

## A Non-stationary Time Series Model and Its Application

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(Received 20 July 2008, accepted 18 September 2008)

**Abstract:** In this paper, a time series analysis model-building and forecast methods are put forward, it is proved that the methods is high precision for building model and application. Using the method, we build the model of Xiang river water level and test model using forecast method. Through application, we know the model is good and useful.

**Keywords:** time series model, identification; forecast; the high order yule-walker estimation

### 1 Time Series Model

Time Series Theory and Methods is a systematic account of linear time series models and their application to the modelling and prediction of data collected sequentially in time. The aim is to provide specific techniques for handling data and at the same time to provide a thorough understanding of the mathematical basis for techniques.

Major Time Series themes will be :Statistical methodology, Applications to economics and in econometrics, Government, Business and Industrial Examples; the Hydroscoences, such as limnology, hydrology, water quality regulation and control, and the modelling of marine environments; the geosciences, especially such areas as oil exploration and seismology; civil engineering and allied disciplines; spatial and space-time processes-their theory and application-especially in geography and related areas, such as city planning or energy demand forecasting; biology and ecology; environmental studies-air and river pollution; medical applications and biomedical engineering; irregularly spaced data; season modelling and adjustment, calendar effects, causality; bayesian approaches; transfer-function, intervention and multivariate modelling; state space; nonlinear modelling; estimation; diagnostic checking; signal extraction; comparative studies; spectral analysis, especially for the physical sciences; business cycle and expectation data; data revisions; computer software and numerical analysis; forecasting; and, no doubt, many other areas of the subject.

Time Series analysis theory and methods are very important for application<sup>[1]-[11]</sup>. In this paper, we put forward a sort of new non-stationary time series model and research its application in water level. Water lever study is impartment more and more in aspect of economic and science study and etc. We give model-building method and forecasting method.

Let  $y_t$  is a random series defined on probability space  $(\Omega, g, P)$ , we have

$$y_t = f(t) + g(t) + X_t$$

where,  $f(t) = \sum_{i=0}^M C_i t^i$

$$g(t) = \sum_{j=1}^P A_j \cos(w_j t + \phi_j), \quad -\pi \leq \phi_j \leq \pi$$

$$X_t \sim AR(P_0)$$

We must obtain these express formular about  $f(t)$ ,  $g(t)$  and  $X_t$ , so as to obtain time series models.

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## 2 High order Yule-walker estimation

Let  $X(t)$  satisfy time series AR( $P_0$ ) model, have

$$\sum_{j=0}^{P_0} a_j X(t-j) = \varepsilon(t)$$

where,  $a_0 = 1, a_{p_0} \neq 0, \varepsilon(t)$  is a white noise,  $E\varepsilon(t)\varepsilon(s) = \sigma^2\delta_{t,s}, E[\varepsilon(t)]^2 < +\infty$ .

Suppose  $A(Z) = \sum_{j=0}^{P_0} a_j z^j, A(Z) \neq 0, (|Z| \leq 1)$

For sample  $X(1), X(2), \dots, X(T)$ . Let

$$R_P = \begin{pmatrix} r(p) & r(p-1) & \dots & r(1) \\ r(p+1) & r(p) & \dots & r(1) \\ \dots & \dots & \dots & \dots \\ r(2p-1) & r(2p-2) & \dots & r(p) \end{pmatrix}$$

$$\text{Let } r(k) = \hat{r}(k) = \begin{cases} \frac{1}{T} \sum_{t=1}^{T-K} x(t)x(t+k), & 0 \leq k \leq T-1 \\ 0, & K \geq T \end{cases}$$

Because  $\hat{r}(-k) = \hat{r}(k), k = 0, 1, 2, \dots$

Hence, we obtain the estimation  $\hat{R}_p$  of  $R_p$ :

$$\hat{R}_p = \begin{pmatrix} \hat{r}(p) & \hat{r}(p-1) & \dots & \hat{r}(1) \\ \hat{r}(p+1) & \hat{r}(p) & \dots & \hat{r}(2) \\ \dots & \dots & \dots & \dots \\ \hat{r}(2p-1) & \hat{r}(2p-2) & \dots & \hat{r}(p) \end{pmatrix}$$

We give a enough large  $P \in N$ , compute characteristic roots of  $\hat{\Gamma}_p = \hat{R}_p^T \hat{R}_p$ , we obtain

$$\hat{\lambda}_1 \geq \hat{\lambda}_2 \geq \dots \geq \hat{\lambda}_p \geq 0$$

and compute function

$$\hat{D}(k) = T - \left( \frac{\hat{\lambda}_1 + \dots + \hat{\lambda}_k}{\hat{\lambda}_1 + \dots + \hat{\lambda}_p} \right)^{\frac{1}{2}}, \quad 1 \leq k \leq P$$

We obtain estimation of  $P_0$ :

$$\hat{P}_0 = \inf_{k \geq 1} \left\{ k : \hat{D}(k) < \left( \frac{\ln T}{T} \right)^{\frac{1}{2}} \right\}$$

and when  $T \rightarrow \infty, \hat{P}_0 \rightarrow P_0$ , a.s, we use high order yule-walker equation

$$\begin{pmatrix} r(p_0) & r(p_0-1) & \dots & r(1) \\ r(p_0+1) & r(p_0) & \dots & r(2) \\ \dots & \dots & \dots & \dots \\ r(2p_0-1) & r(2p_0-2) & \dots & r(p_0) \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ \dots \\ a_{p_0} \end{pmatrix} = - \begin{pmatrix} r(p_0+1) \\ r(p_0+2) \\ \vdots \\ r(2p_0) \end{pmatrix}$$

to obtain estimation  $(\hat{a}_1, \hat{a}_2, \dots, \hat{a}_{p_0})$  of  $(a_1, a_2, \dots, a_{p_0})$

### 3 Applied Analysis

We can obtain  $f(t)$  and  $g(t)$  using the estimating methods in [4].

In fact, we express

$$g(t) = \sum_{j=1}^{2P} a_j e^{it\lambda_j}$$

where

$$\lambda_j = \begin{cases} w_j, & j = 1, \dots, P \\ -w_j, & j = P + 1, \dots, 2P \end{cases}$$

$$a_j = \begin{cases} \frac{1}{2} A_j e^{j\varphi_j}, & j = 1, \dots, P \\ -\frac{1}{2} A_{j-p} e^{-i\phi_{j-p}}, & j = P + 1, \dots, 2P \end{cases}$$

We compute  $P\lambda_j, a_j$

Let

$$J_{y,96}(\lambda) = 96^{-\frac{31}{16}} \left\| \sum_{j=1}^{96} y_j e^{-i\lambda j} \right\|$$

We turn  $[-\pi, \pi]$  into  $2N (=192)$

$$D(k) = J_{y,96} \left( \frac{K\pi}{96} - \pi \right), k = 0, 1, \dots, 192$$

Take

$$E = 96^{\frac{1}{16}} \frac{1}{192} \sum_{K=0}^{192} D(k)$$

We take  $K$  when  $D(k) > E$ , let its be  $k_1, k_2, \dots, k_q$ .

Let

$$A_j = k_{j+1} - k_j - \left[ \frac{\sqrt{96}}{\pi} \right] - 1, j = 1, 2, \dots, q-1$$

$$A_q = 192 + k_1 - k_q - \left[ \frac{\sqrt{96}}{\pi} \right] - 1$$

Then

$$\hat{\lambda}_j = \frac{\pi}{96} j k_{\max}, j = 1, \dots, \hat{P}$$

Then we have

$$\hat{a}_j = \frac{1}{96} \sum_{L=1}^{96} y_L e^{-iL\hat{\lambda}_j}, j = 1, \dots, \hat{P}$$

The last have

$$\hat{w}_j = \hat{\lambda}_j, \hat{A}_j = 2|\hat{a}_j|, \hat{\varphi}_j = \arg \hat{a}_j$$

$$(j = 1, 2, \dots, \hat{P})$$

We test model using forecast methods, we have

$$\hat{y}_t = f(t) + g(t) + \hat{X}_t$$

L-Step forecast of AR(P) model  $X_t$  as follow:

$$\hat{X}_t(L) = a_1 \hat{X}_t(L-1) + a_2 \hat{X}_t(L-2) + \dots + a_p \hat{X}_t(L-p), \quad t \geq 0$$

$$\hat{X}_t(j) = X_{t-j}, \quad j \leq 0$$

Let  $\Phi(B) = \sum_{j=0}^P a_j B^j$ ,  $a_0 = 1$ , then  $\Phi(B)\hat{X}_t(L) = 0, L > 0$ , where,  $\hat{X}_t(0) = X_t, \hat{X}_t(-1) = X_{t-1}, \dots, \hat{X}_t(-p+1) = X_{t-p+1}$

We apply our methods and the method in [2] to water Level forecast of Xiang river in Xiangtan station of Hunan, we take 120 data (one each month from 1. 1994 to 12. 2003) to model-building.

We use model to forecast water level from 1. 2004 to 12. 2004 (one data each months) as follows:

Table 1: use the method in [2]

$X_t$	$\hat{X}_t$	error	$X_t$	$\hat{X}_t$	error
30.2	27.0	3.2	32.5	31.0	1.5
29.0	25.3	3.7	32.4	26.1	6.3
30.5	34.1	3.6	30.6	25.3	5.3
30.8	38.0	7.2	30.2	22.4	7.8
32.7	32.4	0.3	29.9	24.2	5.7
31.0	34.7	3.7	29.2	22.0	7.2

Table 2: use the method in this paper

$X_t$	$\hat{X}_t$	error	$X_t$	$\hat{X}_t$	error
30.2	30.9	0.7	32.5	31.2	1.3
29.0	30.1	1.1	32.4	31.0	1.4
30.5	29.0	1.5	30.6	29.3	1.3
30.8	29.4	1.4	30.2	28.8	1.4
32.7	31.2	0.5	29.9	28.9	1
31.0	31.7	0.7	29.2	27.8	1.4

where error= $|X_t - \hat{X}_t|$

From forecast value  $\hat{y}_t$  compare with original value  $y_t$ , we know that the method is high precision for model-building.

To have superiority with methods of this paper,the following are needed:

- (1)Theory strict,method right,easy to process the sample data.
- (2)Model a with high precisions,and it is easy to be applied.
- (3)Sample data may be very small.

### Acknowledgements

Project supported by the Natural Science Foundation of Hunan Province, China.[08JJ3002]

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