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A Research on the Purchase Quantity Allocation between Contract Market and Power Auction Market

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Abstract: In an open electricity market, electricity purchasers will trade, typical decision-making issue is how to determine purchasing proportion of two markets to reduce purchase expense meanwhile avoid risk. Based on chance constrained programming, a new purchase decision-making optimization model with stochastic constraint of Distribution Company is simplified into a definite model. The solution is genetic algorithm based on stochastic simulation which can coordinate purchase expense and risk successfully. Finally, a numerical simulation demonstrates the optimal distribution strategy . Results indicate that it's helpful for electricity purchaser to seek optimal investment strategy in electricity market.

Keywords: electricity market; electricity distribution; chance-constrained programming; genetic algorithm; risk analysis

1 Introduction

There is variety of different electricity trading forms in Electricity market. The most significant one is the form of contract market and the auction market. Trading quantities and trading price are established by the government's coordination in principle during the contract transaction. However, the auction market bids according to the market clearing principle. The market members obtain the most effective transaction project using the different combination of transaction ways[1]−[7].Speaking of the sole customer's market, a typical decision-making issue is how to determine the purchasing proportion of the two market contract market and auction market.

In the electricity purchaser's risk management of purchasing decision-making, a purchase distribution research [8][9] on the three order markets which are day-ahead market, hour-ahead market and real time market has been done, it discussed the description and impact of risks. Guo Jin and Tan Zhongfu[10] established a target model of profit maximization according to Markowitz theory of the risk investment, variance of profit was considered as an index of measuring risk. Introduction of β (the power company's attention to risk) converted a double- objective programming into a single one, therefore, the value of β determines the purchase of electricity distribution on a large extent, so the key of this model application is to evaluate β correctly. Chi-Keung Woo[11] assessed the risk of purchase cost through cost exposure and VAR which anticipated loss size in future with the possibility of it will occur, it lets the distribution company not only know the loss scale but also know the possibility of it will occur. In [12], a Monte Carlo simulation was used to seek the optimal solution of the contract allocation rate of Monte Carlo has nothing to do with the problem's dimension, moreover Monte Carlo understands easily, it is easy to code, but there is still weakness, its computation load is extremely big, if you to increase a digit precision, you need to increase 100 times of computation loads. Furthermore its convergence rate is slow.

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In this paper, the purchase allocation issue of the two trading markets is transformed to one chance-constrained programming model by using the chance constrained programming theory, through model transformation we can obtain a definite solvable model. This paper also further discusses the mutual influencing relations of the contract market and the spot market(power auction market) .

2 Mathematical Model

2.1 Characteristic of Contract Market and Spot Market

Our model establishment is under such market rules: Purchaser can purchase its electricity quantity needed from the contract market and the spot market. The purchasing quantity and purchasing price are determined through the two sides' coordination in the contract market. However, the power auction market's purchasing quantity and purchasing price are determined by the auction market rules, here according to the market clearing rules.

Any of market members can't control the market's price, moreover there is great fluctuation of the present market price, so it leads the contract purchase and the proportion to be the important content of avoiding risks. The distribution company's expense and risk, we should consider both the demand uncertainty the price and risk of the contract market and spot market. Because any of market participants can't control the market completely, the prices for the two markets are all in random variables, and they are relevant [10].

According to the analysis to power auction market in [13], price of power auction market is a random variable following the normal distribution . We can confirm its basis distribution of price when a future time load has been forecasted. Through the detailed reasoning in [14], we know that price of power auction market follows Logarithmic normal distribution, namely the logarithm of the actual price will be normal distribution. λ_2 is the price of power auction market, and $\lambda_2 \sim N(\mu, \sigma^2)$, so $\ln \lambda_2 \sim N(\mu_2, \sigma_2^2)$. Then the price probability density function of power auction market based on the forecast load is as follows:

$$f(\lambda_2|L) = \begin{cases} \frac{1}{\sqrt{2\pi}\sigma\lambda_2}\epsilon^{-\frac{(ln\lambda-\mu)^2}{2\sigma^2}} & if \ \ \lambda_2 > 0 \\ 0 & if \ \ \lambda_2 < 0 \end{cases} \tag{1}$$

λ_2 denotes a random variable of the power auction market's price; L denotes load demand quantity; μ and σ^2 called the logarithmic mean and logarithmic variance of Logarithmic normal distribution. Then the purchase expense of the distribution company in the power auction market can be represented as:

$$R_s = \lambda_2 \times Q_s \tag{2}$$

Q_s is the purchase quantity in power auction marketThe contract market's results are obtained from the bilateral coordination. According to the beforehand experience, trading price can't been determined during the quantity allocation. We suppose λ_1 to be the contract price and $\lambda_1 \sim N(\mu_1, \sigma_1^2)$, the purchase expense of contract market is given as:

$$R_c = \lambda_1 \times Q_c \tag{3}$$

λ_1 denotes a random variable of contract price which subjects to normal distribution. Q_c is the trading quantity of contract market.

Given that is the total demand quantity of distribution company in a future time, it is obvious that:

$$Q = Q_s + Q_c \tag{4}$$

2.2 Allocation Model of Purchasing Quantity

The purchase allocation model's goal of the distribution company is to minimize the purchase expense, namely the following model

$$\begin{aligned} \min_{q} \quad & f = q.\lambda_1 + (Q - q).\lambda_2 \\ s.t. \quad & 0 \leq q \leq Q \end{aligned} \tag{5}$$

λ_1 denotes a random variable of contract market price; λ_2 denotes power auction market's price, a random variable; Q is the distribution company's total quantity, a certain value; q denotes the purchase quantity of contract market, decision variable of the model.

Because the both are all random variables, purchase expense is also a random variable, this kind of model can't be solved. The usual method is to turn the objective function into the minimum its expectation. Then this article's structure is a stochastic constraint programming further. Concrete model is given as follows:

$$\begin{aligned} &\min_q \quad \overline{f} \\ &s.t. \quad \Pr\left\{f \le \overline{f}\right\} \ge \beta \\ &\qquad 0 \le q \le Q \end{aligned} \tag{6}$$

f is random variable which denotes the purchase expense in formula (5); $\overline{f}$ is a certain value whose meaning is an appointed level of purchase expense; β is an appointed probability which denotes that purchase expense will not bigger than the appointed confidence level. Symbols are similar with formers.

The meaning of above model is to choose the contract purchasing quantity, so it can satisfy that realize probability meets the required confidence level when the purchasing expense is the minimum. It is a typical chance constrained model.

3 Solution to the Model

3.1 The Transformation Based on Chance Constrained Programming Model

According to [18], model (6) can be transformed to the following model:

$$\begin{aligned} &\min_q \quad \overline{f} \\ &s.t. \begin{cases} \overline{f} \ge \sup\left\{K \middle| K = F_f^{-1}(\beta)\right\} \\ 0 \le q \le Q \end{cases} \end{aligned} \tag{7}$$

F_f is the distribution function of $f = q \cdot \lambda_1 + (Q - q) \cdot \lambda_2$. From the detailed reasoning in [14], we can know that:

$$\mu_2 = \epsilon^{\mu + \frac{1}{2}\sigma^2}, \sigma_2^2 = \epsilon^{2\mu + 2\sigma^2} - \epsilon^{2\mu + \sigma^2} \tag{8}$$

Symbol nomenclature see front . So we can deduce that $f \sim N(\overline{\mu}, \overline{\sigma_2}^2)$, and its mean and variance are given as follows:

$$\begin{aligned} \overline{\mu} &= q\mu_1 + (Q - q)\mu_2 \\ \overline{\sigma}^2 &= q^2\sigma_1^2 + (Q - q)^2\sigma_2^2 \\ &\quad + 2\rho q\sigma_1\sigma_2(Q - q) \end{aligned} \tag{9}$$

Model (7) may further simplify for the standard normal distribution form:

$$\begin{aligned} &\min_q \quad \overline{f} \\ &s.t. \begin{cases} \overline{f} \ge \overline{\mu} + \overline{\sigma}\Phi^{-1}(\beta) \\ 0 \le q \le Q \end{cases} \end{aligned} \tag{10}$$

Φ is the standard normal distribution function. The model (7) is a definite model. But then the relation between F_f and q is nonlinear, so the model is a typical nonlinear constraint optimal issue. To this kind of optimized problem, genetic algorithm is an excellent solution.

3.2 Genetic Algorithm Solves Objective Function

Genetic algorithm's Specific flow is shown in [19].

The selection of codes adopts binary system symbols with length $l = 10$ to denote the quantity (q_1, q_2) of spot market and contract market. The entire chromosome expresses to be $[q_1 \mid q_2]$, chromosome's length is 20, the former 10 denotes q_1, and the latter 10 denotes q_2.

We adopts the fitness function which is minimum objective function , $C\max$ is fixed, in order to guarantee the fitness's non-negative, it must be big enough. When the total needed quantity is determinate and also history price is known, total purchasing expense can be forecasted easily, so it is still easy to determine $C\max$, we assume that $C\max = 10000000\$$ in present paper.

$$Fitness(x) = \begin{cases} C\max - f(x) & f(x) < C\max \\ 0 & f(x) \geq C\max \end{cases} \tag{11}$$

Substitute the minimum objective function issue in model (7) for the in model (11), we suppose $f_t = \overline{\mu} + \overline{\sigma}\Phi^{-1}(\beta)$, objective function model of genetic algorithm is given as:

$$\begin{aligned} \min_{q} \;\; & Fitness(q) = \begin{cases} C\max - f_t & f_t < C\max \\ 0 & f_t \geq C\max \end{cases} \\ st \;\; & 0 \leq q \leq Q \end{aligned} \tag{12}$$

Decision variable is q.

4 Example Analysis

On the basis of a certain area distribution company's analysis of historical data[17], the price distribution, correlation coefficient and load forecasting of the contract market and the spot market are estimated as shown in tab.1.

Table 1: Statistical Features of Variable

	Symbol express	Distribution
Contract price	λ_1	N(55,189)
Spot price	$\ln\lambda_2$	N(72,1122)
Load forecasting	Q	N(1800,2803)
correlation coefficient	ρ	0.1
Unit:$\lambda_1 \sim \$/Mwh, \ln\lambda_2 \sim \$/Mwh, Q \sim Mwh$		

The objective function's confidence level $\beta = 0.95, 0.90, 0.85, 0.80$, we can obtain the contract purchase quantity $q$spot purchase quantity $Q - q$, purchasing allocation proportion γ and the expected value of purchase expense $\overline{\mu}$ as shown in Tab.2.

Table 2: Changes Produced by Different β

β	0.99	0.95	0.90	0.85
q	1733.4	1785.6	1798.2	1798.2
$Q - q$	66.6	14.4	1.8	1.8
γ	0.963	0.992	0.999	0.999
$\overline{\mu}$	100132	99244	99030	99030
$\Phi^{-1}(\beta)$	2.33	1.745	1.28	1.14
Unit:$q \sim Mwh, Q - q \sim Mwh, \overline{\mu} \sim \$$				

From Tab. 2, we may see the model's final result, when $\beta = 0.90$, the allocation quantity q of contract market is 1798.2Mwh, spot market $Q - q$ is 1.8Mwh, purchasing allocation proportion γ is 0.999, namely distribution company will choose the relative stable contract market.

In order to narrate the influence of two markets' mean value and standard variance to purchasing allocation strategy better, we change $\mu_1, \mu_2, \sigma_1, \sigma_2$ to analyze the purchasing strategy's variety conditions when $\beta = 0.95$ and others are invariable.

First, change the standard variance σ_1 and σ_2 of the contract market and the spot market, the results are given as Tab. 3 and Tab. 4:

Table 3: Changes Produced by Different σ_1

Change σ_1 when$\beta = 0.95$				
σ_1	q	$Q - q$	γ	$\overline{\mu}$
11	1798.2	1.8	0.999	99030
13	1798.2	1.8	0.999	99030
15	1755	45	0.975	99765
17	1699.2	100.8	0.944	100713
19	1636.2	163.8	0.909	101784
Unit:$q \sim Mwh, Q - q \sim Mwh, \overline{\mu} \sim \$$				

Table 4: Changes Produced by Different σ_2

Change σ_2 when $\beta = 0.95$				
σ_2	q	$Q - q$	γ	$\overline{\mu}$
33	1783.8	16.2	0.991	99275
35	1789.2	10.8	0.994	99183
37	1794.6	5.4	0.997	99091
39	1798.2	1.8	0.999	99030
41	1798.2	1.8	0.999	99030
Unit:$q \sim Mwh, Q - q \sim Mwh, \overline{\mu} \sim \$$				

When the contract market's standard variance σ_1 reduces, then the purchasing quantity of contract market increases, but the most is 1798.2Mwh, and the purchasing quantity spot market reduces, purchasing allocation proportion ascends, total expected purchasing expense reduces. When $\sigma_1 = 15$, purchasing quantity in the contract market is 1755Mwh, in the spot market is 45Mwh, and $\gamma = 0.975$. The variety results of σ_2 can be acquired likewise. This conclusion illuminates that when a market standard variance increases, it represents its market risk increasing, then purchaser will reduce the purchase quantity in this market, meanwhile another market's purchase quantity .When increases purchase quantity in the spot market, it will increase the total expected purchase expense.

The standard mean value μ_1 and μ_2 of the contract market and the spot market, the results are given as Tab. 5 and Tab.6:

Table 5: Changes Produced by Different μ_1

Change μ_1 when $\beta = 0.95$				
μ_1	q	$Q - q$	γ	$\overline{\mu}$
51	1798.2	1.8	0.999	91837
53	1798.2	1.8	0.999	95434
55	1785.6	14.4	0.992	99244
57	1760.4	39.6	0.978	103194
59	1737	63	0.965	107019
Unit:$q \sim Mwh, Q - q \sim Mwh, \overline{\mu} \sim \$$				

When the spot market's price mean value μ_2 increases, then the purchasing quantity in this market reduces, and the purchasing quantity of contract market increases, but the most is 1798.2Mwh, purchasing allocation proportion ascends, total expected purchasing expense reduces.

When $\mu_2 = 70\$/Mwh$, purchasing quantity in the contract market is 1760.4Mwh, in the spot market is 39.6Mwh, and $\gamma = 0.978$. This conclusion illuminates that when a market's price increases, purchaser will reduce the purchase quantity in this market, meanwhile the other market's purchase quantity will increase . When purchase quantity in the spot market reduces, and it will reduce the total expected purchase expense.

In summary, the enlargement of a market risk will lead to electricity purchaser go for another relative stable market; the ascension of a market price will lead to purchasing quantity of another market increases.

Table 6: Changes Produced by Different μ_2

Change $\mu_2$when$\beta = 0.95$				
μ_2	q	$Q-q$	γ	$\overline{\mu}$
68	1737	63	0.965	99819
70	1760.4	39.6	0.978	99594
72	1785.6	14.4	0.992	99244
74	1798.2	1.8	0.999	99034
76	1798.2	1.8	0.999	99037
Unit:$q \sim Mwh, Q-q \sim Mwh, \overline{\mu} \sim \$$				

This conclusion proves that there is relativity between the contract market and the spot market.

5 Conclusion and Expectation

This article gives us the method which transforms the model to a definite optimal model with restraint and also gives the settlement algorithm based on the genetic algorithm. The actual data computation indicates that this method can provide consumers a purchasing quantity allocation decision-making under certain confidence level.

The further work will be how to enhance the efficiency of model's solution and consider all kinds of constraint conditions.

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References

[1] Hung-po Chao:Global Electricity Tranforma-tion'The Critical Need for Integrated Market Design and Risk Management Research'. *Energy* . 31:923-939(2006)

[2] Zhaoming Wang, Yonghui Ling:A Static Negotiation Model of Electronic Commerce. *International Journal of Nonlineat Science* . 5(1):43-50(2008)

[3] Qing Wang, Min Jin, Lifeng Xi:Fitness fo Graph Based on Fractal Dimension.*International Journal of Nonlineat Science* . 4(2), (2007)

[4] Zhensheng Yu:A Descent Algorithm for the Extended Semidefinite Linear Complementarity Problem. *International Journal of Nonlineat Science* . 5(1), (2008)

[5] ZGuoliang Cai, Juanjuan Huang, Lixin Jian, Qingchao Wang:Adaptive Control and Slow Manifold Analysis of a New Chaotiv System. *International Journal of Nonlineat Science* . 2(1):50-60(2006)

[6] Xiaoling Chen, Y.He, Y.H. Song:Study of Impacts of Physical Contracts and Financial Contracts on Bidding Strategies of GENCOs. *Electrical Power and Energy Systems* . 26:715-723(2004)

[7] E.Tanlapco, J.Lawarre, and C.C.Liu:Hedging with Futures Contracts in a Deregulated Electricity Industry. *IEEE PWS* . 17(3):577-582,(2006)

[8] Yaan Liu, Xiaohong Guan:Market Allocation Problem for Energy Demander in Power Market. *Proceedings of CSEE* . 21(10):20-23(2001)

[9] Yaan Liu, Xiaohong Guan:Optimization of Purchase Allocation in Dual Electric Power Markets with Risk Management. *Automation of Electric System* . 26(9):41-44(2002)

[10] Jin Guo, Wei Jiang, Zhongfu Tan:Research on Optimized Power Purchasing of Power Suppliers under Risk Condition. *Power System Technology* . 28(11):18-22(2004)

[11] Chi-Keung Woo, Rouslan I. Karimov, Ira Horow- itz:Managing Electricity Procurement Cost and Risk by a Local Distribution Company. *Energy Policy* . 32:635-645(2004)

[12] I.Vehrilainen and J.Keppo:Managing Electricity Market Price Risk. *Eur. J. Res* . 145(1):136-147(2003)

[13] Zhaoxia Zhu, Bin Zou:The Risk Model of Distribution Utilities and Analyzing the Infection. *Journal of Relay* . (to appear)

[14] Zhaoxia Zhu, Bin Zou:Statistical Analysis of Day-ahead Prices in PJM Market. *Automation of Electric System* . 30(23):53-57(2006)

[15] Jian WangFushuan WenRengang Yang:Optimal maintenance strategy for generation companies based on chance-constrained programming. *Automation of Electric System* . 28(19):27-31(2004)

[16] Yiguo Li, Shen Jiong, Xichui Li:A new bidding strategy for power plants Based on Chance-constrained Programming. *Proceedings of CSEE* . 26(10):120-123(2006)

[17] http://www.pjm-cem.com/markets/energy-market/day-ahead.html.

[18] Baoding Liu, Ruiqing Zhao: Stochastic Programming and Fuzzy Programming. *Tsinghua University Press :Beijing*(2001)

[19] Ming Zhou, Shudong Sun: Genetic Algorithm Theory and Application. *National Defense Industry Press:Beijing*(1999)

[20] S.J. Deng, S.S. Oren:Electricity Derivatives and Risk Management. *Energy* . 31:940-953(2006)

[21] Xinshun Ma, Fushuan Wen, Yixin Nia, Jianxin Liu: Xinshun Ma, Fushuan Wen, Yixin Nia, Jianxin Liu, "Towards the Development of Risk Constrained Optimal Bidding Strategies for Generation Companies in Electricity Markets. *Electric Power Systems Research* . 73:305-312(2005)

[22] I.Vehrilainen and J.Keppo: Managing Electricity Market Price Risk. *Eur. J. Res* . 145(1):136-147(2003)

[23] Min Liu, F.F.Wu: Managing Price Risk in a Multi-market Environment. *IEEE PWS* . 21(4):1512-1519(2006)