

Exact Periodic Wave Solutions to Some Nonlinear Evolution Equations

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Abstract:In this paper, the extended mapping method with symbolic computation is developed to obtain exact periodic wave solutions for nonlinear evolution equations arising in mathematical physics. Limit cases are studied and new solitary wave solutions and triangular periodic wave solutions are obtained. The method is applicable to a large variety of nonlinear partial differential equations, as long as odd and even-order derivative terms do not coexist in the equation under consideration. The method is straightforward and concise, and it can also be applied to other nonlinear evolution equations in mathematical physics.

Key words:extended mapping method; nonlinear evolution equations; new periodic solutions.

1 Introduction

Investigating the exact solutions of nonlinear evolution equations plays an important role in the study of nonlinear physical phenomena. For example, the wave phenomena observed in uid dynamics, plasma and elastic media are often modeled by the bell-shaped sech solutions and kink-shaped tanh solutions. The exact solution, if available, of those nonlinear equations facilitates the verification of numerical solvers and aids in the stability analysis of solutions. Therefore, in recent years, there has been a growing interest in developing and applying a large variety of analytical methods. Among these methods we can cite [1 – 23].

In fact the Jacobian elliptic functions [24] degenerate into hyperbolic functions when the modulus approaches 1, has attracted a lot of interest in the investigation of exact solutions. The three basic Jacobian elliptic functions $sn(\xi, m)$, $cn(\xi, m)$ and $dn(\xi, m)$, where m is the modulus of the elliptic function, satisfy the well known type of trigonometric relations such as $sn^2(\xi) + cn^2(\xi) = 1$, $dn^2(\xi) + m^2 sn^2(\xi) = 1$, $(sn(\xi))' = cn(\xi)dn(\xi)$, $(cn(\xi))' = -sn(\xi)dn(\xi)$, $(dn(\xi))' = -m^2 sn(\xi)cn(\xi)$. When $m \rightarrow 0$, the Jacobi elliptic functions degenerate to the triangular functions, i. e. , $sn(\xi) \rightarrow \sin(\xi)$, $cn(\xi) \rightarrow \cos(\xi)$, $dn(\xi) \rightarrow 1$ and when $m \rightarrow 1$, the Jacobian elliptic functions degenerate to the hyperbolic functions i. e. , $sn(\xi) \rightarrow \tanh(\xi)$, $cn(\xi) \rightarrow \text{sech}(\xi)$, $dn(\xi) \rightarrow \text{sech}(\xi)$. A mapping method and its extensions have been successfully applied to derive a variety of Jacobian elliptic function solutions for nonlinear equations [25 – 27].

For a given nonlinear evolution equation, say, in three independent variables,

$$N(u, u_t, u_x, u_y, \dots) = 0, \tag{1}$$

we seek its travelling wave solution of the form

$$u(x, y, t) = u(\xi), \xi = kx + ly - wt, \tag{2}$$

where k , l and w are constants to be determined later. Substituting Eq.(2) into Eq.(1) yields an ordinary differential equation of $u(\xi)$.

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By virtue of the technique of solution, we assume that the solution in the series form

$$\phi(\xi) = A_0 + \sum_{i=0}^M f^{i-1}(\xi)[A_i f(\xi) + B_i g(\xi)], \quad (3)$$

where A_0 , A_i and B_i are constants to be determined later, M is fixed by balancing the linear term of the highest order derivative with nonlinear term, while $f(\xi)$ and $g(\xi)$ satisfy the system of equations

$$\begin{aligned} f'^2 &= pf^2 + \frac{1}{2}qf^4 + r, f'' = pf + qf^3, \\ g^2 &= c_3 + c_4 f^2, g'' = g(c_1 + c_2 f^2) \\ g' f' &= fg(c_5 + c_6 f^2) \end{aligned} \quad (4)$$

where the prime denotes derivative with respect to ξ , and p , q , r and c_i ($i = 1, \dots, 6$) are constants to be determined.

The aim of this paper is to extend the mapping method [20], to derive a series of new periodic solutions and some of the limiting solutions when the modulus m of the elliptic approaches 0 or 1.

2 New Applications

2.1 Example[1]. The generalized KP-BBM equation

Let us first consider the (3+1)-dimensional KP-BBM equation [28]

$$[u_t + u_x - a(u^2)_x - bu_{xxt}]_x + ku_{yy} = 0, \quad (5)$$

where a , b and k are constants. To look for the travelling wave solution of Eq.(5), we use the gauge transformation

$$u = u(\xi), \xi = x + y - ct \quad (6)$$

where c is constant to be determined. Substituting Eq.(6) into (5), we have

$$[1 + k - c]u - au^2 + bcu'' = 0 \quad (7)$$

Eq.(7) can be rewritten as

$$Au'' + Bu^2 + Cu = 0, \quad (8)$$

$$A = bc, B = -a, C = [1 + k - c]$$

Considering the homogeneous balance between $u''(\xi)$ and $u^2(\xi)$ in Eq.(8), we assume that $u(\xi)$ can be expressed as

$$u(\xi) = A_0 + A_1 f(\xi) + B_1 g(\xi) + A_2 f^2(\xi) + B_2 f(\xi)g(\xi), \quad (9)$$

where A_0 , A_i and B_i are constants to be determined, and $f(\xi)$ and $g(\xi)$ satisfy the system of equations (4). We substitute ansatz (9) into (8), make use of Eq.(4) with computerized symbolic computation, equating to zero the coefficients of all powers of $f^i(\xi)g^j(\xi)$ yields a set of algebraic equations for A_0 , A_i and B_i :

Case[1]:

$$A_0 = -\frac{4pA + C}{2B}, A_1 = 0, A_2 = -\frac{3qA}{B}, B_1 = B_2 = 0, \quad (10)$$

Case[2]:

$$\begin{aligned} A_0 &= -\frac{(p + c_1 + 2c_5)A + C}{2B}, A_1 = 0, B_1 = 0, A_2 = -\frac{(q + c_2 + 2c_6)A}{2B}, \\ B_2^2 &= \frac{(3p - c_1 - 2c_5)(q + c_2 + 2c_6)A^2}{2c_3 B^2}, c_3 = (-5q + c_2 + 2c_6) + 2c_4(3p - c_1 - 2c_5) = 0 \end{aligned} \quad (11)$$

By means of Eqs. (10) and (9), admits to the new exact travelling wave solutions of Eq.(5) as follows

$$u(\xi) = -\frac{4pA + C}{2B} - \frac{3qA}{B} f^2(\xi) \tag{12}$$

where A, B and C are defined in Eq.(8). Using Eqs. (11) and (9), admits to the new exact travelling wave solutions of Eq.(5) as follows

$$u(\xi) = -\frac{(p + c_1 + 2c_5)A + C}{2B} - \frac{(q + c_2 + 2c_6)A}{2B} f^2(\xi) + \sqrt{\frac{(3p - c_1 - 2c_5)(q + c_2 + 2c_6)A^2}{2c_3B^2}} f(\xi)g(\xi), \tag{13}$$

where $f(\xi)$ and $g(\xi)$ satisfy Eq.(4) with the constraint among the parameters

$$c_3 = (-5q + c_2 + 2c_6) + 2c_4(3p - c_1 - 2c_5) = 0,$$

where $\xi = x + y - ct$. Different classes of new periodic wave solutions can be obtained according to the different choice of the two functions $f(\xi)$ and $g(\xi)$ as follows: It is to be noted that, the solution of Eq.(12) can also obtained by means of the mapping method [20]. So we study only the solution of Eq.(13) in what follows.

2.2 The new periodic wave solutions

Case (1). When $p = (2m^2 - 1), q = (-2m^2), r = (1 - m^2), c_1 = m^2, c_2 = -2m^2, c_3 = (1 - m^2), c_4 = m^2, c_5 = m^2, c_6 = -m^2$. We have $f(\xi) = cn(\xi)$ and $g(\xi) = dn(\xi)$. Thus the new periodic wave solution of Eq.(5) is

$$u_1(\xi) = \frac{(5m^2 - 1)bc + [1 + k - c]}{2a} - \frac{2m^2bc}{a} cn^2(\xi) + \sqrt{\frac{9m^2(m^2 - 1)b^2c^2}{(1 - m^2)a^2}} cn(\xi)dn(\xi) \tag{14}$$

For $m \rightarrow 1$, Eq.(14) admits to solitary wave solution

$$u_{11}(\xi) = \frac{4bc + [1 + k - c]}{2a} - \frac{2bc}{a} sech^2(\xi)$$

Case (2). When $p = (2 - m^2), q = -2(1 - m^2), r = -1, c_1 = 1, c_2 = -2(1 - m^2), c_3 = -\frac{1}{m^2}, c_4 = \frac{1}{m^2}, c_5 = 1, c_6 = -(1 - m^2)$. Here, we have $f(\xi) = nd(\xi)$ and $g(\xi) = sd(\xi)$ and the corresponding new periodic wave solution is

$$u_2(\xi) = \frac{((5 - m^2)bc + [1 + k - c])}{2a} + \frac{(-6 + 6m^2)bc}{2a} nd^2(\xi) + \frac{1}{2} \sqrt{-\frac{36(1 - m^2)(-1 + m^2)(bc)^2}{a^2}} nd(\xi)sd(\xi) \tag{15}$$

For $m \rightarrow 0$, Eq.(15) admits to triangular solution

$$u_{20}(\xi) = \frac{5bc + [1 + k - c]}{2a} - \frac{3bc}{a} - \frac{3bc}{a} \sin(\xi)$$

Case (3). If we select $p = -(1 + m^2), q = 2, r = m^2, c_1 = -m^2, c_2 = 2, c_3 = -1, c_4 = 1, c_5 = -m^2, c_6 = 1$. We have $f(\xi) = ns(\xi)$ and $g(\xi) = cs(\xi)$ and we obtain the new periodic wave solution as follows

$$u_3(\xi) = \frac{(-1 - 4m^2)bc + [1 + k - c]}{2a} + \frac{3bc}{a} ns^2(\xi) + 3\sqrt{\frac{b^2c^2}{a^2}} ns(\xi)cs(\xi) \tag{16}$$

When $m \rightarrow 1$, Eq.(16) admits to solitary wave solution as follows

$$u_{31}(\xi) = \frac{-5bc + [1 + k - c]}{2a} + \frac{3bc}{a} coth^2(\xi) + 3\sqrt{\frac{b^2c^2}{a^2}} coth(\xi)csch(\xi)$$

For $m \rightarrow 0$, Eq.(16) admits to triangular wave solution as follows

$$u_{30}(\xi) = \frac{-bc + [1 + k - c]}{2a} + \frac{3bc}{a} csc^2(\xi) + 3\sqrt{\frac{b^2c^2}{a^2}} csc(\xi)cot(\xi)$$

Case (4). Now with $p = -(1 + m^2)$, $q = 2m^2$, $r = 1$, $c_1 = -1$, $c_2 = 2m^2$, $c_3 = 1$, $c_4 = -1$, $c_5 = -1$, $c_6 = m^2$. In this case, we have $f(\xi) = sn(\xi)$ and $g(\xi) = cn(\xi)$ and thus the corresponding new periodic wave solution is

$$u_4(\xi) = \frac{(-4 - m^2)bc + [1 + k - c]}{2a} + \frac{(3m^2)bc}{a}sn^2(\xi) + 3\sqrt{-\frac{m^4b^2c^2}{a^2}}sn(\xi)cn(\xi) \quad (17)$$

For $m \rightarrow 0$, Eq.(17) admits to rational solution

$$u_{40}(\xi) = \frac{-4bc + [1 + k - c]}{2a}$$

In case of $m \rightarrow 1$, Eq.(17) admits to solitary wave solution as follows

$$u_{41}(\xi) = \frac{-5bc + [1 + k - c]}{2a} + \frac{3bc}{a}tanh^2(\xi) + 3\sqrt{-\frac{b^2c^2}{a^2}}tanh(\xi)sech(\xi)$$

Case (5). If $p = -(1 + m^2)$, $q = 2m^2$, $r = 1$, $c_1 = -m^2$, $c_2 = 2m^2$, $c_3 = 1$, $c_4 = -m^2$, $c_5 = -m^2$, $c_6 = m^2$. In this case, we have $f(\xi) = sn(\xi)$ and $g(\xi) = dn(\xi)$ and thus the corresponding new periodic wave solution is

$$u_5(\xi) = \frac{(-1 - 4m^2)bc + [1 + k - c]}{2a} + \frac{3m^2bc}{a}sn^2(\xi) + 3\sqrt{\frac{-m^2b^2c^2}{a^2}}sn(\xi)dn(\xi) \quad (18)$$

When of $m \rightarrow 1$, Eq.(18) admits to solitary wave solution

$$u_{51}(\xi) = \frac{-5bc + [1 + k - c]}{2a} + \frac{3bc}{a}tanh^2(\xi) + 3\sqrt{\frac{-b^2c^2}{a^2}}tanh(\xi)sech(\xi)$$

Case (6). If we select $p = -(1 + m^2)$, $q = 2$, $r = m^2$, $c_1 = -1$, $c_2 = 2$, $c_3 = -m^2$, $c_4 = 1$, $c_5 = -1$, $c_6 = 1$. We have $f(\xi) = ns(\xi)$ and $g(\xi) = ds(\xi)$ and we obtain the new periodic wave solution as follows

$$u_6(\xi) = \frac{(-4 - m^2)bc + [1 + k - c]}{2a} + \frac{2bc}{a}ns^2(\xi) + 3\sqrt{\frac{b^2c^2}{a^2}}ns(\xi)ds(\xi) \quad (19)$$

When of $m \rightarrow 0$, Eq.(19), admits to triangular solution

$$u_{60}(\xi) = \frac{-4bc + [1 + k - c]}{2a} + \frac{2bc}{a}csc^2(\xi) + 3\sqrt{\frac{b^2c^2}{a^2}}csc^2(\xi)$$

When $m \rightarrow 1$, Eq.(19) admits to solitary wave solution as follows

$$u_{61}(\xi) = \frac{-5bc + [1 + k - c]}{2a} + \frac{2bc}{a}coth^2(\xi) + 3\sqrt{\frac{b^2c^2}{a^2}}csch(\xi)coth(\xi)$$

Case (7). In case of $p = -(1 + m^2)$, $q = 2$, $r = m^2$, $c_1 = -1$, $c_2 = 2$, $c_3 = \frac{-1}{(1-m^2)}$, $c_4 = \frac{1}{(1-m^2)}$, $c_5 = -m^2$, $c_6 = 1$. We have $f(\xi) = dc(\xi)$ and $g(\xi) = sc(\xi)$ and we obtain the new periodic wave solution as follows

$$u_7(\xi) = \frac{(-2 - 3m^2)bc + [1 + k - c]}{2a} + \frac{3bc}{a}dc^2(\xi) + \sqrt{\frac{(-6 - 3m^2)(1 - m^2)b^2c^2}{a^2}}dc(\xi)sc(\xi) \quad (20)$$

When of $m \rightarrow 0$, Eq.(20), admits to triangular solution as follows

$$u_{70}(\xi) = \frac{-2bc + [1 + k - c]}{2a} + \frac{3bc}{a}sec^2(\xi) + \sqrt{6}\sqrt{\frac{b^2c^2}{a^2}}sec(\xi)tan(\xi)$$

Case (8). $p = 2m^2$, $q = 2(1 - m^2)$, $r = -m^2$, $c_1 = m^2$, $c_2 = 2(1 - m^2)$, $c_3 = -1$, $c_4 = 1$, $c_5 = m^2$, $c_6 = 1 - m^2$. We have $f(\xi) = nc(\xi)$ and $g(\xi) = sc(\xi)$ and we obtain the new periodic wave solution as follows

$$u_8(\xi) = \frac{5m^2bc + [1 + k - c]}{2a} + \frac{6(1 - m^2)bc}{2a}nc^2(\xi) + \frac{1}{2}\sqrt{\frac{-36m^2(1 - m^2)b^2c^2}{a^2}}nc(\xi)sc(\xi) \quad (21)$$

For of $m \rightarrow 0$, Eq.(21), admits to

$$u_{80}(\xi) = \frac{1 + k - c}{2a} + \frac{3bc}{a} \sec^2(\xi)$$

Case (9). $p = 2 - m^2, q = 2, r = 1 - m^2, c_1 = 1, c_2 = 2, c_3 = 1 - m^2, c_4 = 1, c_5 = 1, c_6 = 1$. We have $f(\xi) = cs(\xi)$ and $g(\xi) = ds(\xi)$ and we obtain the new periodic wave solution as follows

$$u_{90}(\xi) = -\frac{(5 - m^2)bc + [1 + k - c]}{2a} + \frac{3bc}{a} cs^2(\xi) + \sqrt{3} \sqrt{\frac{3(1 - m^2)b^2c^2}{(1 - m^2)a^2}} cs(\xi) ds(\xi) \tag{22}$$

For $m \rightarrow 0, 1$, Eq.(22), admits to

$$u_{90}(\xi) = \frac{5bc + [1 + k - c]}{2a} + \frac{3bc}{a} \cot^2(\xi) + 3\sqrt{\frac{b^2c^2}{a^2}} \cot(\xi) \csc(\xi),$$

$$u_{91}(\xi) = \frac{4bc + [1 + k - c]}{2a} + \frac{3bc}{a} csch^2(\xi)$$

3 Example[3]. The (3+1)-dimensional generalized ZK-BBM equation

A second instructive model is the (3+1)dimensional ZK-BBM equation [28]

$$u_t + u_x - a(u^2)_x - [bu_{xt} + ku_{yt}]_x = 0 \tag{23}$$

where a, b and k are constants. To look for the travelling wave solution of Eq.(23), we use

$$u = u(\xi), \xi = x + y - ct \tag{24}$$

where c is constant to be determined. Substituting Eq.(24) into (23) yields the real system

$$[1 - c]u - au^2 + (b + k)cu'' = 0 \tag{25}$$

Eq.(25) can be rewritten as

$$Au'' + Bu^2 + Cu = 0, \tag{26}$$

Eq.(26) coincides with Eq.(8), where A, B and C are defined by

$$A = (b + k)c, B = -a, C = (1 - c)$$

Our main goal is to solve Eq.(26) using the extended mapping method illustrated above. Considering the homogeneous balance between $u''(\xi)$ and $u^2(\xi)$ in Eq.(26), we assume that $u(\xi)$ can be expressed as

$$u(\xi) = A_0 + A_1f(\xi) + B_1g(\xi) + A_2f^2(\xi) + B_2f(\xi)g(\xi), \tag{27}$$

where A_i and B_i are constants to be determined, and $f(\xi)$ and $g(\xi)$ satisfy the system of Eq.(4). Substituting Eq.(27) into (26) with Eq.(4), equating to zero the coefficients of all powers of $f^i(\xi)g^j(\xi)$ yields a set of algebraic equations for A_0, A_i and B_i . Solving the system of algebraic equations with the aid of Maple, we can determined the coefficients

Case[1]:

$$A_0 = \frac{4p(b + k)c + (1 - c)}{2a}, A_1 = 0, A_2 = \frac{3q(b + k)c}{-a}, B_1 = B_2 = 0, \tag{28}$$

Case[2]:

$$A_0 = \frac{(p + c_1 + 2c_5)(b + k)c + (1 - c)}{2a}, A_1 = 0, B_1 = 0, A_2 = \frac{(q + c_2 + 2c_6)A}{2a}, \tag{29}$$

$$B_2^2 = \frac{(3p - c_1 - 2c_5)(q + c_2 + 2c_6)[(b + k)c]^2}{2c_3a^2},$$

From Eqs. (28) and (27), we have a new exact travelling wave solutions of Eq.(23) as follows

$$u(\xi) = \frac{4p(b + k)c + (1 - c)}{2a} + \frac{3q(b + k)c}{a} f^2(\xi) \tag{30}$$

Using Eqs. (29) and (27), admits to the new exact travelling wave solutions of Eq.(23) as follows

$$u(\xi) = \frac{(p + c_1 + 2c_5)(b + k)c + (1 - c)}{2a} + \frac{(q + c_2 + 2c_6)(b + k)c}{2a} f^2(\xi) + \sqrt{\frac{(3p - c_1 - 2c_5)(q + c_2 + 2c_6)[(b + k)c]^2}{2c_3a^2}} f(\xi)g(\xi), \quad (31)$$

where $f(\xi)$ and $g(\xi)$ satisfy Eq.(4) with the constraint among the parameters

$$c_3 = (-5q + c_2 + 2c_6) + 2c_4(3p - c_1 - 2c_5) = 0,$$

where $\xi = x + y - ct$. A series of new periodic wave solution are obtained according to the different choice of the two functions $f(\xi)$ and $g(\xi)$. We omit this discussion here for simplicity.

4 Example[4]. The generalized Boussinesq wave equation

The Boussinesq wave equation [29] reads

$$u_{tt} + \alpha u_{xx} + \beta(u^2)_{xx} + \gamma u_{xxxx} = 0, \quad (32)$$

where α , β and γ are constants. To look for the travelling wave solution of Eq.(32), we use the gauge transformation

$$u = u(\xi), \xi = x - ct, \quad (33)$$

where c is constant to be determined. Substituting Eq.(33) into (32) reduces to

$$Au'' + Bu^2 + Cu = 0 \quad (34)$$

Eq.(34) coincides with Eq.(8), where A , B and C

$$A = \gamma, B = \beta, C = (c^2 + \alpha) \quad (35)$$

By virtue of the technique of solution, we assume that the solution of Eq.(34), can expressed as

$$u(\xi) = A_0 + A_1f(\xi) + B_1g(\xi) + A_2f^2(\xi) + B_2f(\xi)g(\xi), \quad (36)$$

In the same manner, substituting Eq.(36) into (34), equating to zero the coefficients of all powers of $f^i(\xi)g^j(\xi)$ yields a set of algebraic equations for A_0 , A_i and B_i . Solving the system of algebraic equations, we can determined

Case[1]:

$$A_0 = -\frac{4pA + C}{2B}, A_1 = 0, A_2 = -\frac{3qA}{B}, B_1 = B_2 = 0, \quad (37)$$

Case[2]:

$$vA_0 = -\frac{(p + c_1 + 2c_5)A + C}{2B}, A_1 = 0, B_1 = 0, A_2 = -\frac{(q + c_2 + 2c_6)A}{2B}, \quad (38)$$

$$B_2^2 = \frac{(3p - c_1 - 2c_5)(q + c_2 + 2c_6)A^2}{2c_3B^2}, c_3 = (-5q + c_2 + 2c_6) + 2c_4(3p - c_1 - 2c_5) = 0$$

With the aid of Eq.(37) and (36), admits to the new exact travelling wave solutions of Eq.(32) as follows

$$u(\xi) = -\frac{4p\gamma + (c^2 + \alpha)}{2\beta} - \frac{3q\gamma}{\beta} f^2(\xi) \quad (39)$$

By using Eqs. (38) and (36), admits to

$$u(\xi) = -\frac{(p + c_1 + 2c_5)\gamma + (c^2 + \alpha)}{2\beta} - \frac{(q + c_2 + 2c_6)\gamma}{2\beta} f^2(\xi) + \sqrt{\frac{(3p - c_1 - 2c_5)(q + c_2 + 2c_6)\gamma^2}{2c_3\beta^2}} f(\xi)g(\xi), \quad (40)$$

where $f(\xi)$ and $g(\xi)$ satisfy Eq.(4). where $\xi = x - ct$. Different classes of new periodic wave solutions can be obtained according to the different choice of the two functions $f(\xi)$ and $g(\xi)$. We omit this discussion here for simplicity.

5 Example[5]. The quadratic nonlinear Klein-Gordon equation

In this case, the quadratic nonlinear Klein Godon equation reads [30]

$$u_{tt} - \alpha^2 u_{xx} + \beta u - \gamma u^2 = 0 \tag{41}$$

where α, β and γ are constants. To look for the travelling wave solution of Eq.(41), we use

$$u = u(\xi), \xi = x + wt \tag{42}$$

where w is constant to be determined. Substituting Eq.(42) into (41) reduces to

$$Au'' + Bu^2 + Cu = 0, \tag{43}$$

Eq.(43) coincides with Eq.(8), where

$$A = (w^2 - \alpha^2), B = -\gamma, C = \beta \tag{44}$$

The solution of Eq.(43) for $u(\xi)$ can be expressed as

$$u(\xi) = A_0 + A_1 f(\xi) + B_1 g(\xi) + A_2 f^2(\xi) + B_2 f(\xi)g(\xi) \tag{45}$$

Substituting Eq.(45) into (43), yields a set of algebraic equations for A_0, A_i and B_i . Solving the system of algebraic equations, we can determined

Case[1]:

$$A_0 = \frac{4p(w^2 - \alpha^2) + \beta}{2\gamma}, A_1 = 0, A_2 = \frac{3q(w^2 - \alpha^2)}{\gamma}, B_1 = B_2 = 0, \tag{46}$$

Case[2]:

$$A_0 = \frac{(p + c_1 + 2c_5)(w^2 - \alpha^2) + C}{2\gamma}, A_1 = 0, B_1 = 0, A_2 = \frac{(q + c_2 + 2c_6)(w^2 - \alpha^2)}{2\gamma},$$

$$B_2^2 = \frac{(3p - c_1 - 2c_5)(q + c_2 + 2c_6)(w^2 - \alpha^2)^2}{2c_3\gamma^2}, c_3 = (-5q + c_2 + 2c_6) + 2c_4(3p - c_1 - 2c_5) = 0 \tag{47}$$

From Eq.(46) and (45), admits to the new exact travelling wave solutions of Eq.(41) as follows

$$u(\xi) = \frac{4p(w^2 - \alpha^2) + \beta}{2\gamma} + \frac{3q(w^2 - \alpha^2)}{\gamma} f^2(\xi) \tag{48}$$

Using Eqs. (47) and (45), admits to the new exact travelling wave solutions of Eq.(41) as

$$u(\xi) = \frac{(p + c_1 + 2c_5)(w^2 - \alpha^2) + \beta}{2\gamma} + \frac{(q + c_2 + 2c_6)(w^2 - \alpha^2)}{2\gamma} f^2(\xi)$$

$$+ \sqrt{\frac{(3p - c_1 - 2c_5)(q + c_2 + 2c_6)(w^2 - \alpha^2)^2}{2c_3\gamma^2}} f(\xi)g(\xi), \tag{49}$$

where $\xi = x + wt$. New periodic wave solutions are obtained for a differernt choice of the $f(\xi)$ and $g(\xi)$. We omitt this discussion here for simplicity.

6 Conclusion

The extended mapping method with a computerized symbolic computation is used for constructing wide classes of new periodic travelling wave solutions for nonlinear evolution equations arising in mathematical physics.

The validity of this method has been tested by applying it successfully to the quadratic nonlinear Klein-Gordon equation, generalized Boussinesq wave equation, (3+1)-dimensional generalized ZK-BBM equation and (3+1)-dimensional KP-BBM equation. As a result, new exact travelling wave solutions are obtained which include new solitary wave solutions. The limiting solutions when the modulus m of the elliptic function approach 0 or 1 have also been derived.

Finally, it is worthwhile to mention that the method is straightforward and concise, and it can also be applied to other nonlinear evolution equations in mathematical physics. This is our task in the future work.

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