, we consider the following planar systems:

$$\begin{cases} \dot{x}_1(t) = -a_1(t)x_1(t) + b_1(t)f_1(x_1(t),x_2(t)) + c_1(t)f_1(x_1(t-\tau_{11}(t)), x_2(t-\tau_{12}(t)) \ + I_1(t) \\ \dot{x}_2(t) = -a_2(t)x_2(t) + b_2(t)f_2(x_1(t),x_2(t)) + c_2(t)f_2(x_1(t-\tau_{21}(t)), x_2(t-\tau_{22}(t)) + I_2(t) \end{cases} \tag{3}$$

where $a_i \in C(R,(0,\infty))$, b_i, c_i, $l_i \in C(R,R)$, $i = 1,2$ are ω-periodic functions with respect to the time variable t, $f_i \in C(R^2,R)$, $\tau_{ij} \in C(R,(0,\infty))$, $i,j = 1,2$ being ω-periodic.

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2 Preliminaries

We assume each of the activation functions $f_i(t), i = 1, 2$ is continuous, the set of real numbers possess the following properties.

$(A_1) : |f_i(x_1, x_2)| \le \alpha_i |x_1| + \beta_i |x_2| + M_i$ for all $(x_1, x_2)^\tau \in R^2$, $i = 1, 2$ where α_i, $\beta_i \ge 0$, $M_i > 0$ are constants.

(B_1) : There exist constants $\alpha_i \ge 0$, $\beta_i \ge 0$ such that

$$|f_i(x_1, x_2) - f_i(y_1, y_2)| \le \alpha_i |x_1 - y_1| + \beta_i |x_2 - y_2|$$

for any $(x_1, x_2)^\tau \in R^2$, $(y_1, y_2)^\tau \in R^2$.

For a continuous function $f : [0, w] \mapsto R$, we let

$$f_* = \min_{t\in[0,\omega]} f(t), f^* = \max_{t\in[0,\omega]} f(t), \bar{f} = \frac{1}{\omega}\int_0^w f(t)dt,$$

$$l_i = \frac{|\bar{b}_i|}{\bar{a}_i}\alpha_i + w \cdot |\bar{b}_i| \alpha_i, \quad m_i = (\frac{1}{\bar{a}_i} + w) |\bar{b}_i| \beta_i,$$

$$n_i = (\frac{1}{\bar{a}_i} + w) |\bar{c}_i| \cdot \alpha_i, \quad R_i = (\frac{1}{\bar{a}_i} + w) \cdot |\bar{c}_i| \cdot \beta_i,$$

$$\mathrm{T}_i = (\frac{1}{\bar{a}_i} + w) \cdot |\bar{I}_i| + (|\bar{b}_i| + |\bar{c}_i|)(\frac{1}{\bar{a}_i} + \omega)M_i, i = 1,2.$$

To our best knowledge, the existence and global exponential stability of ω-periodic solution of the system (3) without assuming the smoothness, monotonicity and boundedness of the activation functions has not been studied in pervious works. We shall employ of a periodic solution to (3) in Section 3. Then, in Section 4, we present some sufficient conditions for the global exponential stability of the periodic solution. Example and numerical simulation are given in Section 5.

3 Existence of periodic solution

In this section, we first review some concepts and give the well-known Mawhin continuity theorem [2].

Let $X$and Z be normed vector spaces and L be the identity mapping, $L : DomL \subset X \to Z$ be a linear mapping, and $N : X \to Z$ be a continuous mapping. The mapping L will be called a Fredholm mapping of index zero if $dim\ kerL = co\ dim\ ImL < +\infty$ and ImL is closed in Z. If L is a Fredholm mapping of index zero, there must exist continuous projectors $P : X \mapsto X$ and $Q : Z \mapsto Z$ such that $ImP = KerL$ and $ImL = KerQ = Im(I - Q)$. It follows that $L|_{DomL\cap\ker P} : DomL \cap KerP \to ImL$ is invertible, and the inverse of this map is denoted by K_p. If Ω is an open bounded subset of X, the mapping N with be called L-compact on $\overline{\Omega}$ if $QN(\overline{\Omega})$ is bounded and $K_p(I - Q)N : \bar{\Omega} \to X$ is compact. Since ImQ is isomorphic to $KerL$, there exists an isomorphism $J : ImQ \to KerL$.

Let $\Omega \subset R^n$ be open and bounded$f \in C^1(\Omega, R^n) \cap (\bar{\Omega}, R^n)$ and $y \in R^n\backslash f(\partial\Omega \cup s_f)$, y is a regular value of f. Here, $S_f = \{x \in \Omega : J_f(x) = 0\}$, the critical set of f, and J_f is the Jacobian of f at x. Then the degree $\deg\{f, \Omega, y\}$ is defined by

$$\deg\{f, \Omega, y\} = \sum_{x\in f^{-1}(y)} sgnJ_f(x)$$

with the agreement that the above sum is zero if $f^{-1}(y) = \phi$. For more details about degree theory, we refer to the book by Deming [3].

Lemma 1 *(Continuation Theorem, Gaines and Mawhin [8]) Let $\Omega \subset X$ be an open bounded set and L be a Fredholm mapping of index zero. Assume that $N : X \to Z$ is a continuous operator and is L-compact on $\bar{\Omega}$. Furthermore, suppose that*

(a) $Lx \ne \lambda Nx$ for all $\lambda \in (0, 1)$ and $x \in \partial\Omega \cap DomL$,

(b) $QNx \ne 0$ for $x \in \partial\Omega \cap KerL$,

(c) $\deg\{JQNx, \Omega \cap KerL, 0\} \ne 0$. Then,$Lx = Nx$ has at least one solution in $\bar{\Omega}$.

Theorem 1 *Suppose* (A_1) *hold and*

$$\frac{D_1}{D} > 0, \quad \frac{D_2}{D} > 0,$$

where $D = (m_1{+}l_1)\,(l_2 + n_2){-}(l_1 + n_1 - 1)\,(m_2 + R_2 - 1)$*,* $D_1 = T_2\,(l_1 + n_1 - 1){-}T_1\,(l_2 + n_2)\,, D_2 = T_1\,(m_2 + R_2 - 1) - T_2\,(m_1 + R_1)$*. Then system (3) has a* $w-$*periodic solution.*

Proof. $X = Z = \left\{u = u(t) = (x_1(t), x_2(t)) \in c(R, R^2) : u(t + w) = u(t)\right\}$ with norm $\|x_i(t)\| = \max\limits_{t\in[0,w]} |x_i(t)|$,$\|u\| = \max \|x_i(t)\|$,$i = 1, 2$. Then, X and Z are both Banach spaces. $L : DomL \cap X \to Z$, for all $u \in$ X, $Lu = \dot{u}$. We can see L is a linear mapping.

$$DomL = \{u \in X|\dot{u}(t) \text{ exists and is continuous on R}\}\,.$$

$$N : X \to Z$$

$$Nx_i(t) = -a_i(t)x_i(t) + b_i(t)f_i(x_1(t), x_2(t)) + c_i(t)f_i(x_1(t - \tau_{i1}(t)), x_2(t - \tau_{i2}(t))) + I_i(t), i = 1{,}2.$$

$$KerL = \{\,u|\,u \in X, x_i = h, h \in R, i = 1, 2\} \subset X,$$

$$ImL = \left\{u \left| \int_0^w u(t)dt = 0, u \in Z \right.\right\} \subset Z,$$

$$\dim KerL = Co\dim ImL = 2 < \infty.$$

So ImL is closed in Z.

Define $P : X \to X$, $Pu = \frac{1}{w}\int_0^w u(t)dt$, for all $u \in$ X.

$$Q : Z \to Z\ , Qu = \frac{1}{w}\int_0^w u(t)dt\ ,\ \forall u \in \mathrm{Z}$$

Then, one has $KerL = ImP$, $KerQ = ImL = Im\,(I - Q)$. Then L is a Fredholm mapping of index zero. It follows that $L\,|_{DomL\cap\ker P}\ : DomL \cap KerP \to ImL$ is inversed and the inverse of this map is denoted by K_p.

$$K_p : ImL \to KerP \cap DomL.$$

is given by $K_p(u) = \begin{pmatrix} \int_0^t x_1(s)ds - \frac{1}{w}\int_0^w\int_0^s x_1(v)dvds \\ \int_0^t x_2(s)ds - \frac{1}{w}\int_0^w\int_0^s x_2(v)dvds \end{pmatrix}$. Thus $QN : X \to R$.

$$QNu(t) = \begin{pmatrix} \frac{1}{w}\int_0^w \begin{bmatrix} (-a_1(t)x_1(t) + b_1(t)f_1(x_1(t), x_2(t)) \\ +c_1(t)f_1(x_1(t - \tau_{11}(t)), x_2(t - \tau_{12}(t))) + I_1(t) \end{bmatrix} dt \\ \frac{1}{w}\int_0^w \begin{bmatrix} (-a_2(t)x_2(t) + b_2(t)f_2(x_1(t), x_2(t)) \\ +c_2(t)f_2(x_1(t - \tau_{21}(t), x_2(t - \tau_{22}(t))) + I_2(t) \end{bmatrix} dt \end{pmatrix}$$

$$QN : X \to R, K_p(I - Q)Nu(t) : X \to X.$$

$$K_p(I - Q)Nu(t) = \int_0^t -a_i(s)x_i(s) + b_i(s)f_i(x_1(s), x_2(s)) + c_i(s)f_i(x_1(s - \tau_{i1}(s), x_2(s - \tau_{i2}(s)))$$

$$+I_i(s)ds - \frac{1}{w}\int_0^w\int_0^s \left\{ \begin{array}{l} -a_i(v)x_i(v) + b_i(v)f_i(x_1(v), x_2(v)) \\ +c_i(v)f_i(x_1(v - \tau_{i1}(v)), x_2(v - \tau_{i2}(v))) + I_i(v) \end{array} \right\} dvds$$

$$+((\frac{1}{2} - \frac{t}{w})\int_0^w \begin{array}{l} \{-a_i(v)x_i(v) + b_i(v)f_i(x_1(v), x_2(v)) \\ +c_i(v)f_i(x_1(v - \tau_{i1}(v)), x_2(v - \tau_{i2}(v))) + I_i(v) \end{array} \}dv$$

Then, QN and $K_p(I - Q)N$ are both continuous. Using the Arela-Ascoti Theorem, it is easy to show that $\overline{K_P(I - Q)N(\bar{\Omega})}$ is compact for any open bounded set $\Omega \subset X$.Note that $\overline{K_P(I - Q)N(\bar{\Omega})}$ is a compact operator and $QN(\bar{\Omega})$ is bounded, therefore, N is L-compact on $\bar{\Omega}$ with any bounded open subset $\Omega \subset X$. Since $ImQ = KerL$, we take the isomorphism J of ImQ onto $KerL$ to be the identity mapping.

Now, it needs to show that there exists a domain Ω satisfies all the requirements given in Lemma1.

Consider the operator equation $Lu = \lambda Nu$, $\lambda \in [0, 1]$. We have

$$\begin{cases} \dot{x}_1(t) = \lambda \begin{Bmatrix} -a_1(t)x_1(t) + b_1(t)f_1(x_1(t), x_2(t)) \\ +c_1(t)f_1(x_1(t-\tau_{11}(t)), x_2(t-\tau_{12}(t))) + I_1(t) \end{Bmatrix} \\ \dot{x}_2(t) = \lambda \begin{Bmatrix} -a_2(t)x_1(t) + b_2(t)f_2(x_1(t), x_2(t)) \\ +c_2(t)f_2(x_1(t-\tau_{21}(t)), x_2(t-\tau_{22}(t))) + I_2(t) \end{Bmatrix} \end{cases} \tag{4}$$

Assume that $u = u(t) \in X$ is a solution of system (4) for some $\lambda \in (0, 1)$. We needs to prove that $u = u(t)$ is uniformly bounded with respect to λ to the operator (4), integrating it from 0 to w over the variable t gives

$$0 = \int_0^w \dot{x}_i(t)dt = \lambda \int_0^w [-a_i(t)x_i(t) + b_i(t)f_i(x_1(t), x_2(t)) + c_i(t)f_i(x_1(t-\tau_{i1}(t)), x_2(t-\tau_{i2}(t))) + I_i(t)]dt$$

Hence,

$$\int_0^w a_i(t)x_i(t)dt = \int_0^w [b_i(t)f_i(x_1(t), x_2(t)) + c_i(t)f_i(x_1(t-\tau_{i1}(t)), x_2(t-\tau_{i2}(t))) + I_i(t)]dt$$

From(A_1)we know that

$$|f_i(x_1, x_2)| \le \alpha_i |x_1| + \beta_i |x_2| + M_i$$

$$\le |\bar{b}_i| \cdot (\alpha_i \cdot |x_1(t)|^* + \beta_i |x_2(t)|^*) + |\bar{c}_i| \cdot (\alpha_i |x_1(t-\tau_{i1}(t))|^* + \beta_i |x_2(t-\tau_{i2}(t))|^*) + (|\bar{b}_i| + |\bar{c}_i|)M_i + |\bar{I}_i|$$

$$\le |\bar{b}_i| \cdot \alpha_i |x_1(t)|^* + |\bar{b}_i| \cdot \beta_i |x_2(t)|^* + |\bar{c}_i| \cdot \alpha_i |x_1(t-\tau_{i1}(t))|^* + |\bar{c}_i| \cdot \beta_i |x_2(t-\tau_{i2}(t))|^* + (|\bar{b}_i| + |\bar{c}_i|)M_i + |\bar{I}_i|$$

It follows that

$$|x_i(t)|_* \le \frac{|\bar{b}_i|}{\bar{a}_i} \cdot \alpha_i |x_1(t)|^* + \frac{|\bar{b}_i|}{\bar{a}_i} \cdot \beta_i |x_2(t)|^* + \frac{|\bar{a}|}{\bar{a}_i}\alpha_i |x_1(t-\tau_{i1}(t))|^*$$

$$+ \frac{|\bar{c}_i|}{\bar{a}_i}\beta_i |x_2(t-\tau_{i2}(t))|^* + \frac{(|\bar{b}_i| + |\bar{c}_i|)M_i + |\bar{I}_i|}{\bar{a}_i}, i = 1, 2,$$

As a component of $u(t)$ is continuously differentiable. Then, there exists $t_i \in [0, w]$ such that $|x_i(t_i)| = |x_i(t)|_*$. Then for $t \in [t_i, t_i + w]$, we have

$$|x_i(t)| \le |x_i(t_i)| + \int_{t_i}^t D^+ |x_i(t)|\, dt$$

$$< (\frac{|\bar{b}_i|}{\bar{a}_i}\alpha_i + w \cdot |\bar{b}_i| \cdot \alpha_i) |x_1(t)|^* + \left\{(\frac{1}{\bar{a}_i} + w) \cdot |\bar{b}_i| \beta_i\right\} \cdot |x_2(t)|^*$$

$$+ \left\{(\frac{1}{\bar{a}_i} + w) \cdot |\bar{c}_i| \cdot \alpha_i\right\} \cdot |x_1(t-\tau_{i1}(t))|^* + \left\{(\frac{1}{\bar{a}_i} + w) \cdot |\bar{c}_i| \cdot \beta_i\right\} \cdot |x_2(t-\tau_{i2}(t))|^*$$

$$+ (|\bar{b}_i| + |\bar{c}_i|)(\frac{1}{\bar{a}_i} + w)M_i + \frac{|I_i|}{\bar{a}_i} + (\frac{1}{a_i} + w)|\bar{I}_i|$$

$$= l_i |x_1(t)|^* + m_i |x_2(t)|^* + n_i |x_1(t-\tau_{i1}(t))|^* + R_i |x_2(t-\tau_{i2}(t))|^* + T_i,$$

where $D^+ |x_i(t)|$ denote the right derivative of $|x_i(t)|$ along the solutions of system (4). From $k_1 = \frac{D_1}{D}$, $k_2 = \frac{D_2}{D}$, we find that

$$\begin{cases} k_1 = l_1k_1 + m_1k_2 + n_1k_1 + R_1k_2 + T_1 \\ k_2 = l_2k_1 + m_2k_2 + n_2k_1 + R_2k_2 + T_2 \end{cases},$$

so $k_1 = (l_1 + n_1)k_1 + (m_1 + R_1)k_2 + T_1$, $k_2 = (l_2 + n_2)k_1 + (m_2 + R_2)k_2 + T_2$.

We choose a constant number $d > 1$ and take

$$\Omega = \left\{(x_1, x_2)^T \in (R, R^2), \|x_i\| < dk_i, i = 1{,}2 \right\},$$

we find that

$$|x_i(t)|^* \le l_i\,|x_1(t)|^* + m_i\,|x_2(t)|^* + n_i\,|x_1(t-\tau_{i1}(t))|^* + R_i\,|x_2(t-\tau_{i2}(t))|^* + T_i$$

$$\le l_i \cdot dk_1 + m_i dk_2 + n_i dk_1 + R_i dk_2 + T_i$$

$$< d(l_i k_1 + m_i k_2 + n_i k_1 + R_i k_2 + T_i)$$

$$= dk_i,$$

then$\|x_i(t)\| = |x_i(t)|^* < dk_i$, for $i = 1, 2$. Clearly, dk_i, $i = 1, 2$ are independent of λ. So that $Lu \neq \lambda Nu$ for $\lambda \in (0, 1)$, $u \in \partial\Omega \cap$ DomL. Thus, condition (a) of lemma 1 is satisfied.

For $u \in \partial\Omega \cap kerL = \partial\Omega \cap R^2$, we can see that u is a constant vector in R^2 with $|x_i| = dk_i, i = 1, 2$. For $QNu = JQNu$, so we have

$$QNu = -x_i(t)\int_0^w a_i(t)dt + f_i(x_1, x_2)\int_0^w b_i(t)dt$$

$$+f_i(x_1(t-\tau_{i1}), x_2(t-\tau_{i2}))\int_0^w c_i(t)dt$$

Suppose that there exists u so that $|QNu| = 0$. Thus $\bar{a}_i(t)x_i(t) = f_i(x_1, x_2)\bar{b}_i(t) + f_i(x_1(t-\tau_{i1}), x_2(t-\tau_{i2}))\bar{c}_i(t)$.

So, we have

$$|x_i(t)| = dk_i$$

$$= \frac{\bar{b}_i(t)}{\bar{a}_i(t)} f_i(x_1, x_2) + \frac{\bar{c}_i(t)}{\bar{a}_i(t)} f_i(x_1(t-\tau_{i1}), x_2(t-\tau_{i2}))$$

$$\le \frac{\bar{b}_i(t)+\bar{c}_i(t)}{\bar{a}_i(t)}(\alpha_i dk_i + \beta_i dk_i + u_i)$$

$$< (l_i + n_i)\,k_1 + (m_i + R_i)k_2 + T_i = dk_i$$

We find it is contraction, so $QNu \neq 0$ for all $u \in \partial\Omega \cap \ker L$. Thus, condition (b) of lemma 1 is satisfied.

Now, let $G(\alpha, u) = -\alpha diag(\bar{a}_1, \bar{a}_2)u + (1-\alpha)QNu$ $\alpha \in$ [0,1], where G is the isomorphic map $G : (\bar{\Omega} \cap KerL) * [0, 1] \to \Omega \cap KerL$.

Then

$$deg\,\{JQN, \Omega \cap KerL, 0\} = \deg\{QN, \Omega \cap KerL, 0\}$$

$$= \deg\{G(\cdot, 0), \Omega \cap KerL, 0\}$$

$$= \deg\{G(\cdot, 1), \Omega \cap KerL, 0\}$$

$$= sgn\begin{vmatrix} -\bar{a}_1 \; 0 \\ 0 \text{ - } \bar{a}_2 \end{vmatrix} \neq 0$$

So condition (c) of lemma1 is satisfied. By now, we prove that Ω satisfies all the conditions of lemma1 So the equation $Lx = Nx$ has at least one $w-$periodic solution. ■

4 Global exponential stability of periodic solution

In this section, we will study the global exponential stability of the periodic solution.

Definition 2 *Periodic solution $\tilde{x}(t) = (\tilde{x}_1(t), \tilde{x}_2(t))^T$ is said to be globally exponentially stable, if there exist $\lambda > 0$ and $M \ge 1$, such that for any solution $x(t) = (x_1(t), x_2(t))^T$ of (3). We have $\|x_i(t) - \tilde{x}_i(t)\| \le M\,\|\phi - \tilde{x}(t)\|\,e^{-\lambda t}$ for $t \ge 0$.*

If $\tilde{x}(t)$is a periodic solution to system (3), we denote the sup-norm as follow:

$$\|\Phi(t)-\tilde{x}(t)\| = \sup_{-\tau\le t\le 0} |\Phi(t)-\tilde{x}(t)| \;\; \tau = \max\sup\{\tau_{ij}\}, i,j=1,2,$$

where $\Phi(t)$ is a continuous function, τ is a positive constant.

Theorem 3 *Assume* (B_1) *hold and* $\frac{D_1}{D} > 0$*,* $\frac{D_2}{D} > 0$*. Furthermore,* $a_{i*} > a_i^*(|b_i^*| + |c_i^*|)(\alpha_i + \beta_i)$*, for* $i = 1, 2$*. Then system* (3) *has a global exponentially stable periodic solution.*

Proof. Suppose $a \le b$ and $\psi \in C([-\tau + a, b], \mathrm{R}^2)$, we define $\psi_t \in C$, $C = C\left([-\tau, 0], \mathrm{R}^2\right)$, $\psi_t(\theta) = \psi(t+\theta), \theta \in [-\tau, 0]$. Then for $t \in [a, b]$, we define the sup-norm $\|\psi_t\| = \sup\limits_{\theta\in[-\tau,0]} |\psi_t(\theta)|$.

From(B_1), we can conclude that the system (3) has at least one $w-periodic$ solution, define $\tilde{x}(t)$. Let $x(t)$ be an arbitrary solution of system (3).

Now, we consider $D^+|\tilde{x}_i(t) - x_i(t)|$ of $|\tilde{x}_i(t) - x_i(t)|$ along the solutions of system (3) leads to

$$\begin{aligned} D^+|\tilde{x}_i(t) - x_i(t)| &= D^+\{sgn(\tilde{x}_i(t) - x_i(t))(\tilde{x}_i(t) - x_i(t))\} \\ &\le -a_i(t)|\tilde{x}_i(t) - x_i(t)| + |b_i(t)|[\alpha_i|\tilde{x}_1(t) - x_1(t)| + \beta_i|\tilde{x}_2(t) - x_2(t)|] \\ &\quad + |c_i(t)|[|\alpha_i|\tilde{x}_1(t-\tau_{i1}(t)) - x_1(t-\tau_{i1}(t))| + \beta_i|\tilde{x}_2(t-\tau_{i2}(t)) - x_2(t-\tau_{i2}(t))||] \end{aligned}$$

Let $G_i(t) = |\tilde{x}_i(t) - x_i(t)|$. Then we have

$$D^+G_i(t) \le -a_i(t)G_i(t) + (|b_i(t)| + |c_i(t)|)(\alpha_i + \beta_i)\|G_t\|,$$

$$G_i(t)e^{\int_0^t a_i(s)ds} \le |G_i(0)| + \int_0^t e^{\int_o^u a_i(s)ds}(|b_i(t)| + |c_i(t)|)(\alpha_i + \beta_i)\|G_t\|\,du.$$

So we have

$$\begin{aligned} e^{\int_0^t a_i(s)ds - a_i^*\tau}G_i(t+\theta) &\le e^{\int_0^{t+\theta} a_i(s)ds}G_i(t+\theta) \\ &\le \|G_0\| + \textstyle\int_0^t e^{\int_0^u a_i(s)ds}(|b_i(t)| + |c_i(t)|)(\alpha_i + \beta_i)\|G_t\|\,du. \end{aligned}$$

It follows that

$$e^{\int_0^t a_i(s)ds - a^*\tau}\|G_t\| \le \|G_0\| + \int_0^t e^{\int_0^u a_i(s)ds}(|b_i(t)| + |c_i(t)|)(\alpha_i + \beta_i)\|G_t\|\,du$$

$$e^{\int_0^t a_i(s)ds}\|G_t\| \le e^{a_i^*\tau}\|G_0\| + \int_0^t e^{a_i^*\tau}e^{\int_0^u a_i(s)ds}(|b_i(t)| + |c_i(t)|)(\alpha_i + \beta_i)\|G_t\|\,du.$$

By Gronwall's inequality, we obtain

$$\begin{aligned} \|G_t\| &= e^{a_i^*\tau}\|G_0\|\, e^{\left[\int_0^t e^{a_i^*\tau}(|b_i(t)|+|c_i(t)|)(\alpha_i+\beta_i) - a_i(s)ds\right]} \\ &\le e^{a_i^*\tau}\|G_0\|\, e^{-\left[-a_i^*\tau\left(|b_i^*|+|c_i^*|\right)(\alpha_i+\beta_i)+a_{i*}\right]t} \\ &= M\|G_0\|\, e^{-\lambda t}, \quad for\ all\ \ t \ge 0 \end{aligned}$$

where $M = e^{a_i^*\tau}$ and $\lambda = a_{i*} - a_i^*\tau(|b_i^*| + |c_i^*|)(\alpha_i + \beta_i)$ are positive constants. Therefore $x^*(t)$ is global exponentially stable. ■

5 Example and numerical simulation

In this section, we demonstrate that the main results of this paper . We will find that the theoretical conclusions are in excellent agreement with the numerically observed behavior.

Example1: Consider a planar-neuron network with delays as follows:

$$\begin{cases} \dot{x}_1(t) = -a_1(t)x_1(t) + b_1(t)f_1(x_1(t), x_2(t)) + c_1(t)f_1(x_1(t-\tau_{11}(t)), x_2(t-\tau_{12}(t)) \ + I_1(t), \\ \dot{x}_2(t) = -a_2(t)x_2(t) + b_2(t)f_2(x_1(t), x_2(t)) + c_2(t)f_2(x_1(t-\tau_{21}(t)), x_2(t-\tau_{22}(t)) + I_2(t), \end{cases}$$

where

$$a_1(t) = 2 + \sin t, \quad a_2(t) = 3 + \cos t, \quad b_1(t) = \frac{\pi}{10}\sin t, \quad b_2(t) = \frac{\pi}{20}\cos t,$$

$$c_1(t) = \frac{\pi}{50}\sin t, \quad c_2(t) = \frac{\pi}{30}\cos t, \quad I_1(t) = \frac{\pi}{30}\cos t, \quad I_2(t) = \frac{\pi}{40}\sin t,$$

$$f_2(x_1, x_2) = \cos(\frac{1}{2}x_2) - \frac{1}{6}\left|x_1\right|, \quad f_1(x_1, x_2) = \sin(\frac{1}{4}x_1) - \frac{1}{8}\left|x_2\right|, \quad 0 \le \tau_{ij}(t) \le \frac{1}{2}$$

are 2π-periodic continuous functions for $i, j = 1, 2$.

By simple calculations, we obtain

$$\alpha_1 = \frac{1}{4}, \quad \alpha_2 = \frac{1}{6}, \quad \beta_1 = \frac{1}{8}, \quad \beta_2 = \frac{1}{2},$$

$$\bar{a}_1 = 2, \quad \bar{a}_2 = 3, \quad a_1^* = 3, \quad a_2^* = 4,$$

$$\left|\bar{b}_1\right| = \frac{1}{5}, \quad \left|\bar{b}_2\right| = \frac{1}{10}, \quad \left|\bar{c}_1\right| = \frac{1}{25}, \quad \left|\bar{c}_2\right| = \frac{1}{15},$$

$$\left|\bar{I}_1\right| = \frac{1}{15}, \quad \left|\bar{I}_2\right| = \frac{1}{20}, \quad l_1 = \frac{1+4\pi}{40}, \quad l_2 = \frac{1+6\pi}{60},$$

$$m_1 = \frac{1+4\pi}{60}, \quad m_2 = \frac{1+6\pi}{120}, \quad n_1 = \frac{1+\pi}{200}, \quad n_2 = \frac{1+6\pi}{90},$$

$$R_1 = \frac{1+4\pi}{300}, \quad R_2 = \frac{1+6\pi}{180}, \quad T_1 = \frac{2+8\pi}{15}, \quad T_2 = \frac{1+6\pi}{20}.$$

It is easy to find that $D > 0$, $D_1 > 0$, $D_2 > 0$. Therefore $\frac{D_2}{D} > 0$, $\frac{D_1}{D} > 0$. Furthermore, when $0 \le \tau_{ij} \le \frac{1}{2}$, $\quad a_{i*} > a_i^*(|b_i^*| + |c_i^*|)(\alpha_i + \beta_i)$, for $i = 1, 2$ equations are also true. Applying theorem 3, there exits a unique 2π-periodic solution and it is globally exponentially stable.

6 Conclusions

In the paper, the global exponential stability and periodicity have been investigated for PDPNN's. Several sufficient conditions have been derived for checking the global exponential stability and the existence of periodic solution for the considered system .In addition, an example is given to show the effectiveness of the proposed method and results.

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