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Control and Synchronization of a New Hyperchaotic System with Unknown Parameters

Dianchen Lu[1] *,Aicheng Wang[1], Xiandong Tian[2]
[1]Department of Mathematics,Jiangsu University, Zhenjiang, Jiangsu,212013,P.R.China
[2] Linyi Normal University
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Abstract: This article introduces a new hyperchaotic system named hyperchaotic Lorenz system and the effective controllers are designed to suppress this hyperchaotic system to its unstable equilibrium immediately based on the Lyapunov stability theory and adaptive control theory. Then, adaptive control laws are derived to make the states of two identical hyperchaotic Lorenz systems with unknown parameters asymptotically synchronize rapidly. Finally, numerical simulations are presented to demonstrate the effectiveness of the proposed chaos control and synchronization schemes.

Key words: Lyapunov stability theory; adaptive control; control of hyperchaotic system; synchronization of hyperchaotic system

1 Introduction

Since chaos phenomenon was observed by Lorenz firstly in 1961, a large number of chaos phenomena and chaos behavior have been discovered in natural and social systems. In light of the theoretical and practical applications of chaos in many areas, such as biological system, chemical reaction, economics, secure communication and so on, the control and synchronization of chaotic systems have received increased research attention. Since the classical work on chaos control was first presented by Ott, Grebogi and Yorke, and the pioneering work on the synchronization was introduced by Pecora and Carroll, various effective techniques, such as backstepping design, adaptive control, time-delay feedback control, linear feedback control, and active control, variable structure control[1-10] and so on, have been successfully applied to chaos control and synchronization.

However, most of the works mentioned above considered the low-dimensional chaos system with only one positive Lyapunov exponent. Hyperchaotic system, owning more than one positive Lyapunov exponent, has more complex chaos behavior and abundant dynamics than chaotic system. Control and synchronization of hyperchaotic system have attracted a great deal of attention from various fields and become a challenging work.

In this article, a fully new hyperchaotic Lorenz system with unknown parameters constructed by Qiang Jia is studied. Stabilization of hyperchaotic system to the unstable equilibrium is achieved. And synchronization of this hyperchaos with four unknown parameters is realized based on the Lyapunov stability theory and adaptive control.

2 The new hyperchaotic Lorenz system description

Very recently, Qing Jia constructed a new hyperchaotic system reported in[10] by introducing state feedback control based on the familiar Lorenz system. The new hyperchaotic system is described by

*Corresponding author. *E-mail address*: dclu@ujs.edu.cn

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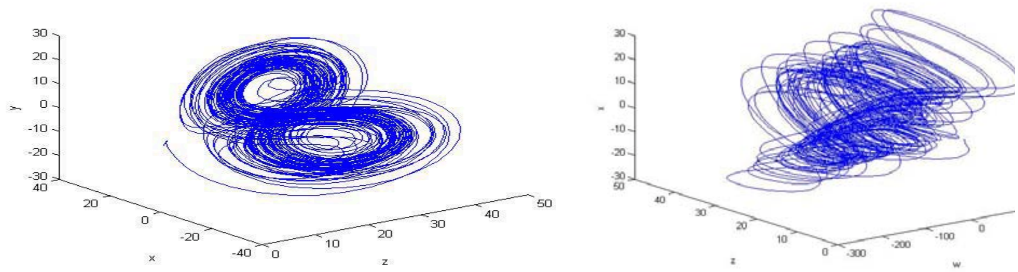
$$\begin{cases} \dot{x} = a(y - x) + w \\ \dot{y} = -xz + rx - y \\ \dot{z} = xy - bz \\ \dot{w} = -xz + dw \end{cases} \tag{2.1}$$

in which a, r, b and d are constant parameters. When parameters $a = 10, r = 28, b = 8/3$ and $d = 1.3$, the system (2.1) has two positive Lyapunov exponents and the Kaplan-Yorke dimension is $D_{KY} = 3.05$. Thus, the system (2.1) shows complex hyperchaotic behavior. The hyperchaotic attractor is given in Fig.1.



(a) (b)

Figure 1: Figures of the hyperchaotic attractor of the system (2.1). $(a)xyz$; $(b)xzw$.

3 Controlling the hyperchaotic system to equilibrium point

For arbitrary parameters a, r, b and d, $E_0 = (0, 0, 0, 0)$ is an equilibrium of the new system. When parameters $a = 10, r = 28, b = 8/3$ and $d = 1.3$, the characteristic values of the Jacbian matrix at the equilibrium E_0 are $-22.8277, 11.8277, 1.3000$ and -2.667. Thus, the only equilibrium E_0 is an unstable saddle-node point.

Suppose the four parameters a, r, b and d are unknown, adaptive control theory is employed to suppress the new hyperchaotic system to the unstable equilibrium.

Consider the controlled system as follow

$$\begin{cases} \dot{x} = a(y - x) + w + u_1 \\ \dot{y} = -xz + rx - y + u_2 \\ \dot{z} = xy - bz + u_3 \\ \dot{w} = -xz + dw + u_4 \end{cases} \tag{3.1}$$

in which a, r, b and d are unknown parameters, and $u_i (i = 1, 2, 3, 4)$ are the controllers to design.

According to the Lyapunov stability theory, we choose the following Lyapunov function

$$V = \frac{1}{2}(x^2 + y^2 + z^2 + w^2 + \tilde{a}^2 + \tilde{r}^2 + \tilde{b}^2 + \tilde{d}^2)$$

in which $\tilde{a} = a - \bar{a}, \tilde{r} = r - \bar{r}, \tilde{b} = b - \bar{b}$ and $\tilde{d} = d - \bar{d}$. And $\bar{a}, \bar{r}, \bar{b}, \bar{d}$ are the estimate values of these unknown parameters, respectively.

In order to ensure that the controlled system (3.1) converges to the origin asymptotically, the following $u_i (i = 1, 2, 3, 4)$ are chosen

$$\begin{cases} u_1(t) = \bar{a}(x - y) - w - x \\ u_2(t) = xz - \bar{r}x \\ u_3(t) = -xy + \bar{b}z - z \\ u_4(t) = xz - \bar{d}w - w \end{cases} \tag{3.2}$$

At the same time, we chose the following parameter estimation update laws

$$\begin{cases} \dot{\bar{a}} = yx - x^2 \\ \dot{\bar{r}} = xy \\ \dot{\bar{b}} = -z^2 \\ \dot{\bar{d}} = w^2 \end{cases} \tag{3.3}$$

With above choices (3.2) and (3.3), the time derivation of the Lyapunov function along the trajectory has the following form

$$\begin{aligned} \frac{dV}{dt} &= x\dot{x} + y\dot{y} + z\dot{z} + w\dot{w} + \tilde{a}\dot{\tilde{a}} + \tilde{r}\dot{\tilde{r}} + \tilde{b}\dot{\tilde{b}} + \tilde{d}\dot{\tilde{d}} \\ &= x(a(y-x) + w + u_1) + y(-xz + rx - y + u_2) \\ &+ z(xy - bz + u_3) + w(-xz + dw + u_4) + \tilde{a}(-\dot{\bar{a}}) + \tilde{r}(-\dot{\bar{r}}) + \tilde{b}(-\dot{\bar{b}}) + \tilde{d}(-\dot{\bar{d}}) \\ &= x(\bar{a}(y-x) + w + u_1) + y(-xz + \bar{r}x - y + u_2) \\ &+ z(xy - \bar{b}z + u_3) + w(-xz + \bar{d}w + u_4) \\ &+ \tilde{a}(-\dot{\bar{a}} + yx - x^2) + \tilde{r}(-\dot{\bar{r}} + xy) + \tilde{b}(-\dot{\bar{b}} - z^2) + \tilde{d}(-\dot{\bar{d}} + w^2) \\ \frac{dV}{dt} &= -x^2 - y^2 - z^2 - w^2 < 0 \end{aligned}$$

According to the Lyapunov stability theory, the condition ensure that the controlled system (3.1) can converge to the unstable equilibrium E_0 with the controllers (3.2) and the parameter estimation update laws (3.3), though the system is in hyperchaotic state. Fig.2 shows the hyperchaotic system converges to the unstable equilibrium E_0 immediately.

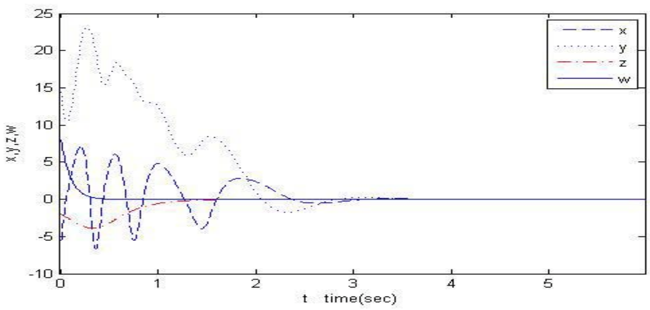


Figure 2: Time responses of the controlled system (3.1).

4 Adaptive synchronization of the new hyperchaotic system

In this section, adaptive synchronization of two identical hyperchaotic systems with unknown parameters is achieved based on the Lyapunov stability theory and adaptive control theory. Let the hyperchaotic system (2.1) be the drive system, and the response system is defined as follow

$$\begin{cases} \dot{x_1} = a(y_1 - x_1) + w_1 + u_1 \\ \dot{y_1} = -x_1 z_1 + r x_1 - y_1 + u_2 \\ \dot{z_1} = x_1 y_1 - b z_1 + u_3 \\ \dot{w_1} = -x_1 z_1 + d w_1 + u_4 \end{cases} \tag{4.1}$$

in which $u_i (i = 1, 2, 3, 4)$ are the control functions to design.

Assume that parameters a, r, b and d are unknown constants. To realize the synchronization between the drive system (2.1) and the response system (4.1), adaptive control theory is employed. Subtracting the system (2.1) from the system (4.1), we obtain the error dynamical system (4.2) between the drive system and the response system as follow

$$\begin{cases} \dot{e_1} = a(e_2 - e_1) + e_4 + u_1(t) \\ \dot{e_2} = -e_1e_3 - xe_3 - ze_1 + re_1 - e_2 + u_2(t) \\ \dot{e_3} = e_1e_2 + xe_2 + ye_1 - be_3 + u_3(t) \\ \dot{e_4} = -e_1e_3 - xe_3 - ze_1 + de_4 + u_4(t) \end{cases} \tag{4.2}$$

where: $e_1 = x_1 - x, e_2 = y_1 - y, e_3 = z_1 - z$, and $e_4 = w_1 - w$.

Choose the following Lyapunov function

$$V = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2 + e_4^2 + \tilde{a}^2 + \tilde{r}^2 + \tilde{b}^2 + \tilde{d}^2)$$

in which $\tilde{a} = a - \bar{a}, \tilde{r} = r - \bar{r}, \tilde{b} = b - \bar{b}$ and $\tilde{d} = d - \bar{d}$. $\bar{a}, \bar{r}, \bar{b}$ and $\bar{d}$ are the estimate values of these unknown parameters a, r, b and d, respectively.

The time derivation of the Lyapunov function along the trajectory is

$$\begin{aligned} \frac{dV}{dt} &= e_1\dot{e_1} + e_2\dot{e_2} + e_3\dot{e_3} + e_4\dot{e_4} + \tilde{a}\dot{\tilde{a}} + \tilde{r}\dot{\tilde{r}} + \tilde{b}\dot{\tilde{b}} + \tilde{d}\dot{\tilde{d}} \\ &= e_1(a(e_2 - e_1) + e_4 + u_1) + e_2(-e_1e_3 - xe_3 - ze_1 + re_1 - e_2 + u_2) \\ &+ e_3(e_1e_2 + xe_2 + ye_1 - be_3 + u_3) + e_4(-e_1e_3 - xe_3 - ze_1 + de_4 + u_4) \\ &+ \tilde{a}(-\dot{\bar{a}}) + \tilde{r}(-\dot{\bar{r}}) + \tilde{b}(-\dot{\bar{b}}) + \tilde{d}(-\dot{\bar{d}}) \end{aligned}$$

When $\bar{a}, \bar{r}, \bar{b}, \bar{d}$ take the place of a, r, b, d, respectively, that is

$$\begin{aligned} \frac{dV}{dt} &= e_1\dot{e_1} + e_2\dot{e_2} + e_3\dot{e_3} + e_4\dot{e_4} + \tilde{a}\dot{\tilde{a}} + \tilde{r}\dot{\tilde{r}} + \tilde{b}\dot{\tilde{b}} + \tilde{d}\dot{\tilde{d}} \\ &= e_1(\bar{a}(e_2 - e_1) + e_4 + u_1) + e_2(-e_1e_3 - xe_3 - ze_1 + \bar{r}e_1 - e_2 + u_2) \\ &+ e_3(e_1e_2 + xe_2 + ye_1 - \bar{b}e_3 + u_3) + e_4(-e_1e_3 - xe_3 - ze_1 + \bar{d}e_4 + u_4) \\ &+ \tilde{a}(e_1(e_2 - e_1) - \dot{\bar{a}}) + \tilde{r}(e_2e_1 - \dot{\bar{r}}) + \tilde{b}(-e_3^2 - \dot{\bar{b}}) + \tilde{d}(e_4^2 - \dot{\bar{d}}) \end{aligned}$$

According to the Lyapunov stability theory, to ensure the error dynamical system converges to the origin asymptotically, the condition $\dot{V} < 0$ should be satisfied. So we choose the following controllers

$$\begin{cases} u_1(t) = -(\bar{a}(e_2 - e_1) + e_4) - e_1 \\ u_2(t) = -(-e_1e_3 - xe_3 - ze_1 + \bar{r}e_1 - e_2) - e_2 \\ u_3(t) = -(e_1e_2 + xe_2 + ye_1 - \bar{b}e_3) - e_3 \\ u_4(t) = -(-e_1e_3 - xe_3 - ze_1 + \bar{d}e_4) - e_4 \end{cases} \tag{4.3}$$

and the parameter estimation update laws

$$\begin{cases} \dot{\bar{a}} = e_1(e_2 - e_1) + \tilde{a} \\ \dot{\bar{r}} = e_2e_1 + \tilde{r} \\ \dot{\bar{b}} = -e_3^2 + \tilde{b} \\ \dot{\bar{d}} = e_4^2 + \tilde{d} \end{cases} \tag{4.4}$$

With the choices of (4.3) and (4.4), the time derivation of the Lyapunov function $\dot{V}$ becomes

$$\frac{dV}{dt} = -e_1^2 - e_2^2 - e_3^2 - e_4^2 - \tilde{a}^2 - \tilde{r}^2 - \tilde{b}^2 - \tilde{d}^2 < 0$$

In light of the Lyapunov stability theory, the error dynamical system can converge to the origin asymptotically. Consequently, the drive system (2.1) is synchronous asymptotically with the response system (4.1) with the controllers (4.3) and the parameter estimation update laws (4.4).

To verify the effectiveness and feasibility of (4.3) and (4.4), we simulate the dynamics of the drive system and the response system. In the simulation, we choose the parameters $a = 10, b = 28, c = 8/3$ and $d = 1.3$ to ensure the system (2.1) and (4.1) are hyperchaotic without control. Runge-Kutta method of order four is employed with the time step $\triangle t = 0.001$. The initial condition of the drive system and the response system are $(-5, -3, 20, 10)$ and $(5, 3, 35, -10)$, respectively. The initial condition of the parameter update law is $(15, 25, 5, -4)$. Fig.3 shows the time evolutions of the drive system and the response system, and Fig.4 gives the dynamics of the parameter estimation errors and the system errors of the two hyperchaoses. From Fig.3 and Fig.4, we can see that the two hyperchaotic systems starting from different conditions synchronize each other immediately, and the four parameter estimation errors and the state errors between two systems vanish rapidly with the evolution of time t.

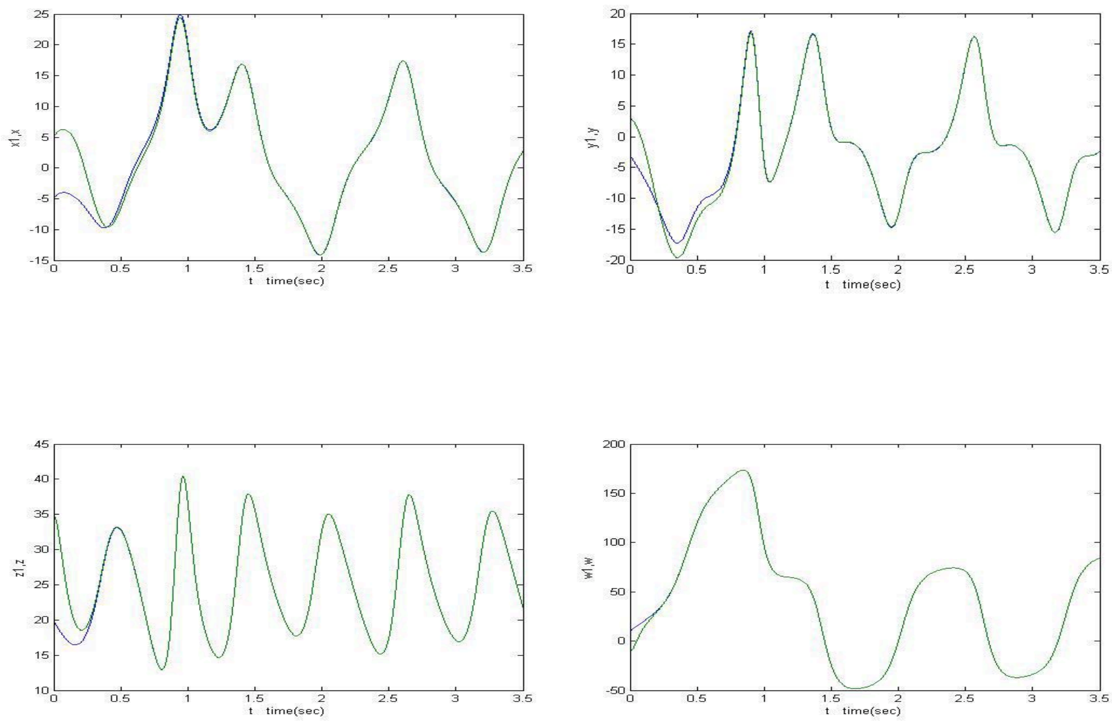
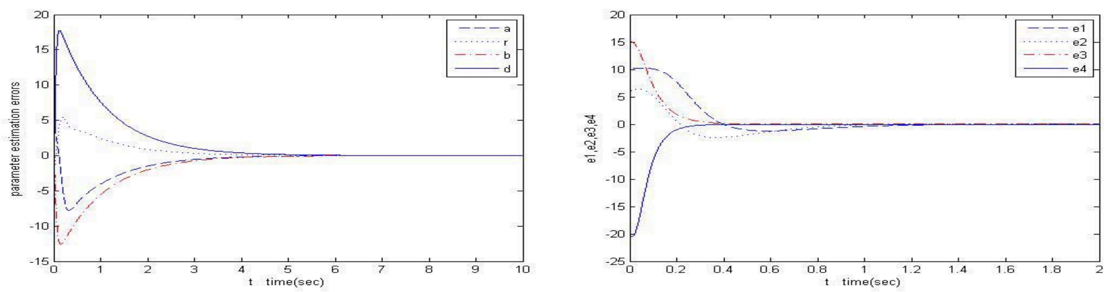


Figure 3: State trajectories of the drive system (2.1) and the response systems (4.1)



(a) (b)

Figure 4: (a) Time responses of the parameter estimation errors; (b) Time responses of the error states.

5 Conclusion

In this paper, based on the Lyapunov stability theory and adaptive control theory, we realized the control and synchronization of a new hyperchaotic system with unknown parameters. First, controllers designed by adaptive control method are used to control the new hyperchaotic system to the unstable equilibrium point successfully. Further, synchronizations between two identical hyperchaotic systems with unknown parameters are achieved via the Lyapunov stability theory and adaptive control theory. Finally, numerical simulations show the effectiveness of the proposed chaos control and synchronization schemes.

Acknowledgements

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