

The Modified Extended Direct Algebraic Method for Solving Nonlinear Partial Differential Equations

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Abstract: By means of modified extended direct algebraic (MEDA) method the multiple exact complex solutions of some different kinds of nonlinear partial differential equations are presented and implemented in a computer algebraic system. New complex solutions for nonlinear equations such as one-dimensional Burgers, KdV-Burgers, coupled Burgers and two-dimensional Burgers' equations are obtained.

Keywords: Burgers' equation; KdV-Burgers' equation; coupled Burgers' equations; two-dimensional Burgers' equations; the MEDA method

1 Introduction

Recently many new approach to obtain the exact solutions of nonlinear differential equations have been proposed. Among these are variational iteration method [1-7], tanh function method [8,9], modified extended tanh function method [10-16], sine-cosine method [17,18], Exp-method [19], inverse scattering method [20], Hirota's bilinear method [21], the homogeneous balance method [22], the Riccati expansion method with constant coefficients [23,24]. The Burgers' equation has been found to describe various kinds of phenomena such as a mathematical model of turbulence [25] and the approximate theory of flow through a shock wave traveling in viscous fluid [26]. Fletcher using the Hopf-Cole transformation [27] gave an analytic solution for the system of two dimensional Burgers' equations. Several numerical methods of this equation system have been given such as algorithms based on cubic spline function technique [28], The explicit-implicit method [29], and implicit finite-difference scheme [30]. Soliman [31] used the similarity reductions for the partial differential equations to develop a scheme for solving the Burgers' equation. High-order accurate schemes for solving the two-dimensional Burgers' equations have been used [32, 33]. The fourth-order accurate two point compact scheme, and the fourth-order accurate Dufort Frankel scheme have been derived, and then numerical stability and convergence have been discussed [33].

The coupled system was derived by Esipov [34]. It is a simple model of sedimentation or evolution of scaled volume concentrations of two kinds of particles in fluid suspensions or colloids, under the effect of gravity [35]. The variational iteration method was used to solve the one-dimensional Burgers and coupled Burgers' equations [1], the solution was obtained in a series of initial conditions and was turned to a closed form.

Recently, the direct algebraic method and symbolic computation have been suggested to obtain the exact complex solutions of nonlinear partial differential equations [36,37].

The aim of this paper is to extend the modified extended direct algebraic (MEDA) method to solve four different types of nonlinear differential equations such as the Burgers, KdV-Burgers, coupled Burgers and two-dimensional Burgers' equations [10].

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2 Modified extended direct algebraic method

To illustrate the basic concepts of the modified extended direct algebraic (MEDA) method. We consider a given PDE in two independent variables given by

$$F(u, u_x, u_t, u_{xx}, \dots) = 0, \quad (1)$$

We first consider its travelling solutions $u(x, t) = u(z)$, $z = i(x + ct)$ or $z = i(x - ct)$, $i = \sqrt{-1}$, then Eq.(1) becomes an ordinary differential equation

$$H(u, uu', -icu', -u'', \dots) = 0, \quad (2)$$

where $u' = \frac{du}{dz}$.

In order to seek the solutions of Eq.(1), we introduce the following ansatz

$$u(z) = a_0 + \sum_{j=1}^M (a_j \phi^j + b_j \phi^{-j}), \quad (3)$$

$$\phi' = b + \phi^2, \quad (4)$$

where b is a parameter to be determined, $\phi = \phi(z)$, $\phi' = \frac{d\phi}{dz}$. The parameter M can be found by balancing the highest-order derivative term with the nonlinear terms [38]. Substituting (3) into (2) with (4) will yield a system of algebraic equations with respect to a_j , b_j , b and c (where $j = 1 \dots M$) because all the coefficients of ϕ^j have to vanish. we can then determine a_0 , a_j , b_j , b , and c . Eq.(4) has the general solutions:

(I) If $b < 0$

$$\phi = -\sqrt{-b} \tanh(\sqrt{-b}z), \text{ or } \phi = -\sqrt{-b} \coth(\sqrt{-b}z),$$

it depends on initial conditions.

(II) If $b > 0$

$$\phi = \sqrt{b} \tan(\sqrt{b}z), \text{ or } \phi = -\sqrt{b} \cot(\sqrt{b}z).$$

it depends on initial conditions.

(III) If $b = 0$

$$\phi = \frac{-1}{z}. \quad (5)$$

Substituting the results into (3), then we obtain the exact travelling wave solutions of Eq. (1).

To illustrate the procedure, four examples related to the one dimensional Burgers, KdV-Burgers, coupled Burgers, and two-dimensional Burgers' equations are given in the following.

3 Applications

3.1 One-dimensional Burgers' equation

Let us first consider the one-dimensional Burgers' equation which has the form [10]

$$u_t + \alpha u u_x - \nu u_{xx} = 0, \quad (6)$$

where α and ν are arbitrary constants. In order to solve Eq. (6) by the MDA method, we use the wave transformation $u(x, t) = U(z)$ with wave complex variable $z = i(x - ct)$, Eq. (6) takes the form of an ordinary differential equation as

$$-ciU' + \frac{\alpha i}{2}(U^2)' + \nu U'' = 0. \quad (7)$$

Integrating Eq. (7) once with respect to z and setting the constant of integration to be zero, we obtain

$$-ciU + \frac{\alpha i}{2}(U^2) + \nu U' = 0. \quad (8)$$

Balancing the order of U^2 with the order of U' in Eq. (8), we find $M = 1$. So the solution takes the form

$$U(z) = a_0 + a_1\phi(z) + b_1\phi(z)^{-1}. \quad (9)$$

Inserting Eq. (9) into Eq. (8) and making use of Eq. (4), using the Maple Package, we get a system of algebraic equations, for a_0, a_1, b_1 and b in the form

$$\begin{aligned} \frac{1}{2}\alpha a_1 i - \nu b_1 &= 0, \\ -c + \alpha a_0 &= 0, \\ -ca_0 i + \nu a_1 b + \frac{1}{2}\alpha a_0^2 i - \nu b_1 + \alpha a_1 b_1 i &= 0, \\ \alpha a_0 - c &= 0, \\ \frac{1}{2}\alpha b_1 i - \nu b &= 0. \end{aligned} \quad (10)$$

These equations give the following three cases:

Case (I): $b_1 = 0, a_1 = \frac{2\nu}{\alpha}i, b = \frac{a_0^2\alpha^2}{4\nu^2}$ and $c = \alpha a_0$, with a_0 being arbitrary constant, the travelling wave solution is given by

$$u(x, t) = a_0 \left\{ 1 + i \tan \left(\frac{1}{2} \frac{\alpha a_0 i}{\nu} (x + \alpha a_0 t) \right) \right\}. \quad (11)$$

Case (II): $b_1 = -\frac{1}{2} \frac{\alpha a_0^2}{\nu} i, a_1 = 0, b = \frac{a_0^2\alpha^2}{4\nu^2}$ and $c = \alpha a_0$, with a_0 being arbitrary constant, the travelling wave solution is given by

$$u(x, t) = a_0 \left\{ 1 - i \cot \left(\frac{1}{2} \frac{\alpha a_0 i}{\nu} (x + \alpha a_0 t) \right) \right\}. \quad (12)$$

Case (III) : $b_1 = -\frac{1}{8} \frac{\alpha a_0^2}{\nu} i, a_1 = \frac{2\nu}{\alpha}i, b = \frac{a_0^2\alpha^2}{16\nu^2}$ and $c = \alpha a_0$, with a_0 being arbitrary constant, the travelling wave solution is given by

$$u(x, t) = a_0 \left\{ 1 + \frac{a_0 i}{2} \tan \left(\frac{1}{4} \frac{\alpha a_0 i}{\nu} (x + \alpha a_0 t) \right) - \frac{a_0 i}{2} \cot \left(\frac{1}{4} \frac{\alpha a_0 i}{\nu} (x + \alpha a_0 t) \right) \right\} \quad (13)$$

All solutions are new exact solutions for the Burgers equation.

3.2 KdV-Burgers' equation

A second important example is the KdV-Burgers' equation [10], which can be written as

$$u_t + \varepsilon u u_x - \nu u_{xx} + \mu u_{xxx} = 0, \quad (14)$$

where ε, ν and μ are arbitrary constants. In order to solve Eq. (14) by the MDA method, we use the wave transformation $u(x, t) = U(z)$ with wave complex variable $z = i(x + ct)$. Eq. (14) takes the form of an ordinary differential equation

$$ciU' + \frac{\varepsilon i}{2}(U^2)' + \nu U'' - i\mu U''' = 0. \quad (15)$$

Integrating Eq. (15) once with respect to z and setting the constant of integration to zero, we obtain

$$ciU + \frac{\varepsilon i}{2}U^2 + \nu U' - \mu iU'' = 0. \tag{16}$$

Balancing the order of U^2 with the order of U'' in Eq. (16), we find $M = 2$. So the solution takes the form

$$U(z) = a_0 + a_1\phi(z) + a_2\phi(z)^2 + b_1\phi(z)^{-1} + b_2\phi(z)^{-2}. \tag{17}$$

Substituting Eq. (17) into Eq. (16) and making use of Eq. (4), we obtain a system of algebraic equations, for a_0, a_1, a_2, b_1, b_2 and b in the form

$$\begin{aligned} -2\mu b_2i + \varepsilon a_2b_2i + \varepsilon a_1b_1i + \nu a_1b - 2\mu a_2b^2i - \nu b_1 + \frac{1}{2}\varepsilon a_0^2i + ca_0i &= 0, \\ \varepsilon a_0a_2i - 8\mu a_2bi + \frac{1}{2}\varepsilon a_1^2i + \nu a_1 + ca_2i &= 0, \\ \varepsilon a_0a_1i + \varepsilon a_2b_1i + 2\nu a_2b - 2\mu a_1bi + ca_1i &= 0, \\ \varepsilon a_0b_1i + \varepsilon a_1b_2i - 2\nu b_2 - 2\mu b_1bi + cb_1i &= 0, \\ \varepsilon a_0b_2i + \nu b_1b - 8\mu b_2bi + \frac{1}{2}\varepsilon b_1^2i + cb_2i &= 0, \\ -2\nu b_2b - 2\mu b_1b^2i + \varepsilon b_1b_2i &= 0, \\ \varepsilon a_2a_1i + 2\nu a_2 - 2\mu a_1i &= 0, \\ \frac{1}{2}\varepsilon b_2^2 - 6\mu b_2b^2 &= 0, \\ \frac{1}{2}\varepsilon a_2^2 - 6\mu a_2 &= 0. \end{aligned} \tag{18}$$

The travelling wave solutions from the output of the Maple packages are as follows:

Case (I): $a_0 = \frac{9\nu^2}{25\mu\varepsilon}, a_1 = 0, a_2 = 0, b_1 = -\frac{3\nu^3}{125\mu^2\varepsilon}i, b_2 = \frac{3\nu^4}{2500\mu^3\varepsilon}, b = \frac{\nu^2}{100\mu^2}, c = \frac{-6\nu^2}{25\mu}$. The travelling wave solution is given by

$$u(x, t) = \frac{\nu^2}{25\varepsilon\mu} \left[9 - 6i \cot\left(\frac{\nu i}{10\mu}(x - \frac{6\nu^2}{25\mu}t)\right) - 3 \cot^2\left(\frac{\nu i}{10\mu}(x - \frac{6\nu^2}{25\mu}t)\right) \right]. \tag{19}$$

Case (II): $a_0 = \frac{9\nu^2}{25\mu\varepsilon}, a_1 = \frac{12\nu i}{5\varepsilon}, a_2 = \frac{12\mu}{\varepsilon}, b_1 = 0, b_2 = 0, b = \frac{\nu^2}{100\mu^2}, c = \frac{-6\nu^2}{25\mu}$. The travelling wave solution is given by

$$u(x, t) = \frac{\nu^2}{25\varepsilon\mu} \left[9 + 6i \tan\left(\frac{\nu i}{10\mu}(x - \frac{6\nu^2}{25\mu}t)\right) - 3 \tan^2\left(\frac{\nu i}{10\mu}(x - \frac{6\nu^2}{25\mu}t)\right) \right]. \tag{20}$$

Case(III) : $a_0 = \frac{3\nu^2}{10\mu\varepsilon}, a_1 = \frac{12\nu i}{5\varepsilon}, a_2 = \frac{12\mu}{\varepsilon}, b_1 = \frac{-3\nu^3 i}{500\mu^2\varepsilon}, b_2 = \frac{3\nu^4}{40000\mu^3\varepsilon}, b = \frac{\nu^2}{400\mu^2}, c = \frac{-6\nu^2}{25\mu}$. The travelling wave solution is given by

$$\begin{aligned} u(x, t) &= \frac{3\nu^2}{10\varepsilon\mu} + \frac{3\nu^2}{100\varepsilon\mu} \left[4i \left\{ \tan\left(\frac{\nu i}{20\mu}(x - \frac{6\nu^2}{25\mu}t)\right) - \cot\left(\frac{\nu i}{20\mu}(x - \frac{6\nu^2}{25\mu}t)\right) \right\} \right. \\ &\quad \left. + \left\{ \tan^2\left(\frac{\nu i}{20\mu}(x - \frac{6\nu^2}{25\mu}t)\right) + \cot^2\left(\frac{\nu i}{20\mu}(x - \frac{6\nu^2}{25\mu}t)\right) \right\} \right]. \end{aligned} \tag{21}$$

All the solutions of the KdV Burgers equation are new.

3.3 Coupled Burgers' equations

The third instructive example to illustrate of the MEDA method is the homogeneous form of a coupled Burgers' equations [10]. We will consider the following system of equations

$$u_t - u_{xx} + 2u u_x + \alpha(uv)_x = 0, \quad (22)$$

$$v_t - v_{xx} + 2vv_x + \beta(uv)_x = 0. \quad (23)$$

In order to solve Eqs. (22,23) by the MEDA method. We use the wave transformations $u(x, t) = U(z)$ and $v(x, t) = V(z)$ with wave complex variable $z = i(x + ct)$. Eqs. (22,23) take the form of ordinary differential equations

$$ciU' + U'' + i(U^2)' + \alpha i(UV)' = 0, \quad (24)$$

$$ciV' + V'' + i(V^2)' + \beta i(UV)' = 0. \quad (25)$$

Integrating Eqs. (24,25) once with respect to z and setting the constant of integration to zero, we obtain

$$c i U + U' + i U^2 + \alpha i UV = 0, \quad (26)$$

$$c i V + V' + i V^2 + \beta i UV = 0. \quad (27)$$

Balancing the order of U^2 with the order of U' and the order of V^2 with V' in Eqs. (26,27), we find $M = 1$. So the solutions take the form

$$U(z) = a_0 + a_1\phi(z) + b_1\phi(z)^{-1}, \quad (28)$$

$$V(z) = s_0 + s_1\phi(z) + r_1\phi(z)^{-1}. \quad (29)$$

Inserting Eqs. (28,29) into Eqs. (26,27) and making use of Eq. (4), we obtain a system of algebraic equations, for $a_0, a_1, b_1, s_0, s_1, r_1, c$ and b in the form

$$\begin{aligned} a_1 + a_1^2 i + \alpha a_1 s_1 &= 0, \\ ca_1 i + 2a_0 a_1 i + \alpha a_0 s_1 i + \alpha a_1 s_0 i &= 0, \\ ca_0 i + a_1 b - b_1 + a_0^2 i + 2a_1 b_1 i + \alpha a_0 s_0 i + \alpha a_1 r_1 i + \alpha b_1 s_1 i &= 0, \\ cb_1 + 2a_0 b_1 + \alpha a_0 r_1 + \alpha b_1 s_0 &= 0, \\ -b_1 b + b_1^2 i + \alpha b_1 r_1 i &= 0, \\ s_1 + s_1^2 i + \beta a_1 s_1 i &= 0, \\ cs_1 + 2s_0 s_1 + \beta a_0 s_1 + \beta a_1 s_0 &= 0, \\ cs_0 i + s_1 b - r_1 + s_0^2 i + 2s_1 r_1 i + \beta a_0 s_0 i + \beta a_1 r_1 i + \beta b_1 s_1 i &= 0, \\ cr_1 + 2s_0 r_1 + \beta a_0 r_1 + \beta b_1 s_0 &= 0, \\ -r_1 b + r_1^2 i + \beta b_1 r_1 i &= 0. \end{aligned} \quad (30)$$

We solve the above system by the Maple Packages and select three kinds of solutions,

Case(I): $s_0 = \frac{a_0(\beta - 1)}{\alpha - 1}, a_1 = \frac{\alpha - 1}{\alpha\beta - 1}i, s_1 = \frac{\beta - 1}{\alpha\beta - 1}i, c = -2\frac{a_0(-1 + \alpha\beta)}{\alpha - 1}, b_1 = 0, r_1 = 0,$
 $b = -\frac{1}{16}\frac{a_0^2(\alpha^2\beta^2 - 2\alpha\beta + 1)^2}{(\alpha - 1)^2},$ with a_0 being an arbitrary constant. The travelling wave solutions are

given by

$$u(x, t) = a_0 - \frac{A}{B} i \{ \tan(A i (x - 2a_0 B t)) \}, \quad (31)$$

$$v(x, t) = a_0 \frac{\beta - 1}{\alpha - 1} + \frac{A}{C} i \{ \tan(A i (x - 2a_0 B t)) \} \quad (32)$$

where $A = \sqrt{\frac{a_0^2(\alpha^2\beta^2 - 2\alpha\beta + 1)}{(\alpha - 1)^2}}$, $B = \frac{(\alpha\beta - 1)}{\alpha - 1}$, $C = \frac{\alpha\beta - 1}{\beta - 1}$.

Case(II) : $a_0 = \frac{(\alpha - 1)s_0}{\beta - 1}$, $a_1 = 0$, $s_1 = 0$, $c = \frac{s_0(\alpha\beta - 1)}{\beta - 1}$, $b_1 = -\frac{s_0^2(\alpha\beta - 1)(\alpha - 1)}{(-1 + \beta)^2} i$, $r_1 = -\frac{s_0^2(-1 + \alpha\beta)}{-1 + \beta} i$, $b = \frac{s_0^2(-1 + \alpha\beta)^2}{(-1 + \beta)^2}$, with s_0 being an arbitrary constant. The travelling wave solutions are given by

$$u(x, t) = \frac{(\alpha - 1)s_0}{\beta - 1} - \frac{A_1}{B} i \{ \cot(A_1 i (x - 2A_1 t)) \}, \tag{33}$$

$$v(x, t) = s_0 - \frac{A_1}{C} i \{ \cot(A_1 i (x - 2A_1 t)) \}, \tag{34}$$

where $A_1 = \frac{s_0(\alpha\beta - 1)}{\beta - 1}$.

Case (III): $s_0 = \frac{a_0(\beta - 1)}{\alpha - 1}$, $a_1 = \frac{\alpha - 1}{\alpha\beta - 1} i$, $s_1 = \frac{\beta - 1}{\alpha\beta - 1} i$, $c = -2 \frac{(\alpha\beta - 1) a_0}{\alpha - 1}$, $b_1 = \frac{1}{4} \frac{a_0^2(\alpha\beta - 1)}{(\alpha - 1)}$, $r_1 = \frac{B^2 a_0^2}{4C} i$, $b = \frac{1}{4} \frac{a_0^2(\alpha\beta - 1)^2}{(\alpha - 1)^2}$, with a_0 being an arbitrary constant. The traveling wave solutions are given by

$$u(x, t) = a_0 \left(1 + \frac{1}{2} i \{ \tan(\frac{1}{2} Ba_0 i (x - Ba_0 t)) - \cot(\frac{1}{2} Ba_0 i (x - Ba_0 t)) \} \right) \tag{35}$$

$$v(x, t) = \frac{Ba_0}{C} \left(1 + \frac{1}{2} i \{ \tan(\frac{1}{2} Ba_0 i (x - Ba_0 t)) - \cot(\frac{1}{2} Ba_0 i (x - Ba_0 t)) \} \right) \tag{36}$$

All the solutions of the coupled Burgers' equations are new.

3.4 The two-dimensional Burgers' equations

Our last example is a system of 2D-Burgers' equations [10]

$$u_t + u u_x + v u_y - \frac{1}{Re}(u_{xx} + u_{yy}) = 0, \tag{37}$$

$$v_t + u v_x + v v_y - \frac{1}{Re}(v_{xx} + v_{yy}) = 0, \tag{38}$$

where Re is the Reynolds number. To solve the system of Eqs. (37,38) by means of the MEDA method, we use the transformations $u(x, t) = U(z)$ and $v(x, t) = V(z)$ with wave complex variable $z = i(x + y + ct)$, Eqs. (37,38) take the form of ODEs

$$c i U' + i U U' + i V U' + 2\nu U'' = 0, \tag{39}$$

$$c i V' + i U V' + i V V' + 2\nu V'' = 0, \tag{40}$$

where $\nu = \frac{1}{Re}$. Balancing the order of $U U'$ with the order of U'' and the order of $V V'$ with V'' in Eqs. (39,40), we find $M = 1$. So the solutions take the form

$$U(z) = a_0 + a_1 \phi(z) + b_1 \phi(z)^{-1}, \tag{41}$$

$$V(z) = s_0 + s_1 \phi(z) + r_1 \phi(z)^{-1}. \tag{42}$$

Inserting Eqs. (41,42) into Eqs. (39,40) and making use of Eq. (4), and by using the Maple Package, we get a system of algebraic equations, for $a_0, a_1, b_1, s_0, s_1, r_1, c$ and b in the form

$$\begin{aligned}
 a_1^2 + s_1 a_1 - 4\nu a_1 i &= 0, \\
 ca_1 + a_0 a_1 + s_0 a_1 &= 0, \\
 -b(-a_1^2 - s_1 a_1 + 4\nu a_1 i) + r_1 a_1 - s_1 b_1 &= 0, \\
 -b(-ca_1 - a_0 a_1 - s_0 a_1) - cb_1 - a_0 b_1 - s_0 b_1 &= 0, \\
 -b(-r_1 a_1 + s_1 b_1) - 4\nu b_1 b i - b_1^2 - r_1 b_1 &= 0, \\
 -b(cb_1 + a_0 b_1 + s_0 b_1) &= 0, \\
 -b(4\nu b_1 b i + b_1^2 + r_1 b_1) &= 0, \\
 cs_1 + a_0 s_1 + s_0 s_1 &= 0, \\
 s_1^2 + s_1 a_1 - 4\nu s_1 i &= 0, \\
 -b(-s_1^2 - s_1 a_1 + 4\nu s_1 i) + s_1 b_1 - r_1 a_1 &= 0, \\
 -b(-cs_1 - a_0 s_1 - s_0 s_1) - cr_1 - s_0 r_1 - a_0 r_1 &= 0, \\
 -b(-s_1 b_1 + r_1 a_1) - 4\nu r_1 b i - r_1 b_1 - r_1^2 &= 0, \\
 -b(cr_1 + s_0 r_1 + a_0 r_1) &= 0, \\
 -b(4\nu r_1 b i + r_1 b_1 + r_1^2) &= 0.
 \end{aligned} \tag{43}$$

We solve the above system and obtain three kinds of solutions,

Case (I): $a_0 = -c - s_0, a_1 = -s_1 + 4\nu i, b_1 = 0, r_1 = 0,$

with s_0, s_1, b and c being arbitrary constants. The traveling wave solutions are given by

$$u(x, y, t) = -c - s_0 + (s_1 i + 4\nu)\sqrt{-b} \tan(\sqrt{-b}(x + y + ct)), \tag{44}$$

$$v(x, y, t) = s_0 - s_1 \sqrt{-b} i \tan(\sqrt{-b}(x + y + ct)). \tag{45}$$

Case (II): $a_0 = -c - s_0, a_1 = 0, s_1 = 0, r_1 = -2\nu b i - b_1,$ with s_0, b_1, b and c being arbitrary constants. The traveling wave solutions are given by

$$u(x, y, t) = -c - s_0 + \frac{b_1 i}{\sqrt{-b}} \cot(\sqrt{-b}(x + y + ct)), \tag{46}$$

$$v(x, y, t) = s_0 + \frac{(4\nu b - b_1 i)}{\sqrt{-b}} \cot(\sqrt{-b}(x + y + ct)). \tag{47}$$

Case (III) : $a_0 = -c - s_0, s_1 = -a_1 - 2\nu i, b_1 = -a_1 b, r_1 = b a_1 - 4\nu b i,$ with s_0, a_1, b and c being arbitrary constants. The traveling wave solutions are given by

$$u(x, y, t) = -c - s_0 - a_1 i \sqrt{-b} \left(\frac{\tan(\sqrt{-b}(x + y + ct)) -}{\cot(\sqrt{-b}(x + y + ct))} \right), \tag{48}$$

$$v(x, y, t) = s_0 + (a_1 i + 4\nu)\sqrt{-b} \left(\frac{\tan(\sqrt{-b}(x + y + ct)) -}{\cot(\sqrt{-b}(x + y + ct))} \right). \tag{49}$$

All the solutions of the two dimensional Burgers' equations are new.

4 Conclusions

In this paper, the MEDA method has been successfully applied to find the solution for four nonlinear partial differential equations such as the one-dimensional Burgers, KdV-Burgers, coupled Burgers and two-dimensional Burgers' equations. The modified extended direct algebraic method is used to find a new

complex travelling wave solutions. The results show that the modified extended direct algebraic method is a powerful mathematical tool to solve the one-dimensional Burgers, KdV-Burgers, coupled Burgers and two-dimensional Burgers' equations, it is also a promising method to solve other nonlinear equations.

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