

The Model of Multiple Stage Decision Making for Venture Capital

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Abstract: In the activities of market economy, by discussing the problem of re-production and capital operating investment for venture enterprises, from introducing a traditional Ramsey Model to Venture Investment, a Multiple Stage Decision Model is set up, which maximizes the total profit of venture enterprise in multi-periods. Using the stability theory, the solution stability of the model is investigated, and it discusses the distribution of control right at the equilibrium points. Conclusion of this paper could have a guiding meaning on economic activities.

Key words: venture investment; ramsey model; equilibrium; multiple stage decision

1 Introduction

The development of high-tech industries calls for venture capital investment, but there exist both high risk and high return in venture capital investment. Venture capitalists how to conduct a reasonable investment in the higher receipts At the same time, risk control in the moderate range, which calls for a reasonable investment cooperation between the two sides and that the path can be controlled. In the actual operation, as the same investors, in order to avoid risks, will have conducted a number of risk investment decision-making, and because of the limitation of funds, in many of its investment in the investment decision-making is associated. Therefore, It is quite necessary to study the issue that the investment in various stages of the relationship between the multi-stage investment portfolio. Based on this consideration, this paper proposed the topic of multi-stage venture investment decision-making.

There are many models and conclusions about single stage of investment portfolio[2], but the study of the investment in various stages of the investment decision-making is not yet mature. Therefore, the study of the multi-stage investment decisions has some practical significance. Li Zhenhua, Wang Huanchen presented a multi-stage venture capital model of multi-objective decision-making and security regional of investment cooperation. This has a guiding significance for investment cooperation[1]. Liu Heng obtains the optimal solution of Ramsey consumption model by Hamiltonian function[3]. The optimal path of Ramsey model on individual consumer has been extensively studied by Reyer Gerlagha Michiel A. Keyzerb[4]. Xu Jiuping and Hu Zhineng introduces some dynamic theories of economic management[5]. Robert A. Becker discussed the stability of Ramsey model under various holders of the two departments[6]. A research on the allocation of control rights between capitalists and entrepreneurs based on Game Theory is studied by An-Shi[7]. Based on China's entry WTO, Hongxing Yao, Chengyao Wu PaPer sets up a dynamics model which describes the investing in state-owned enterprises and brings the duoPoly model with bounded rationality into the financial field[8]. Economic growth has been studied by Honglin Yang, Lixin Tian[9]. Method of Optimal Control and Stability Analysis is introduced by reference [10,11].

The remainder of this paper is organized in the following way. In Section 2, we discuss existence of solution and impact of single stages over Multiple-stage investment decision by establishing multi-stage decision model. In section 3, the relation of degree of venture entrepreneur's effort and control rights is studied. And finally, we draw the conclusion of the paper in Section 4.

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Figure 1: Graph of t-period dynamic financial

2 Establishment of the Model

Venture capitalist pours the fund into the venture enterprise through stage and the investment decision-making occurs in the time-discrete cycle. Graphical representation of the investment is as follow: In order to be accurate, we made a series of assume

(1) The investment process is continuous; venture enterprise will get all funds which are used to develop the venture business at t stage.

(2) We assume that the total revenue of venture entrepreneur at t stage is π and revenue function π has the nature of Rumsey model: $\pi = \max \sum_{i=1}^t \beta^i u(k_i)$. It is determined by utility function $u(k_i)$ and discount rate β , here, $0 < \beta < 1$, k_i is capital input to the venture enterprise by venture capitalist at i -stage.

(3) Investor have investment decision-making in every investment stage, so the decision-making of multi-stage investment is constrained by the single -stage. $I_{i-1} + (r - p)k_i \leq B(I_i)$, here, I_i is amount of capital possessed by venture enterprise at the end of i -stage, r is the rate of average return, p is average loss ratio of venture capital, We assume the formal of effort level's function is as follow: $B(I_i) = I_i a^\lambda$, here, a is the effort level of venture entrepreneur and λ is the venture entrepreneur's control weight of I_i which has the function of motivation, $0 < \lambda < 1$, if $\lambda = \frac{1}{2}$, this means the control weight is equal between venture entrepreneur and venture investor.

(4) We assume the utility function is upper convex and monotone increasing, thus, $u' > 0$, $u'' < 0$, $B''(I_i) < 0$

(5) The cost function of venture entrepreneur meet the condition as follows: $c(a) = \frac{1}{2h} a^2, \frac{\partial c(a)}{\partial a} > 0$

From assumption above, theorem 1 set up

Theorem 1 Profit function converges to a stable fixed point, and the point is a saddle stable point.

Proof.

$$\pi = \max \sum_{i=1}^t \beta^i u(k_i) \tag{2.1}$$

St:

$$I_{i-1} + (r - p)k_i \leq B(I_i)$$

From the constraint condition (2), we have:

$$k_i \leq \frac{1}{r - p} [B(I_i) - I_{i-1}] \tag{2.2}$$

Lead (2.3) into (2.1):

$$\pi = \max \sum_{i=1}^t \beta^i u \left[\frac{1}{r - p} (B(I_i) - I_{i-1}) \right]$$

Let :

$$\pi' = \max \sum_{i=1}^t \beta^i u [(B(I_i) - I_{i-1})]$$

Establish the Euler equation

$$-u'(c)(B(I_i) - I_{i-1}) + \beta B'(I_{i+1})u'(c)(B(I_{i+1}) - I_i) = 0 \quad (2.3)$$

Equilibrium point meets the condition as follow:

$$\beta B'(I) = 1 \quad (2.4)$$

Consider the stability of equilibrium, linear equation (2.3) at I

$$-u''(c)[B'(I)(I_i - I) + (I_{i-1} - I)] + \beta B''(I)u'(c)(I_{i+1} - I) + \beta B'(I)u''(c)[B'(I)(I_{i+1} - I) + (I_i - I)] = 0 \quad (2.5)$$

Leading (2.4) into (2.5),we get:

$$-u''(c)[\beta^{-1}(I_i - I) + (I_{i-1} - I)] + \beta B''(I)u'(c)(I_{i+1} - I) + u''(c)[\beta^{-1}(I_{i+1} - I) + (I_i - I)] = 0$$

Let:

$$x_j = I_j - I \quad j = \{i - 1, i, i + 1\} \quad j = \{i - 1, i, i + 1\}$$

So we get the equation (2.6)

$$-u''x_{i-1} - u''(\beta^{-1} - 1)x_i + [u''\beta^{-1} + \beta B''(I)u']x_{i+1} = 0 \quad (2.6)$$

$$\begin{pmatrix} x_{i-1} \\ x_i \end{pmatrix} = \begin{pmatrix} 1 - \beta^{-1} & \beta^{-1} + \beta B''(I)\frac{u'}{u''} \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x_i \\ x_{i+1} \end{pmatrix} \quad i = \{1, 2, 3, \dots, t\}$$

Characteristic equation is a follows:

$$f(\mu) = \mu^2 - (1 - \beta^{-1})\mu - [\beta^{-1} + \beta B''(I)\frac{u'}{u''}]$$

Utility function is upper convex function from assumption 4, we have:

$$f(0) = -[\beta^{-1} + \beta B''(I)\frac{u'}{u''}] < 0$$

$$f(1) = \beta B''(I)\frac{u'}{u''} > 0$$

Based on the two inequalities above, the following conclusions can be obtained that at least one solution μ_1 exist in $(0, 1)$. Through the study of equation (2.6), we know there still exist another characteristic root u_2 , and $\mu_2 < 0$

So the equilibrium point is saddle point, and for any known value, it has an unique value, making revenue function (2.1) convergence in a stable condition, namely, the existence of an optimal path makes venture capital to achieve this equilibrium. ■

3 Analyses at Equilibrium Point

In the actual economic activities, Venture companies are not completely controlled but partly controlled by the venture entrepreneurs. The arrangement of control rights is core in the investment process. The distribution of control rights can meet the spiritual and material needs of entrepreneurs, also can inspire and restraint entrepreneurs.

We will discuss the distribution of control rights at equilibrium point: first, we assume that multi-stage investment decision-making reached in a balanced state at i stage, beside this, quantity of capital that venture entrepreneur hold is I , and profit π is maximum.

Theorem 2 Degree of venture entrepreneur's effort a^* increased with the proportion of control rights.

Proof.

$$\pi' = \max \sum_{i=1}^T \beta^i u[(B(I) - I_{i-1})]$$

From the constraint condition 4, π' is increased with $B(I)$ when $u' > 0$ $B(I)$ has the following form:

$$B(I) = Ia^\lambda \tag{3.1}$$

Venture capitalist face the participation constraint from venture entrepreneur, that is the expected utility which venture entrepreneur accept the investment contract from venture capitalist should be more than not to accept the contract. While the entrepreneur does not accept contract, the most expectation utility which decides by marketplace opportunity cost is defined reservation utility and denoted by \underline{u} .

$$st : \quad \lambda B(I) - c(a) \geq \underline{u}$$

Lead into Kuhn- Take multiplication η :

$$\begin{aligned} \Delta &= B(I) + \eta(\lambda B(I) - c(a) - \underline{u}) \\ &= (1 + \eta\lambda)B(I) - \eta c(a) - \eta \underline{u} \end{aligned} \tag{3.2}$$

Lead (3.1) into (3.2):

$$\Delta = (1 + \eta\lambda)Ia^\lambda - \frac{\eta}{2h}a^2 - \eta \underline{u}$$

$$\frac{\partial \Delta}{\partial a} = (1 + \eta\lambda)I\lambda a^{\lambda-1} - \frac{\eta}{h}a = 0$$

$$a^* = \left(\frac{Ih(1 + \eta\lambda)\lambda}{\eta} \right)^{\frac{1}{2-\lambda}} \quad I, h \text{ and } \eta \text{ are constant respectively}$$

Assign value to I, h, η . Let: $I = 100,000, h = 1, \eta = 1$

From above, we obtain (3.3):

$$a^* = [10(1 + \lambda)\lambda]^{\frac{1}{2-\lambda}} \tag{3.3}$$

Figure about relation between the degree of venture entrepreneur's effort a^* , and the proportion of control rights is plotted by Matlab.

Conclusion that degree of venture entrepreneur's effort increased with the proportion of control rights can be seen from fig2. ■

In the actual economic activities, control rights of venture entrepreneur change vary with enterprise business performance. As it is showed in figure 2.

(1) It can be seen Obviously from fig2. curve is comparatively mild when the control rights range from 0 to 0.5 , It is indicate that the degree of venture entrepreneur's efforts increase modest with increasing of control rights, Therefore, Venture capitalist's domination occupies the main position. This phase suitable for venture business in the stage of mature or in the difficult period.

(2) At this stage, Curve rise steeply when the control rights range from 0.5 to 1 , It is indicate that control rights of the venture capitalist decreases gradually. This phase suitable for venture business in the stage of start-up period or comparatively well operated. In order to gain higher profit and stimulate the venture entrepreneur to work hard, venture capitalist granted control rights to venture entrepreneur.

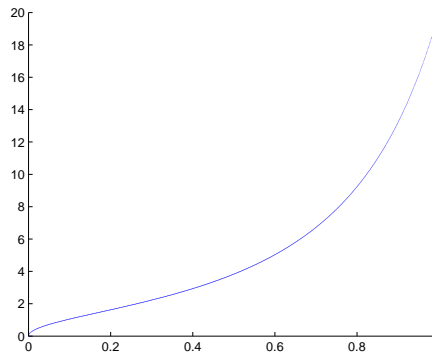


Figure 2: Relation between the degree of venture entrepreneur's effort and the proportion of control rights

4 Conclusions

Based on the background of multi-stage venture investment decision-making, this paper introduces a traditional Ramsey Model to venture investment and set up a Multiple Stage Decision Model which maximizes the total profit of venture enterprise in multi-periods. As we pointed out in Section 2, there exist an optimal path which make the profit function converges to a stable fixed point. From the Theorem 2, Venture capitalists assign control rights according to operation situation of Venture Enterprise. Conclusion of this paper can be used as a mean that venture capitalists encourage venture enterprise.

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