

The Cauchy Problem of the Degasperis-Procesi Equation with the Dispersive Term

Shu Wen *

Department of Computing Science, Huaiyin Institute of Technology
 Huaian, Jiangsu, 223002, P.R.China

(Received 5 September 2007, accepted 18 March 2008)

Abstract: This paper is concerned with several aspects of the Cauchy problem and the local well-posedness for the Degasperis-Procesi equation with the dispersive term.

Keywords: Cauchy problem; shallow water equation; dispersive term; well posedness

1 Introduction

Our aim is to study the Cauchy problem and the local well-posedness of the D-P equation with the dispersive term:

$$\partial_t u - \partial_{txx}^3 u + \partial_{xxx}^3 u - \partial_{xxxxx}^5 u + 4u\partial_x u = 3\partial_x u \partial_{xx}^2 u + u \partial_{xxx}^3 u \quad (1.1)$$

with an initial data $u(x, 0) = u_0$.

For the existence and uniqueness of the Cauchy problem of (1.1), [5,6] used the method introduced by Bourgain to deal with the periodic KdV equation and the Schrödinger equation; Kenig, Ponce and Vega improved this method [7,8] to deal with the non-periodic case.

2 Preliminaries

As above and henceforth, we denote the convolution by $*$. We write \widehat{f} as the Fourier transform of f . $\psi, \sigma \in C_0^\infty(\mathbb{R})$, $\text{supp } \psi \in (-1, 1)$ and $\psi = 1$ in $[-1/2, 1/2]$, $\sigma = 1$ in $\text{supp } \psi$ and $\text{supp } \sigma \in (-1, 1)$, $0 < \sigma < 1$. $\|\cdot\|_s$ stand for the norm in the classical Sobolev space. Let $\xi \in \mathbb{R}$, $\langle \xi \rangle = 1 + |\xi|$, $X^{s,b}$ denotes as the completion of the Schwartz space with respect to the norm:

$$\|f\|_{X^{s,b}}^2 = \int_{\mathbb{R}} \int_{\mathbb{R}} \langle \tau - \xi^3 \rangle^{2b} \langle \xi \rangle^{2s} |\widehat{f}(\xi, \tau)|^2 d\xi d\tau \quad (2.1)$$

Rewrite D-P equation

$$\begin{cases} \partial_t u + \partial_{xxx}^3 u + p = 0 \\ u(x, 0) = u_0 \end{cases} \quad (2.2)$$

where the nonlinear term p is given by:

$$p = \frac{1}{2} \partial_x (u^2) + (1 - \partial_{xx}^2)^{-1} \left(\partial_x \left(\frac{3}{2} u^2 \right) \right) \quad (2.3)$$

First we consider the corresponding homogeneous linear equation

$$\begin{cases} \partial_t v + \partial_{xxx}^3 v = 0, & x, t \in \mathbb{R} \\ v(x, 0) = v_0 \end{cases} \quad (2.4)$$

* Corresponding author. E-mail address: Wenshu119@163.com

$W(t)$ denotes the corresponding unitary group; the solution of (2.4) can be represented by:

$$v(x, t) = W(t) v_0(x) = S_t * v_0(x),$$

where $S_t(\cdot)$ is defined as the oscillatory integral equation:

$$S_t(x) = c \int e^{ix\xi} e^{it\xi^3} d\xi$$

Then (2.2) is equivalent to the integral equation:

$$u(x, t) = W(t) u_0(x) - \int_0^t W(t-t') g(x, t') dt'$$

We briefly collect the needed results from [2] in order to pursue our goal.

Lemma 2.1 Let $\widehat{H}_\rho(\xi, \tau) = \frac{h(\xi, \tau)}{(\tau - \xi^3)^\rho}$. There for $\rho > 1/2$, $\|H_\rho\|_{L_x^2 L_t^\infty} \leq C \|h\|_{L_\xi^2 L_\tau^2}$.

Lemma 2.2 Let $\widehat{\mathcal{F}}_\rho(\xi, \tau) = \lambda(\xi) \widehat{F}_\rho(\xi, \tau)$, where $\widehat{F}_\rho(\xi, \tau) = \frac{f(\xi, \tau)}{(\tau - \xi^3)^\rho}$, $\lambda \in C_0^\infty(\mathbb{R})$, $\lambda(\xi) = 1$ if $|\xi| > 1$, $\lambda(\xi) = 0$ if $|\xi| > 2$. Then for $\rho < 1/2$, $\|F_\rho\|_{L_x^2 L_t^\infty} \leq C \|f\|_{L_\xi^2 L_\tau^2}$.

Lemma 2.3 If $|\xi| > 3/8$, $0 < \theta < 1/8$, then $\|D_x^\theta \widehat{\mathcal{F}}_\rho\|_{L_x^4 L_t^4} \leq C \|f\|_{L_\xi^2 L_\tau^2}$.

Lemma 2.4 If $\rho > 5/12$, then $\|F_\rho\|_{L_x^4 L_t^6} \leq C \|f\|_{L_\xi^2 L_\tau^2}$.

Lemma 2.5 If $\rho > \theta/12$, where $\theta \in [0, 1]$, then $\|D_x^\theta F_\rho\|_{L_x^{2/(1-\theta)} L_t^2} \leq C \|f\|_{L_\xi^2 L_\tau^2}$.

Lemma 2.6 If $\rho > 1/3$, then $\|D_x^{1/4} F_\rho\|_{L_x^4 L_t^3} \leq C \|f\|_{L_\xi^2 L_\tau^2}$.

Lemma 2.7 $\|\psi(\delta^{-1}t) W(t) u_0\|_{X^{s,t}} \leq C_0 \delta^{(1-2b)/2} \|u_0\|_{H^s}$.

Lemma 2.8 If $1/2 < b < 1$, then $\|\psi(\delta^{-1}t) h\|_{X^{s,t}} \leq C_0 \delta^{(1-2b)/2} \|h\|_{X^{s,b}}$.

Lemma 2.9 If $1/2 < b < 1$, then $\|\psi(\delta^{-1}t) \int_0^t W(t-t') \omega(t')\|_{X^{s,b}} \leq C \delta^{(1-2b)/2} \|\omega\|_{X^{s,b-1}}$.

Lemma 2.10 $\|\psi(\delta^{-1}t) \int_0^t W(t-t') \omega(t')\|_{H^s} \leq C \delta^{(1-2b)/2} \|\omega\|_{X^{s,b-1}}$.

3 Nonlinear estimates

We now describe an estimate needed to handle the nonlinear term in the KP equation. We first recall from some [3, 4] the following statement:

Lemma 3.1 (3, Lemma 3.4) Let $q(x, t) = \sigma^2 (\delta^{-1}t) (1 - \partial_{xx}^2)^{-1} \partial_x ((\partial_x u(x, t))^2)$. Suppose

$$\begin{cases} 1/2 < \varepsilon < 5/8, 1/2 < b < 7/12, b \leq \varepsilon, \varepsilon + b \leq 9/8, \\ \gamma_0/4b + 1 - 2b = \theta_0, \text{ where } \gamma_0 = 1/2 - \beta_0, \\ 3/8 < \beta_0 < 1/2. \end{cases}$$

then $\|q\|_{X^{-\varepsilon, b-1}} \leq C \delta^{\theta_0} \|u\|_{X^{-\varepsilon, b}}^2$.

Lemma 3.2 (4, Theorem 3.1) Assume

$$\begin{cases} 3/8 < s < 1/2, 1/2 < b < 7/12, 0 < b - s \leq 1/8, \\ \gamma_0/4b + 1 - 2b = \theta_0 > 0, \text{ where } \gamma_0 = 1/2 - \beta_0, \\ 3/8 < \beta_0 < 1/2. \end{cases}$$

then $\|q\|_{X^{s, b-1}} \leq C \delta^{\theta_0} \|u\|_{X^{s, b}}^2$.

Theorem 3.1 Let $P_1(x, t) = \sigma^2(\delta^{-1}t)\partial_x(u^2(x, t))$. Suppose $s \geq 0, 1/2 < b < 7/12$, then there exists $\theta_0 > 0$, such that $\|p_1\|_{X^{s,b-1}} \leq C\delta^{\theta_0}\|u\|_{X^{s,b}}^2$.

Proof. Set $v(x, t) = \sigma(\delta^{-1}t)u(x, t)$. Then we have

$$\widehat{p}_1(\xi, \tau) = \xi \int \int \widehat{v}(\xi - \xi_1, \tau - \tau_1) d\xi_1 d\tau_1 = \xi \widehat{v} * \widehat{v}(\xi, \tau).$$

Let $h(\xi, \tau) = \langle \tau - \xi \rangle^{b-1} \langle \xi \rangle^s \widehat{p}_1(\xi, \tau)$, then $\|p_1\|_{X^{s,b-1}} \leq \|h\|_{L_\xi^2 L_\tau^2}$. For $g \geq 0$ with $\|g\|_{L_\xi^2 L_\tau^2} \leq 1$

$$\|h\|_{L_\xi^2 L_\tau^2} = \sup_g \left| \int \int g(\xi, \tau) \langle \tau \xi^3 \rangle^{b-1} \langle s \rangle^s \xi \widehat{v} * \widehat{v}(\xi, \tau) d\xi d\tau \right|$$

Let $f(\xi, \tau) = \langle \tau - \xi^3 \rangle^b \langle \xi \rangle^s \widehat{v}(\xi, \tau)$.

$$\|p_1\|_{X^{s,b-1}} = \sup_g \left| \int_{R^4} \frac{g(\xi, \tau) \xi \langle \xi \rangle^s}{\langle \tau - \xi^3 \rangle^{1-b}} \frac{f(\xi_1, \tau_1)}{\langle \tau_1 - \xi_1^3 \rangle^b \langle \xi_1 \rangle^s} \frac{f(\xi - \xi_1, \tau - \tau_1)}{\langle \tau - \tau_1 - (\xi - \xi_1)^3 \rangle^b \langle \xi - \xi_1 \rangle^s} d\mu \right|$$

where $d\mu = d\xi d\tau d\xi_1 d\tau_1$. Let

$$X = \left| \int_{R^4} \frac{g(\xi, \tau) \xi \langle \xi \rangle^s}{\langle \tau - \xi^3 \rangle^{1-b}} \frac{f(\xi_1, \tau_1)}{\langle \tau_1 - \xi_1^3 \rangle^b \langle \xi_1 \rangle^s} \frac{f(\xi - \xi_1, \tau - \tau_1)}{\langle \tau - \tau_1 - (\xi - \xi_1)^3 \rangle^b \langle \xi - \xi_1 \rangle^s} d\mu \right|$$

Since $\langle \xi \rangle^{s+\varepsilon} \leq \langle \xi_1 \rangle^{s+\varepsilon} \langle \xi - \xi_1 \rangle^{s+\varepsilon}$, where ε satisfies Proposition 3.1,

$$X \leq \int \frac{g(\xi, \tau) |\xi|}{\langle \tau - \xi^3 \rangle^{1-b} \langle \xi \rangle^\varepsilon} \frac{|f(\xi_1, \tau_1)| \langle \xi_1 \rangle^\varepsilon}{\langle \tau_1 - \xi_1^3 \rangle^b} \frac{|f(\xi - \xi_1, \tau - \tau_1)| \langle \xi - \xi_1 \rangle^\varepsilon}{\langle \tau - \tau_1 - (\xi - \xi_1)^3 \rangle^b} d\mu$$

By Lemma 3.1, $X \leq C\delta^{\theta_0}\|u\|_{X^{-\varepsilon,b}}$, we have

$$\|p_1\|_{X^{s,b-1}} \leq C\delta^{\theta_0}\|u\|_{X^{-\varepsilon,b}}^2 \leq C\delta^{\theta_0}\|u\|_{X^{s,b}}^2$$

■

Theorem 3.2 Let $p_2(x, t) = \sigma^2(\delta^{-1}t)(1 - \partial_{xx}^2)^{-1}\partial_x(u^2(x, t))$. Suppose $s \geq 0, 1/2 < b < 7/12$, then there exists $\theta_0 > 0$, such that $\|p_2\|_{X^{s,b-1}} \leq C\delta^{\theta_0}\|u\|_{X^{s,b}}^2$.

Proof.

$$|\widehat{p}_2(\xi, \tau)| = \frac{|\xi|}{1 + \xi^2} |\widehat{v} * \widehat{v}(\xi, \tau)| \leq |\xi| |\widehat{v} * \widehat{v}(\xi, \tau)| = |\widehat{p}_1(\xi, \tau)|,$$

and then proceed as in the proof of Theorem 3.1. ■

Proposition 3.1 (4, Corollary 3.1) Given $s \in (1/4, 3/8]$, there exists $b \in (1/2, 1)$, such that, for $b' \in (1/2, b)$ satisfying $b - b' \leq \min\{1/4 + s/3, s - 1/6\}$, then

$$\|B(F, F)\|_{X^{s,b-1}} \leq \|F\|_{X^{s,b'}},$$

where constant C depends on ρ, b and $b - b'$.

Proposition 3.2 (4, Theorem 3.5) For $s \geq 0, b \in (1/2, 3/4), b' \in (1/2, b)$, there exists $c > 0$, such that

$$\|\partial_x(p^2)\|_{X^{s,b-1}} \leq C\|P\|_{X^{s,b}}^2$$

and

$$\|(1 - \partial_{xx}^2)^{-1}\partial_x(p^2)\|_{X^{s,b-1}} \leq C\|P\|_{X^{s,b}}^2$$

4 Results

Theorem 4.1 *Let $s \geq 0$, then there exists $b > \frac{1}{2}$ such that, for any $u_0 \in H^s(\mathbb{R})$, there are $T = T(\|u_0\|_{H^s(\mathbb{R})}) > 0$ and a unique solution $u(t)$ of (2.4) in the time interval $[0, T]$, satisfying*

$$u \in C([0, T]; H^s(\mathbb{R})), \tag{4.1}$$

$$u \in X^{s,b}, \tag{4.2}$$

$$\partial_x(u^2), (1 - \partial_{xx}^2)^{-1}(\partial_x(\partial_x u)^2) \in X^{s,b-1},$$

$$\partial_t u \in X^{s-3,b-1}.$$

Moreover, for any $T' \in (0, T)$, there exists a neighborhood \mathfrak{V} of u_0 in $H^s(\mathbb{R})$, such that the map, $\tilde{u}_0 \rightarrow \tilde{u}(t)$ from \mathcal{V} into the class defined by (4.1) and (4.2) with \tilde{T} instead of T , is Lipschitz.

Proof. Let $u_0 \in H^s(\mathbb{R})$, $s \in (3/8, 1/2)$. For $p \in X^{s,b-1}$, we define a map

$$\begin{aligned} \Phi_{u_0}(p) &= \Phi(p) = \psi(\delta^{-1}t)W(t-t')u_0 - \psi(\delta^{-1}t) \int_0^t W(t-t')p(t')dt' \\ &= \psi(\delta^{-1}t)W(t-t')u_0 - \psi(\delta^{-1}t) \int_0^t W(t-t')\sigma^2(\delta^{-1}t')p(t')dt' \end{aligned}$$

Let p be defined by (2.3). It suffices to prove $\Gamma(u) = \Phi(p)$ is a contraction in \mathcal{B} :

$$\mathcal{B} = \{u \in X^{s,b} : \|u\|_{X^{s,b}} \leq 2c_0\delta^{(1-2b)/2}\|u_0\|_{H^s}\}.$$

We have

$$\begin{aligned} \|\Gamma(u)\|_{X^{s,b}} &= \|\Phi(p)\|_{X^{s,b}} \\ &\leq \|\phi(\delta^{-1}\cdot)W(\cdot)u_0\|_{X^{s,b}} \\ &\quad + c\|\phi(\delta^{-1}t) \int_0^t W(t-t')\sigma^2(t')\partial_x(u^2(t'))dt'\|_{X^{s,b}} \\ &\quad + c\|\phi(\delta^{-1}t) \int_0^t W(t-t')\sigma^2(t')(1 - \partial_{xx}^2)^{-1}\partial_x(u^2(t'))dt'\|_{X^{s,b}} \\ &\leq c_0\delta^{(1-2b)/2}\|u_0\|_{H^s} + c\delta^{(1-2b)/2}\|p_1\|_{X^{s,b-1}} + c\delta^{(1-2b)/2}\|p_2\|_{X^{s,b-1}} \\ &\leq c_0\delta^{(1-2b)/2}\|u_0\|_{H^s} + c\delta^{(1-2b)/2}\delta^{\theta_0}\|u\|_{X^{s,b}}^2 \\ &\leq c_0\delta^{(1-2b)/2}\|u_0\|_{H^s} + 4c_0^2c\delta^{(1-2b)/2}\delta^{\theta_0}\delta^{(1-2b)}\|u\|_{H^s}^2 \\ &\leq c_0\delta^{(1-2b)/2}\|u_0\|_{H^s} + 4c_0^2c\delta^{(1-2b)/2}\delta^{r_0}\|u\|_{H^s}^2 \end{aligned}$$

follows from Lemma 2.7, Lemma 2.9, Theorem 3.1 and Theorem 3.2, where $r_0 = \theta_0 + (1 - 2b) > 0$. Choosing δ sufficiently small, such that $4c_0c\delta^{r_0}\|u\|_{H^s} < \frac{1}{2}$, then

$$\|\Gamma(u)\|_{X^{s,b}} \leq \frac{3}{2}\delta^{(1-2b)/2}\|u_0\|_{H^s}$$

so $\Gamma(\mathcal{B}) \subset \mathcal{B}$. By same arguments as in Section 3 we have

$$\begin{aligned} &\|\Gamma(u) - \Gamma(\tilde{u})\|_{X^{s,b}} \\ &\leq c\delta^{(1-2b)/2}(\|\sigma^2(\delta^{-1}t)\partial_x(u^2)\|_{X^{s,b-1}} \\ &+ \|\sigma^2(\delta^{-1}t)(1 - \partial_{xx}^2)^{-1}\partial_x(u^2)\|_{X^{s,b-1}}) \leq c\delta^{(1-2b)/2}\delta^{\theta_0}\|u + \tilde{u}\|_{X^{s,b}}\|u - \tilde{u}\|_{X^{s,b}} \\ &\leq c\delta^{(1-2b)/2}\delta^{\theta_0}4c_0\delta^{(1-2b)}\|u_0\|_{H^s}\|u - \tilde{u}\|_{X^{s,b}} \\ &= 4cc_0\delta^{r_0}\|u_0\|_{H^s}\|u - \tilde{u}\|_{X^{s,b}} \\ &\leq \frac{1}{2}\|u - \tilde{u}\|_{X^{s,b}} \end{aligned}$$

Γ is a contraction on \mathcal{B} , so there exists $u \in \mathcal{B}$ such that

$$u(t) = \psi(\delta^{-1}t)W(t)u_0(x) - \psi(\delta^{-1}t) \int_0^t W(t-t')\sigma^2(\delta^{-1}t')p(t')dt'$$

By the definition of ψ and σ , for $t \in [-\delta/2, \delta/2]$, i.e. $T = \frac{\delta}{2}$, we have ,

$$u(t) = W(t)u_0 - \int_0^t W(t-t')p(t')dt'$$

where δ depends only on $\|u_0\|_{H^s}$. Let u and \tilde{u} be solutions corresponding to the initial u_0 and \tilde{u}_0 , respectively. By similar arguments, we have

$$\|u - \tilde{u}\|_{X^{s,b}} \leq c\|u_0 + \tilde{u}_0\|_{X^{s,b}} + \frac{1}{2}\|u - \tilde{u}\|_{X^{s,b}}$$

so

$$\|u - \tilde{u}\|_{X^{s,b}} \leq c\|u_0 + \tilde{u}_0\|_{X^{s,b}}$$

Now we prove $u \in C([-T, T]; H^s(\mathbb{R}))$. For $0 \leq \tilde{t} < t < T$, $t - \tilde{t} < \Delta t$, by

$$u(x, t) = \psi(\delta^{-1}t)W(t - \tilde{t})u(x, \tilde{t}) - \psi(\delta^{-1}t) \int_{\tilde{t}}^t W(t-t')\sigma^2(\delta^{-1}t')p(t')dt'$$

and the definition of σ , we have

$$\begin{aligned} & \|\Gamma(t) - \Gamma(\tilde{t})\|_{H^s} \\ & \leq \|W(t - \tilde{t})u(\tilde{t})\|_{H^s} + \|\psi(\delta^{-1}t) \int_{\tilde{t}}^t W(t-t')\sigma^2(\delta^{-1}t')p(t')dt'\|_{H^s} \\ & \leq \|W(t - \tilde{t})u(\tilde{t})\|_{H^s} + \|\psi(\delta^{-1}t) \int_{\tilde{t}}^t W(t-t')\sigma^2(\frac{t' - \tilde{t}}{2\Delta t})p(t')dt'\|_{H^s} \\ & \leq \|W(t - \tilde{t})u(\tilde{t})\|_{H^s} + c\|\sigma^2(\frac{\cdot - \tilde{t}}{2\Delta t})p\|_{X^{s,b-1}} \\ & \leq \|W(t - \tilde{t})u(\tilde{t})\|_{H^s} + c(\Delta t)^{\theta_0}\|u\|_{X^{s,b}}^2 \end{aligned}$$

so when $\Delta t \rightarrow 0$, we have $\|u(t) - u(\tilde{t})\| \rightarrow 0$, which completes the proof of the theorem. ■

Theorem 4.2 Let $s \in (1/4, 3/8]$, then there exists $b > 1/2$, for any $u_0 \in H^s(\mathbb{R})$, whose norm is sufficiently small, and then the Cauchy problem (2.2) has a unique solution $u(t)$ in $(0, 1)$ satisfying

$$\begin{aligned} u & \in BC((0, 1); H^s(\mathbb{R})), \\ u & \in X^{s,b}, \\ \partial_x(u^2), (1 - \partial_{xx}^2)^{-1}(\partial_x(\partial_x u)^2) & \in X^{s,b-1}, \\ \partial_t u & \in X^{s-3,b-1}. \end{aligned}$$

Proof. For $u_0 \in H^s(\mathbb{R})$, $s \in (1/4, 1)$, and $\|u_0\|_{H^s} = r$ sufficiently small, define a map

$$\Phi_{u_0}(p) = \Phi(p) = \psi(t)W(t-t')u_0 - \psi(t) \int_0^t W(t-t')\sigma^2(t')p(t')dt'$$

where $p = p(t)$ is given by (2.3). Let $\Gamma(u) = \Phi(p)$. Now we prove that Γ is a contraction on $\mathcal{B}(2c_0r) = \{u \in X^{s,b} : \|u\|_{X^{s,b}} \leq 2c_0r\}$. By Lemma 2.7, Proposition 3.1-3.2, we have

$$\begin{aligned} \|\Gamma(u)\|_{X^{s,b}} & \leq \|\phi(t)W(t)u_0\|_{X^{s,b}} \\ & + \|\phi(t) \int_0^t W(t-t')\sigma^2(t')(\frac{1}{2}\partial_x(u^2(t')) + (1 - \partial_{xx}^2)^{-1}\partial_x(\frac{3}{2}u^2(t'))dt'\|_{X^{s,b}} \\ & \leq c_0\|u_0\|_{H^s} + c\|\phi(t)u\|_{X^{s,b}} \\ & \leq c_0r + c(2c_0r)^2. \end{aligned}$$

Choosing r satisfying $4c_0cr < 1$, then $\|\Gamma(u)\|_{X^{s,b}} \leq 2c_0r$. By the same argument, letting $u, \tilde{u} \in \mathcal{B}(2c_0r)$, we have

$$\begin{aligned} & \|\Gamma(u) - \Gamma(\tilde{u})\|_{X^{s,b}} \\ & \leq \frac{1}{2} \|\phi(t) \int_0^t W(t-t') \sigma^2(t') \partial_x(u^2(t') - \tilde{u}^2(t')) dt'\|_{X^{s,b}} \\ & + \frac{3}{2} \|\phi(t) \int_0^t W(t-t') \sigma^2(t') (1 - \partial_{xx}^2)^{-1} \partial_x(u^2(t') - \tilde{u}^2(t')) dt'\|_{X^{s,b}} \\ & \leq c \|u + \tilde{u}\|_{X^{s,b}} \|u - \tilde{u}\|_{X^{s,b}} \\ & \leq 4c_0cr \|u - \tilde{u}\|_{X^{s,b}}. \end{aligned}$$

Γ is a contraction on $\mathcal{B}(2c_0c)$, so there exists a unique $u \in \mathcal{B}(2c_0c)$ such that

$$u(t) = \psi(t)W(t)u_0 - \psi(t) \int_0^t W(t-t') \sigma^2(t') p(t') dt'$$

By the definition of ψ, σ , we have, on $[-1/2, 1/2]$,

$$u(t) = W(t)u_0 - \int_0^t W(t-t') \sigma^2(t') p(t') dt'$$

where $p = p(u)$ is given by (2.3). ■

References

- [1] Yue Liu, Zhaoyang Yin: Global existence and blow-up phenomena for the degasperis-procesi equation. *Communications in Mathematical Physics*.267,801-820(2006)
- [2] Octavian G Mustafa: A note on the Degasperis-Procesi equation. *Journal of Nonlinear Mathematical Physics*.12(1),10-14(2005)
- [3] Giuseppe M. Coclite, Kenneth H. Karlsen. On the well-posedness of the Degasperis-Procesi equation. *Journal of Functional Analysis*.233,60-91(2006)
- [4] A. Degasperis, D. D. Holm, and A. N. W. Home: Integrable and non-integrable equations with peakons. *In Nonlinear Physics: Theory and Experiment*, (Gallipoli,2002),pages 37-43. World Sci. Publishing , River Edge, NJ, 2003.
- [5] Bourgain.J.: Fourier restriction phenomena for certain lattice subsets and applications to nonlinear evolution equations I,Schrödinger equations. *GAF*.3,107-156(1993)
- [6] Bourgain.J.: Fourier restriction phenomena for certain lattice subsets and applications to nonlinear evolution equations II,The Periodic KdV Equation. *GAF*.3,209-262(1993)
- [7] Keing, C. E., Ponce, G., Vega. L.: The Cauchy problem for the Korteweg-de Vries equation in Sobolev spaces of negative indices. *Duke Math. J*.71,1-21(1993)
- [8] Keing, C. E., Ponce, G., Vega. L.: A bilinear estimate with applications to the KdV equation. *J. Amer.Math*.9(2),573-603(1996)
- [9] Danping Ding, Lixin Tian: The Study of Solution of Dissipative Camassa-Holm Equation on Total Space. *International Journal of Nonlinear Science*. 1(1),37-42(2006)
- [10] Lixin Tian, Shulai Zhang, Guangjuan Yang: Some Results of a Class of Systems and Their Application in Chaotic Synchronization. *International Journal of Nonlinear Science*.2(3), 159-165(2006)

- [11] Xinghua Fan, Lixin Tian, Lihong Ren: New Compactons in Nonlinear Atomic Chain Equations with first-and second-neighbour. Interactions. *International Journal of Nonlinear Science*. 1(2),105-110(2006)
- [12] Haixia Cao, Dianchen Lu, Lixin Tian, Yueling Cheng: Boundary Control of the Kuramoto-Sivashinsky Equation with an Exteral Excitation. *International Journal of Nonlinear Science*. 1(2),67-81(2006)