

Non-wandering Property of Differentiation Operator

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Abstract:we provide a sufficient condition for an unbounded operator to be non-wandering operator, and then apply the condition to the differentiation operator on the Bargmann space F and the Hardy space H^2 . Finally, we give a sufficient condition for the operator $g(D)$ defined by means of a functional calculus to be non-wandering operator.

Keywords:non-wandering operator, unbounded operator, Bargmann space, Hardy space

1 Introduction

It is well known that linear operators in finite-dimensional linear spaces can't be chaotic but the nonlinear operator may be. Only in infinite-dimensional linear spaces can linear operators have chaotic properties. This has attracted widely attention (see [1-5]). Lixin Tian and other researchers introduced non-wandering operators in infinite-dimensional Banach space, which are the generalization of Axiom A dynamic system but different from it. They are new linear chaotic operators and relative to hypercyclic operators, but different from them (see [2]). In recent years, Jiangbo Zhou discussed the hereditarily hypercyclic decomposition of non-wandering operators in infinite dimensional Frechet space (see [3]); Shaoguang Shi obtained non-wandering operator sequences on Banach space (see[4]) ; Lihong Ren studied n-multiple non-wandering operator (see [5])and Minggang Wang studied the pseudo orbit tracing property of invertible non-wandering operator (see[6]).

In this paper the Bargmann space is denoted by F and the Hardy space is denoted by H^2 . These spaces have been studied by many authors(see[9-14]). Bargmann space's roots can be found in mathematical problem of relativistic physics (see[9]) or in quantum optics (see[10]). In physics the Bargmann space contains the canonical coherent states, so it is the main tool for studying the bosonic coherent state theory of radiation field (see[11]) and for other application (see[12]). In the same, the Hardy space is also the important space (see[13][14]).

In finite-dimensional separable Banach space, for the bounded linear operators, Lixin Tian and other researchers have given the definition of non-wandering operator (see[2]). However, this definition is restricted for the bounded linear operators. In this paper, we consider the non-wandering property of the unbounded operators. Let T be an unbounded operator on a separable infinite dimensional Banach space X . It may happen that a vector x is in the domain of T , but Tx fails to be in the domain of T . For this reason, in order to consider the non-wandering property of the unbounded operator, we should firstly suppose that if x in the domain of T then for every integer $n \geq 1$ the vector $T^n x$ is in the domain of T .

On the basis of the above research, in this paper, we first provide a sufficient condition for an unbounded operator to be non-wandering operator(see Theorem 1), and then apply the condition to the differentiation operator on the Bargmann space F (see Theorem 3) and the Hardy space H^2 (see Theorem 5). Finally, we give a sufficient condition for the operator $g(D)$ defined by means of a functional calculus to be non-wandering operator (see Theorem 6).

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2 Basic notation and definitions

Definition 1 ([2]) Let $(X, \|\cdot\|)$ be an infinite dimensional separable Banach space. Suppose $T \in L(X)$
 (1) Assume that there exists a closed subspace $E \subset X$, which has hyperbolic structure: $E = E^u \oplus E^s$, $TE^u = E^u, TE^s = E^s$, where E^u, E^s are closed subspaces. In addition, there exists constants $\tau (0 < \tau < 1)$ and $C > 0$, such that for any $\xi \in E^u, k \in N, \|T^k \xi\| \geq C\tau^{-k} \|\xi\|$, and for any $\eta \in E^s, k \in N, \|T^k \eta\| \leq C\tau^k \|\eta\|$;

(2) Assume also that $Per(T)$ is dense in E . Then T is said to be a non-wandering operator relative to E .

Definition 2 Suppose $T \in L(X)$ and $\{e_i\}_1^\infty$ is a basis in X , then T is called a unilateral backward shift operator relative to $\{e_i\}_1^\infty$ if $Te_n = e_{n-1} (n > 1)$ and $Te_1 = 0$.

3 Main results

Theorem 1 Let $(X, \|\cdot\|)$ be an infinite dimensional separable Banach space. T is an unbounded operator, if for $\forall n \geq 1, T^n$ is the closed operator and T satisfy (1) there exists a closed subspace $E \subset X$, which has hyperbolic structure; (2) $Per(T)$ is dense in E . Then T is a non-wandering operator relative to E .

Proof. By the Closed Graph Theorem, we can easily obtain this result. ■

Remark 1 In fact, from Theorem 1, if an unbounded operator T has non-wandering property, then T need to satisfy: (1) T^n is the closed operator, $\forall n \geq 1$; (2) there exists a closed subspace $E \subset X$, which has hyperbolic structure relative to T ; (3) $\overline{Per(T)} = E$

In the following, we will apply Theorem 1 to discuss the Non-wandering property of differentiation operator.

3.1 Non-wandering property of differentiation operators in Bargmann space

Let $\{w_n\}_{n \in N}$ be an arbitrary weight sequence, we define the iterated unbounded back-ward shift T^n in Bargmann space by

$$T^n \left(\sum_{k \geq 0} C_k \frac{x^k}{\sqrt{k!}} \right) = \sum_{k \geq 0} \left(\prod_{j=k}^{n+k-1} w_j \right) C_{n+k} \frac{x^k}{\sqrt{k!}}$$

with its domain in F , and we define

$$D(T^n) = \left\{ f(x) = \sum_{k \geq 0} C_k \frac{x^k}{\sqrt{k!}} \mid \sum_{k \geq 0} |C_k|^2 < \infty; \sum_{k \geq 0} \left| \prod_{j=k}^{m+k-1} w_j \right|^2 |C_{k+m}|^2 < \infty \right\}$$

for all $m \in N, 1 \leq m \leq n$.

Theorem 2 A linear unbounded backward shift operator $T: F \rightarrow F$ is non-wandering operator if the positive series

$$\sum_{n=0}^{\infty} \prod_{j=0}^{n-1} \frac{1}{|w_j|^2}$$

converges.

Proof. From Theorem 1, we need three steps to proof the theorem:

(1) We proof that for $\forall n \in N, T^n$ is closed.

By the definition, we choose $\{f_j\} \in D(T^n)$, since F is a Hilber space, then $\{f_j\} \rightarrow f_0$ in F , so

$$f_j(x) = \sum_{k \geq 0} C_{k,j} \frac{x^k}{\sqrt{k!}} \rightarrow f_0(x) = \sum_{k \geq 0} C_k^0 \frac{x^k}{\sqrt{k!}} \quad j \rightarrow \infty$$

$$\Rightarrow \left(\prod_{j=k}^{n+k-1} w_j \right) C_{n+k,j} \rightarrow \left(\prod_{j=k}^{n+k-1} w_j \right) C_{n+k}^0$$

Let $T^n f_j \rightarrow g_0$, then $g_0(x) = \sum_{k \geq 0} \xi_k^0 \frac{x^k}{\sqrt{k!}}$

We can conclude that

$$\xi_k^0 = \left(\prod_{j=k}^{n+k-1} w_j \right) C_{n+k}^0$$

This proves that $f_0 \in D(T^n)$ and $T^n f_0 = g_0$.

(2) Since the positive series $\sum_{n=0}^{\infty} \prod_{j=0}^{n-1} \frac{1}{|w_j|^2}$ converges, then $\prod_{j=0}^{n-1} \frac{1}{|w_j|^2} \rightarrow 0$, that is, $\prod_{j=0}^n w_j \rightarrow \infty \quad n \rightarrow \infty$

We choose w_j is an increasing sequence.

For $\forall \lambda \in C$, let $f_\lambda(x) = \sum_{k \geq 0} \left(\prod_{j=0}^{k-1} \frac{\lambda}{w_j} \right) \frac{x^k}{\sqrt{k!}}$, then for $\forall 0 < v < 1$ and $n \in N$ large enough, we have $|\lambda| \leq v |w_n|$, so

$$\begin{aligned} \sum_{k \geq 0} \prod_{j=0}^{k-1} \left| \frac{\lambda}{w_j} \right|^2 &= \sum_{k=0}^n \prod_{j=0}^{k-1} \left| \frac{\lambda}{w_j} \right|^2 + \sum_{k=n+1}^{\infty} \prod_{j=0}^{k-1} \left| \frac{\lambda}{w_j} \right|^2 \\ &\leq \sum_{k=0}^n \prod_{j=0}^{k-1} \left| \frac{\lambda}{w_j} \right|^2 + \left(\frac{1}{1-v} \right) \prod_{j=0}^n \left| \frac{\lambda}{w_j} \right|^2 < \infty \end{aligned}$$

Thus we get $f_\lambda(x) \in F$ and

$$T f_\lambda(x) = \sum_{k \geq 0} w_k \left(\prod_{j=0}^k \frac{\lambda}{w_j} \right) \frac{x^k}{\sqrt{k!}} = \lambda \sum_{k \geq 0} \left(\prod_{j=0}^{k-1} \frac{\lambda}{w_j} \right) \frac{x^k}{\sqrt{k!}} = \lambda f_\lambda(x)$$

Therefore, $f_\lambda(x)$ is the eigenvector corresponding to the eigenvalue λ , furthermore, by the arbitrariness of λ , we can construct the sets:

$$V_1 = \{ \lambda : |\lambda| > 1 \quad \lambda \in \delta_P(T) \}$$

$V_2 = \{ \lambda : 0 < |\lambda| < 1 \quad \lambda \in \delta_P(T) \}$, where $\delta_P(T)$ is the spectrum of T .

Let $E^u = \overline{\text{span} \{ f_\lambda; \lambda \in V_1 \}}$ for $\forall \xi \in E^u$, then $\xi = \sum_{i=1}^{\infty} \alpha_i f_{\lambda_i} = \sum_{i=1}^{\infty} \alpha_i \sum_{k \geq 0} \left(\prod_{j=0}^{k-1} \frac{\lambda_i}{w_j} \right) \frac{x^k}{\sqrt{k!}}$ and for $\forall k \in N$, we have

$$\begin{aligned} \|T^k(\xi)\| &= \left\| T^k \left(\sum_{i=1}^{\infty} \alpha_i f_{\lambda_i} \right) \right\| = \left\| \sum_{i=1}^{\infty} \alpha_i \lambda_i^k \sum_{k \geq 0} \left(\prod_{j=0}^{k-1} \frac{\lambda_i}{w_j} \right) \frac{x^k}{\sqrt{k!}} \right\| \\ &\geq \mu^k \left| \sum_{i=1}^{\infty} \sum_{k \geq 0} \left(\prod_{j=0}^{k-1} \frac{\lambda_i}{w_j} \right) \frac{x^k}{\sqrt{k!}} \right| = \mu^k |\xi| \end{aligned} \tag{1}$$

where $\mu = \min \{ |\lambda_i| \mid \lambda_i \in V_1 \} > 1$. Let $\tau = \frac{1}{\mu}$, then we can easily get $0 < \tau < 1$. So by (1) we have

$$\|T^k(\xi)\| = \left\| T^k \left(\sum_{i=1}^{\infty} \alpha_i f_{\lambda_i} \right) \right\| = \left\| \sum_{i=1}^{\infty} \alpha_i \lambda_i^k f_{\lambda_i} \right\| \geq \tau^{-k} |\xi|.$$

Next, we will prove E^u is the invariant subspace of T .

Since for $\forall \xi \in E^u$, then $\xi = \sum_{i=1}^{\infty} \alpha_i f_{\lambda_i} = T \sum_{i=1}^{\infty} \frac{\alpha_i}{\lambda_i} f_{\lambda_i} = TE^u \Rightarrow \xi \in TE^u$, so $E^u \subset TE^u$. In the other hand, for $\forall \eta \in TE^u$, then there exists $\varphi \in E^u \Rightarrow \varphi = \sum_{i=1}^{\infty} \beta_i f_{\lambda_i}$, such that, $\eta = T\varphi = \sum_{i=1}^{\infty} \lambda_i \beta_i f_{\lambda_i} \in E^u$. So we can get $TE^u \subset E^u$, therefore, $TE^u = E^u$.

Similarly, let $E^s = \overline{span \{f_{\lambda}; \lambda \in V_2\}}$, then $TE^s = E^s$, for $\forall \eta \in E^s$, we have $\eta = \sum_{i=1}^{\infty} \beta_i f_{\lambda_i} = \sum_{i=1}^{\infty} \beta_i \sum_{k \geq 0} \left(\prod_{j=0}^{k-1} \frac{\lambda}{w_j} \right) \frac{x^k}{\sqrt{k!}}$, so for $\forall k \in N, \|T^k(\eta)\| = \left\| T^k \left(\sum_{i=1}^{\infty} \beta_i f_{\lambda_i} \right) \right\| = \left\| \sum_{i=1}^{\infty} \beta_i \lambda_i^k f_{\lambda_i} \right\| \leq \tau^k |\eta|$ where $0 < \tau = \max \{|\lambda_i| \mid \lambda_i \in V_2\} < 1$ and $TE^s = E^s$

Let $E = E^u \oplus E^s$, then we can easily get E has hyperbolic structure.

(3) From the definition, we have $T^n \left(\sum_{k \geq 0} C_k \frac{x^k}{\sqrt{k!}} \right) = \sum_{k \geq 0} \left(\prod_{j=k}^{n+k-1} w_j \right) C_{n+k} \frac{x^k}{\sqrt{k!}}$, if T has the N -periodic point, then we have $\left(\prod_{j=k}^{n+k-1} w_j \right) C_{n+k} = C_k, \forall k \geq 0$, so for $\forall l = 0, 1 \dots n-1, k \geq 1$, we have $C_{kn+l} = \left(\prod_{j=l}^{kn+l-1} \frac{1}{w_j} \right) C_l$, thus for $\forall v \geq 0, n \geq v, n \in N$, we can construct

$$g_{v,n}(x) = \frac{x^v}{\sqrt{v!}} + \sum_{k=1}^{\infty} \left(\prod_{j=v}^{kn+v-1} \frac{1}{w_j} \right) \frac{x^{kn+v}}{\sqrt{(kn+v)!}}$$

Since the series $\sum_{n=0}^{\infty} \prod_{j=0}^{n-1} \frac{1}{|w_j|^2}$ converges, then we have $\sum_{k=1}^{\infty} \prod_{j=v}^{kn+v-1} \frac{1}{w_j} < \infty$, so, we get $g_{v,n} \in F$ and

$$T^N g_{v,n}(x) = T^N \left(\frac{x^v}{\sqrt{v!}} + \sum_{k=1}^{\infty} \left(\prod_{j=v}^{kn+v-1} \frac{1}{w_j} \right) \frac{x^{kn+v}}{\sqrt{(kn+v)!}} \right) = \frac{x^v}{\sqrt{v!}} + \sum_{k=1}^{\infty} \left(\prod_{j=v}^{kn+v-1} \frac{1}{w_j} \right) \frac{x^{kn+v}}{\sqrt{(kn+v)!}}$$

Thus $g_{v,n}(x)$ is the N -periodic point of T .

Let $E_0 = span \{g_{v,n}(x)\}$, in the following we will prove that E_0 is dense on F .

Since for $\forall f(x) \in F$, let $f(x) = \sum_{v=0}^m C_v \frac{x^v}{\sqrt{v!}}$, by the definition of Bargmann space, we have

$$|C_v \prod_{j=0}^{v-1} w_j| < \infty.$$

Suppose $|C_v \prod_{j=0}^{v-1} w_j| < 1$, then there exists $g(x) \in E_0$ and

$$g(x) = \sum_{v=0}^m C_v g_{v,n}(x)$$

so that

$$\begin{aligned} \|g - f\| &= \left\| \sum_{v=0}^m C_v (g_{v,n}(x) - \frac{x^v}{\sqrt{v!}}) \right\| = \left\| \sum_{v=0}^m (C_v \prod_{j=0}^{v-1} w_j) \sum_{k=1}^{\infty} \left(\prod_{j=0}^{kn+v-1} \frac{1}{w_j} \right) \frac{x^{kv+n}}{\sqrt{(kn+v)!}} \right\| \\ &\leq \sum_{v=0}^m \left\| \sum_{k=1}^{\infty} \left(\prod_{j=0}^{kn+v-1} \frac{1}{w_j} \right) \frac{x^{kv+n}}{\sqrt{(kn+v)!}} \right\| \end{aligned} \tag{2}$$

Furthermore, from the series $\sum_{n=0}^{\infty} \prod_{j=0}^{n-1} \frac{1}{|w_j|^2}$ converges, then there exists $n \geq m$ such that for $\forall \varepsilon > 0$, we have

$$\sum_{k \geq n+1} \left(\prod_{j=0}^k \frac{1}{w_j} \varepsilon_k \frac{x^k}{\sqrt{k!}} \right) < \frac{\varepsilon}{m+1}$$

where ε_k taking values 0 or 1, so we can get when $n \geq m$, then (2) $< \varepsilon$.

Therefore, E_0 is dense on F . ■

Theorem 3 The operator of differentiation $D : f \rightarrow f'$ defined on

$$\delta = \left\{ f \in F \mid f' \in F \right\}$$

is the non-wandering operator on F .

Proof. Since $\frac{x^k}{\sqrt{k!}}$ is an orthonormal basis in F , then we have

$$D \left(\frac{x^k}{\sqrt{k!}} \right) = k \cdot \frac{x^{k-1}}{\sqrt{k!}} = \sqrt{k} \frac{x^{k-1}}{\sqrt{(k-1)!}} = w_{k-1} \frac{x^{k-1}}{\sqrt{(k-1)!}}$$

where $w_k = \sqrt{k+1}$,

So, we can get the operator of differentiation $D : f \rightarrow f'$ is the weighted backward shift operator, therefore,

$$D \left(\sum_{k \geq 0} C_k \frac{x^k}{\sqrt{k!}} \right) = \sum_{k \geq 0} w_k C_{k+1} \frac{x^k}{\sqrt{k!}}, \quad w_k = \sqrt{k+1}$$

so the series

$$\sum_{n=0}^{\infty} \prod_{j=0}^{n-1} \frac{1}{|w_j|^2} = \sum_{n=0}^{\infty} \prod_{k=0}^{n-1} \frac{1}{k+1}$$

converges in F , thus by the Theorem 2, we have D is the non-wandering operator on F . ■

3.2 Non-wandering property of differentiation operators in Hardy space

Denote Hardy space as H^2 , then

$$H^2 = \{ f \in H(D) : \|f\| < \infty \}$$

where D is the unit disk which centers at zero and $H(D)$ is made of all the analytic function which defined in D . Suppose $f(x) \in H(D)$, then we have $f(x) = \sum_{n=0}^{\infty} \overline{f(n)} x^n$

Lemma 4 ([8]) For each positive integer n , the operator D^n is the closed operator on its domain $\varepsilon_r = \{ f \in H^2; f^{(n)} \in H^2 \}$.

Theorem 5 The operator D of differentiation defined on $\varepsilon = \{ f \in H^2 \mid f' \in H^2 \}$ of H^2 by $D : f \rightarrow f'$ is the non-wandering operator.

Proof. Since D is the chaotic operator on ε (see[8]), so D has the dense set of periodic points on ε . Thus, from Theorem 1, we only need to construct the hyperbolic structure.

In the following, we will construct the hyperbolic structure.

Firstly, we pick a weighted sequence (w_j) on the unit disk. Suppose (w_j) is the decreasing sequence which satisfy:

$$1 > w_1 > w_2 > \dots > 0 \text{ and } \lim_{n \rightarrow \infty} w_n = 0$$

Let $r_n = \frac{1}{n!} \prod_{i=1}^n \frac{1}{w_i}$, $n \geq 1$, we construct $\varepsilon = \{f(x) : f(x) = \sum_{n=0}^{\infty} a_n x^n\}$ and define $\|f\| = \left(\sum_{n=0}^{\infty} \frac{|a_n|^2}{r_n^2}\right)^{1/2} < \infty$.

It is simple to check that ε is a Hilbert space having norm $\|\cdot\|$ and orthonormal basis $e_n = r_n x^n$.

Since $De_n = D(r_n x^n) = nr_n x^{n-1} = n \cdot \frac{1}{n!} \prod_{i=1}^n \frac{1}{w_i} x^{n-1} = \frac{1}{(n-1)!} \prod_{i=1}^n \frac{1}{w_i} x^{n-1} = \frac{1}{w_n} e_{n-1}$, so, the operator of differentiation D is the weighted backward shift operator on ε .

For $\forall \lambda_i \in C$, we construct $f_{\lambda_i}(x) = \sum_{n=0}^{\infty} \prod_{i=0}^{n-1} \lambda_i w_i e_n$, then $\lambda_i w_i \rightarrow 0 \quad n \rightarrow \infty$,

So, when n large enough, we have

$$\left\| \sum_{n=0}^{\infty} \prod_{i=0}^{n-1} \lambda_i w_i \right\| < \infty, \text{ so } f_{\lambda_i}(x) \in \varepsilon \text{ and}$$

$$Df_{\lambda_i}(x) = \sum_{n=0}^{\infty} \frac{1}{w_n} \prod_{i=0}^n \lambda_i w_i e_n = \lambda_i \sum_{n=0}^{\infty} \prod_{i=0}^{n-1} \lambda_i w_i e_n = \lambda f_{\lambda_i}(x)$$

Therefore, $f_{\lambda_i}(x)$ is the eigenvector corresponding to the eigenvalue λ_i . So we can construct

$$E^u = \overline{\text{span} \{f_{\lambda_i}(x) : |\lambda_i| > 1\}}$$

For $\forall \xi \in E^u$, then $\xi = \sum_{i=1}^{\infty} \alpha_i f_{\lambda_i} = \sum_{i=1}^{\infty} \alpha_i \sum_{n=0}^{\infty} \prod_{i=0}^{n-1} \lambda_i w_i e_n$ and for $\forall k \in N$, we have

$$\begin{aligned} \|D^k(\xi)\| &= \left\| D^k \left(\sum_{i=1}^{\infty} \alpha_i f_{\lambda_i} \right) \right\| = \left\| \sum_{i=1}^{\infty} \alpha_i \lambda_i^k \sum_{n=0}^{\infty} \prod_{i=0}^{n-1} \lambda_i w_i e_n \right\|, \\ &\geq \mu^k |\xi| \end{aligned}$$

where $\mu = \min \{|\lambda_i| > 1\}$, let $\tau = \frac{1}{\mu}$, then $0 < \tau < 1$, so we get

$$\|D^k(\xi)\| = \left\| D^k \left(\sum_{i=1}^{\infty} \alpha_i f_{\lambda_i} \right) \right\| = \left\| \sum_{i=1}^{\infty} \alpha_i \lambda_i^k \sum_{n=0}^{\infty} \prod_{i=0}^{n-1} \lambda_i w_i e_n \right\| \geq \tau^{-k} |\xi|$$

Next, we prove E^u is the invariant subspace of D .

For $\forall \eta \in E^u$, then $\xi = \sum_{i=1}^{\infty} \alpha_i f_{\lambda_i} = D \sum_{i=1}^{\infty} \frac{\alpha_i}{\lambda_i} f_{\lambda_i} = DE^u \Rightarrow \xi \in DE^u$.

So $E^u \subset DE^u$, otherwise, for $\forall \eta \in DE^u$, there exists $\varphi \in E^u \Rightarrow \varphi = \sum_{i=1}^{\infty} \beta_i f_{\lambda_i}$ such that $\eta = D\varphi = \sum_{i=1}^{\infty} \lambda_i \beta_i f_{\lambda_i} \in E^u$, thus $DE^u \subset E^u$, therefore, $DE^u = E^u$.

Similarly, let $E^s = \overline{\text{span} \{f_{\lambda_i}(x) : |\lambda_i| < 1\}}$

For $\forall \eta \in E^s$, then $\eta = \sum_{i=1}^{\infty} \beta_i f_{\lambda_i} = \sum_{i=1}^{\infty} \beta_i \sum_{n=0}^{\infty} \prod_{i=0}^{n-1} \lambda_i w_i e_n$ and for $\forall k \in N$

$$\|D^k(\eta)\| = \left\| D^k \left(\sum_{i=1}^{\infty} \beta_i f_{\lambda_i} \right) \right\| = \left\| \sum_{i=1}^{\infty} \beta_i \lambda_i^k \sum_{n=0}^{\infty} \prod_{i=0}^{n-1} \lambda_i w_i e_n \right\| \leq \tau^k |\eta|,$$

where $\tau = \max_{i=1 \dots n} \{|\lambda_i| < 1\}$, $0 < \tau < 1$ and $DE^s = E^s$.

Let $E = E^u \oplus E^s$, then E has hyperbolic structure. ■

Theorem 6 If ∂U is the boundary of the open unit disk U and if g is a nonconstant analytic function for which $g(U) \cap \partial U \neq \emptyset$, then the operator $T : f \rightarrow g(D)f$ is the non-wandering operator on H^2 .

Proof. Since $g(U) \cap \partial U \neq \emptyset$, then there exists $x_0 \in U$ such that $|g(x_0)| = 1$, furthermore, since g is nonconstant and analytic on U , there exists $0 < \varepsilon < \frac{1-|x_0|}{4}$ so that the sets V_1, V_2 and V_3 given by

$$V_1 = \{\lambda; |\lambda - x_0| < \varepsilon \quad |g(\lambda)| < 1\}$$

$$V_2 = \{ \lambda; \quad | \lambda - x_0 | < \varepsilon \quad | g(\lambda) | > 1 \}$$

and

$$V_3 = \{ \lambda \in U; \quad \exists n \geq 1 \quad s.t. \quad g(\lambda)^n = 1 \}$$

are nonempty open subsets of U . Next, let $\tau = |x_0| + 2\varepsilon$, and pick a sequence of positive numbers (w_n) with $1 > w_1 > w_2 > \dots > \tau$ and $\lim_{n \rightarrow \infty} w_n = \tau$.

Also, let $r_0 = 1$ and $r_n = \frac{1}{n!} \prod_{i=1}^n w_i$ ($n \geq 1$). We consider the space ε of analytic functions on U of the form $f(z) = \sum_{n=0}^{\infty} a_n z^n$ that satisfy

$$\|f\| = \left(\sum_{n=0}^{\infty} \frac{|a_n|^2}{r_n^2} \right)^{1/2} < \infty.$$

It is simple to check that ε is a Hilbert space having norm $\|\cdot\|$ and orthonormal basis $e_n = r_n x^n$.

Since $De_n = D(r_n x^n) = nr_n x^{n-1} = n \cdot \frac{1}{n!} \prod_{i=1}^n w_i x^{n-1} = \frac{1}{(n-1)!} \prod_{i=1}^n w_i x^{n-1} = w_n e_{n-1}$, so the operator of differentiation D is the weighted backward shift operator on ε . Since $\|D\| = \sup_{n \geq 1} (w_n) = w_1 < 1$, and furthermore, g is analytic, so $g(D)$ is an absolutely convergent power series in D , and hence the operator $T : f \rightarrow g(D)f$ is bounded on ε . Also, notice that for each $|\lambda| < \tau$, we have $e^{\lambda x} \in \varepsilon$ and is an eigenfunction of T with eigenvalue $g(\lambda)$. Let $E^s = \overline{\text{span}(e^{\lambda x}; \quad \lambda \in V_1)}$, then we have $TE^s = E^s$ and for $\forall \xi \in E^s$ then $\xi = \sum_{i=1}^n \alpha_i e^{\lambda_i x}$

$$\|T^k(\xi)\| = \left\| T^k \sum_{i=1}^n \alpha_i e^{\lambda_i x} \right\| = \left\| \sum_{i=1}^n \alpha_i (g(\lambda_i)^k) e^{\lambda_i x} \right\| \leq \tau^k \|\xi\|$$

where $0 < \tau = \max_{i=1 \dots n} |g(\lambda_i)| < 1$.

Similarly, let $E^u = \overline{\text{span}(e^{\lambda x}; \quad \lambda \in V_2)}$, then $TE^u = E^u$ and for $\forall \eta \in E^u \quad \eta = \sum_{i=1}^n \beta_i e^{\lambda_i x} \quad \|T^k(\eta)\| = \left\| T^k \sum_{i=1}^n \beta_i e^{\lambda_i x} \right\| = \left\| \sum_{i=1}^n \beta_i (g(\lambda_i)^k) e^{\lambda_i x} \right\| \geq \tau^{-k} \|\eta\|$, where $\mu = \min_{i=1 \dots n} |g(\lambda_i)| > 1, 0 < \tau = \frac{1}{\mu} < 1$.

Let $E = E^u \oplus E^s$, then E has hyperbolic structure.

Finally we prove that the periodic points of T are dense on E .

We consider $V_3 = \{ \lambda \in U; \quad \exists n \geq 1 \quad s.t. \quad g(\lambda)^n = 1 \}$, then there exists the accumulation point of λ in U .

Denote $E_0 = \text{span} \{ e^{\lambda_i x}; \quad \lambda_i \in V_3 \}$, then E_0 is set which made of all the periodic points of $g(D)$. So, we can get E_0 is dense in H^2 , otherwise, E_0 is not dense in H^2 , then by $H - B$ Theorem, there exists $\varphi \in (H^2)^* = H^2$, for $\forall x \in E_0$, we have $\langle \varphi, x \rangle = 0$, so for $\forall x \in E_0$, $g_\varphi(\lambda) = \langle \varphi, x \rangle = 0$, furthermore, there exists the accumulation point of λ in U and $g_\varphi(\lambda)$ is analytic on U , so $g_\varphi(\lambda) \equiv 0$ on U . Thus it is contrary to the condition. ■

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References

[1] Godefroy G, Shapiro J H: Operators with dense, invariant cyclic vector manifolds. *J.Funct Anal.*98:229-269(1991)

- [2] Lixin Tian, Jiangbo Zhou, Xun Liu, Guangsheng Zhong: Nonwandering operators in Banach Space. *International Journal Of Mathematics and Mathematical Sciences*. 24 :3895-3908(2005)
- [3] Jiangbo Zhou, Lixin Tian, Dianchen Lu: The hereditarily decomposition of non-wandering operators in infinite dimensional Frechet space. *Journal of Jiangsu University(Natural Science Edition)*. 22(6):88 - 91(2001)
- [4] Shaoguang Shi, Guangsheng Zhong: Nonwandering operator sequences in Banach space. *International Journal of Nonlinear Science*. 1(3):164-171(2006)
- [5] Lixin Tian, Lihong Ren: N-multiple Nonwandering Unilateral Weighted Backward Shift Operators and the Property of Direct Sum Operators in Banach Space. *International Journal of Nonlinear Science*. 2(2): 104-110 (2006)
- [6] Lixin Tian, Minggang Wang: Pseudo orbit tracing property of non-wandering operator. *International Journal of Nonlinear Science*. 1(3): 3-7 (2007)
- [7] H. Emamirad, G.S. Heshmati: Chaotic weighted shifts in Bargmann space, *J. Math. Anal. Appl.* 308:36-46 (2005)
- [8] J. Bés, K.C. Chan, S.M. Seubert: Chaotic unbounded differentiation operators. *Integral Equations Operator Theory*. 40:257-267(2001)
- [9] I.E. Segal: Mathematical Problems of Relativistic Physics. *Amer. Math. Soc, Providence, RI*. (1963)
- [10] J.R. Klauder, E. C. Sudarshan: Fundamentals of Quantum Optics. *Benjamin, New York*. (1968)
- [11] J.R. Klauder, B. Skagerstam: Coherent States, Application in Physics and Mathematical Physics, *World Scientific, Singapore*. (1985)
- [12] H.Emamirad, R. Holtz: Vecteurs d'état coherent et image de la transformee de Husimi C. R. Acad. Sci. Paris. 324:1295-1300(1997)
- [13] C. R. MacCluer, Chaos in linear distributed systems. *Proc. Of the 29th IEEE Conf. on Decision and Control, Honolulu, Hawaii, December* (1990)
- [14] Godefroy G, Shapiro J H: Operators with dense, invariant cyclic vector manifolds. *J.Funct Anal*, 98:229-269(1991)